Unit 5 Vectors Pretest

① a) $[-3,-6] = -3\hat{i} - 6\hat{j}$

b) $[0,-8] = -8\hat{j}$

c) $[-6,0] = -6\hat{i}$

② a) $-4\hat{i} = [-4,0]$

b) $7\hat{i} - 4\hat{j} = [7,-4]$

c) $2\hat{j} = [0,2]$

③ $\vec{AB} = [4-2, 1-6] = [2,-5]$; $|\vec{AB}| = \sqrt{(2)^2+(-5)^2} = \sqrt{29}$ units

$\vec{CD} = [0-(-5), 3-2] = [5,1]$; $|\vec{CD}| = \sqrt{(5)^2+(1)^2} = \sqrt{26}$ units

$\vec{EF} = [6-6, 2-(-5)] = [0,7]$; $|\vec{EF}| = \sqrt{(0)^2+(7)^2} = 7$ units

$\vec{GH} = [-4-(-2), 6-(-3)] = [-2,9]$; $|\vec{GH}| = \sqrt{(-2)^2+(9)^2} = \sqrt{85}$ units

④ $x-(-2) = 5$
$x = 3$

$y-(-7) = -1$
$y = -8$

$\boxed{\therefore Q(3,-8)}$

⑤ a) $\vec{QP} = [-6-(-2), 1-(-1)]$
$= [-4, 2]$

b) $\vec{RP} = [-6-(-3), 1-4]$
$\vec{RP} = [-3,-3]$

$|\vec{RP}| = \sqrt{(-3)^2+(-3)^2}$
$|\vec{RP}| = \sqrt{18}$
$|\vec{RP}| = 3\sqrt{2}$ units

c) $|\vec{QP}| = \sqrt{(-4)^2+(2)^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

$\vec{RQ} = [-2-(-3), -1-4]$
$= [1,-5]$
$|\vec{RQ}| = \sqrt{(1)^2+(-5)^2}$
$= \sqrt{26}$

Perimeter $= 2\sqrt{5} + \sqrt{26} + 3\sqrt{2}$
$\simeq 13.8$ units

⑥ $\vec{u} = [4,-1]$ $\vec{v} = [2,7]$

a) $8\vec{u} = 8[4,-1]$
$= [32,-8]$

b) $-8\vec{u} = -8[4,-1]$
$= [-32,8]$

c) $\vec{u}+\vec{v} = [4+2, -1+7]$
$= [6,6]$

d) $\vec{v} - \vec{u} = [2-4, 7-(-1)]$
$= [-2,8]$

e) $5\vec{u} - 3\vec{v} = 5[4,-1] - 3[2,7]$
$= [20,-5] - [6,21]$
$= [20-6, -5-21]$
$= [14,-26]$

f) $-4\vec{u} + 7\vec{v} = -4[4,-1] + 7[2,7]$
$= [-16,4] + [14,49]$
$= [-16+14, 4+49]$
$= [-2,53]$

⑦ Does $\vec{a} = k\vec{b}$?

$6 = k(-9)$ $-4 = k(6)$
$k = -\frac{2}{3}$ $k = -\frac{2}{3}$

$\vec{a} = -\frac{2}{3}\vec{b}$

$\therefore$ $\vec{a}$ and $\vec{b}$ are collinear

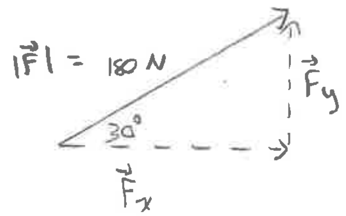
Does $\vec{a} = k\vec{c}$

$6 = k(-6)$ $-4 = k(-4)$
$k = -1$ $k = 1$

$\therefore \vec{a} \neq k\vec{c}$

$\therefore$ $\vec{a}$ and $\vec{c}$ are not collinear

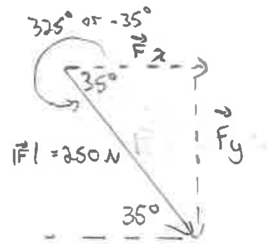
(8)



$$\vec{F} = [180\cos(30^\circ),\ 180\sin(30^\circ)]$$

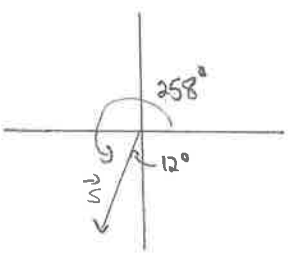
$$= [90\sqrt{3},\ 90]$$

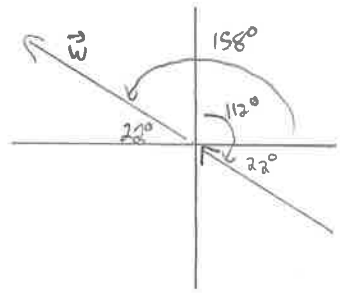
(9)



$$\vec{F} = [250\cos(325^\circ),\ 250\sin(325)]$$

(10)





$|\vec{s}| = 30$

$\vec{s} = [30\cos(258),\ 30\sin(258)]$

$|\vec{w}| = 14$

$\vec{w} = [14\cos(158),\ 14\sin(158)]$

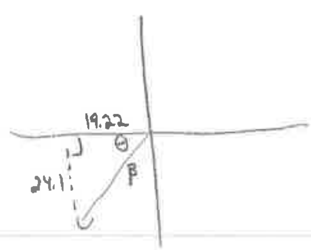
$$\vec{r} = \vec{s} + \vec{w}$$

$$\vec{r} = [30\cos(258) + 14\cos(158),\ 30\sin(258) + 14\sin(158)]$$

$$\vec{r} \simeq [-19.22,\ -24.1]$$

$$|\vec{r}| \simeq 30.8\ Km/h$$

$$\tan\theta = \frac{-24.1}{-19.22}$$



$\theta \simeq 51.4^\circ$

$\beta \simeq 38.6^\circ$

30.8 Km/h on a true bearing of 218.6°

11 a) $\vec{a} \cdot \vec{b} = 70(115)\cos(70°)$

≈ 2753.26

b) $\vec{c} \cdot \vec{d} = 8(12)\cos(150)$

≈ -83.14

c) $\vec{e} \cdot \vec{f} = 0$

d) $\vec{u} \cdot \vec{v} = [2,4] \cdot [3,-1]$

$= 2(3) + 4(-1)$

$= 2$

e) $\vec{s} \cdot \vec{t} = [9,-3] \cdot [3,-3]$

$= 9(3) + (-3)(-3)$

$= 36$

f) $\vec{a} \cdot \vec{b} = [2,3] \cdot [9,-7]$

$= 2(9) + 3(-7)$

$= -3$

12 $\vec{u} = [3,-5]$ $\vec{v} = [-6,1]$ $\vec{w} = [4,7]$

a) $\vec{u} \cdot (\vec{v} + \vec{w}) = [3,-5] \cdot [-6+4, 1+7]$

$= [3,-5] \cdot [-2,8]$

$= 3(-2) + (-5)(8)$

$= -46$

b) $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$

$= [3+(-6), -5+1] \cdot [3-(-6), -5-1]$

$= [-3,-4] \cdot [9,-6]$

$= -3(9) + (-4)(-6)$

$= -3$

c) $\vec{u} + \vec{v} \cdot \vec{w}$

vector + scalar is not possible to do.

d) $-3\vec{v} \cdot (2\vec{w})$

$= -3[-6,1] \cdot 2[4,7]$

$= [18,-3] \cdot [8,14]$

$= 18(8) + (-3)(14)$

$= 102$

e) $(\vec{u}+2\vec{v})\cdot(3\vec{w}-\vec{u})$

$= ([3,-5]+2[-6,1])\cdot(3[4,7]-[3,-5])$

$= [-9,-3]\cdot[9,26]$

$= -9(9)+(-3)(26)$

$= -159$

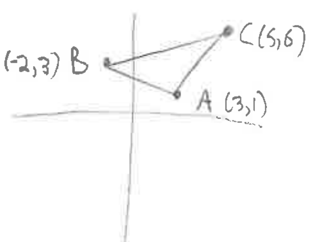
f) $\vec{v}\cdot\vec{v}+\vec{w}\cdot\vec{w}$

$= [-6,1]\cdot[-6,1]+[4,7]\cdot[4,7]$

$= -6(-6)+1(1)+4(4)+7(7)$

$= 102$

13)



$\vec{AB} = [-2-3, 3-1] = [-5,2]$

$\vec{AC} = [5-3, 6-1] = [2,5]$

$\vec{AB}\cdot\vec{AC} = [-5,2]\cdot[2,5]$

$= -5(2)+2(5)$

$= 0$

$\therefore$ $\vec{AB}$ and $\vec{AC}$ are perpendicular.

$\therefore$ ΔABC is a RIGHT triangle.

$\angle A$ is the right angle.

(14) $\vec{u} = [9,2]$ Let $\vec{v}$ be a perpendicular vector:

$\vec{v} = [-2,9]$

Verify they are perpendicular by checking if $\vec{u} \cdot \vec{v} = 0$

$$\vec{u} \cdot \vec{v} = [9,2] \cdot [-2,9]$$
$$= 9(-2) + 2(9)$$
$$= 0$$

(15)
$$\vec{u} \cdot \vec{v} = 0$$
$$[2,5] \cdot [k,4] = 0$$
$$2k + 5(4) = 0$$
$$k = -10$$

(16)
$$\vec{u} \cdot \vec{v} = 0$$
$$[k,3] \cdot [k,2k] = 0$$
$$k(k) + 3(2k) = 0$$
$$k^2 + 6k = 0$$
$$k(k+6) = 0$$
$$k_1 = 0 \quad k_2 = -6$$

(17) a) $W = \vec{F} \cdot \vec{s}$
$$= [5,2] \cdot [7,4]$$
$$= 5(7) + 2(4)$$
$$= 43 \text{ J}$$

b) $W = [100,400] \cdot [12,27]$
$$= 100(12) + 400(27)$$
$$= 12\,000 \text{ J}$$

c) $W = 241(45.2)\cos(80)$
$$\simeq 1891.6 \text{ J}$$

(18) a) $\cos\theta = \dfrac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}$

$\cos\theta = \dfrac{7(4)+8(3)}{\left(\sqrt{7^2+8^2}\right)\left(\sqrt{4^2+3^2}\right)}$

$\cos\theta = \dfrac{52}{\left(\sqrt{113}\right)(5)}$

$\theta \simeq 11.94°$

b) $\cos\theta = \dfrac{\vec{t} \cdot \vec{u}}{|\vec{t}||\vec{u}|}$

$\cos\theta = \dfrac{-7(6)+2(11)}{\left(\sqrt{(-7)^2+(2)^2}\right)\left(\sqrt{6^2+11^2}\right)}$

$\cos\theta = \dfrac{-20}{\left(\sqrt{53}\right)\left(\sqrt{157}\right)}$

$\theta \simeq 102.67°$

(19) a) $proj_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\ (\hat{v})$

$= [56\cos(125)](\hat{v})$

$\simeq -32.12\hat{v}$

b) $proj_{\vec{v}}\vec{u} = \dfrac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}(\vec{v})$

$= \dfrac{7(9)+1(-3)}{9^2+(-3)^2}[9,-3]$

$= \dfrac{60}{90}[9,-3]$

$= \dfrac{2}{3}[9,-3]$

$= [6,-2]$

(20)

$$|\text{proj}_{\vec{b}}\vec{a}| = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \frac{6(11)+(-1)(5)}{\sqrt{11^2+5^2}}$$

$$= \frac{61}{\sqrt{146}}$$

$$\text{proj}_{\vec{b}}\vec{a} = |\text{proj}_{\vec{b}}\vec{a}|\,(\hat{b})$$

$$= \frac{61}{\sqrt{146}}\left(\frac{1}{\sqrt{146}}\right)[11, 5]$$

$$= \frac{61}{146}[11, 5]$$

$$= \left[\frac{671}{146}, \frac{305}{146}\right]$$

(21)

$$\vec{W} = 15(500)\cos(12)$$

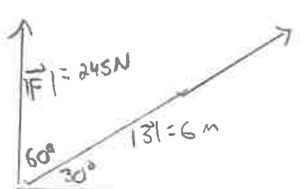
$$\approx 7336.11\text{ J}$$

(22)

$$W = 3(30)\cos(25°) + 4(30)\cos(5°) + 5(30)\cos(25°)$$

$$W \approx 337.06\text{ J}$$

(23)

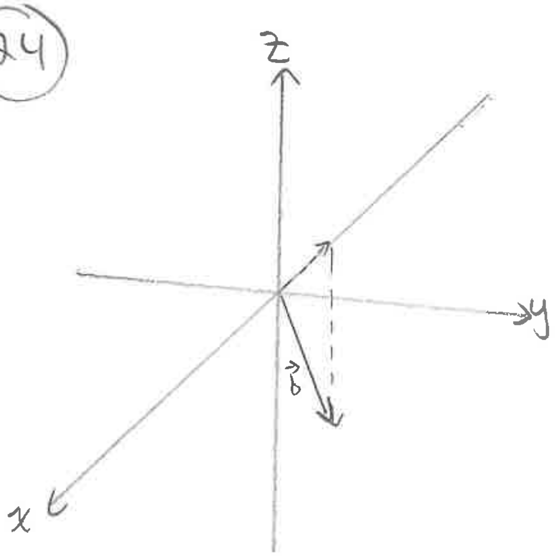


$$W = \vec{F}\cdot\vec{s}$$

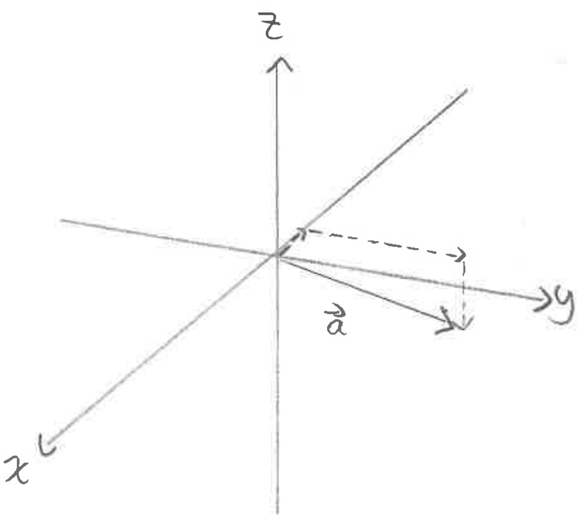
$$= 245(6)\cos(60°)$$

$$= 735\text{ J}$$

(24)



$$|\vec{b}| = \sqrt{(-2)^2 + (-4)^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$



$$|\vec{a}| = \sqrt{(-1)^2 + (5)^2 + (-2)^2}$$
$$= \sqrt{30}$$

(25) check if $\vec{u} = k\vec{v}$

x: $6 = k(-12)$, $k = \frac{-1}{2}$

y: $-2 = k(4)$, $k = \frac{-1}{2}$

z: $-5 = k(10)$, $k = \frac{-1}{2}$

$\vec{u} = \frac{-1}{2}\vec{v}$; so $\vec{u}$ and $\vec{v}$ are collinear.

(26) Let $\vec{u} = k\vec{v}$; $[a, 3, 6] = k[-8, 12, b]$

x:

$a = k(-8)$

$a = \frac{1}{4}(-8)$

$a = -2$

y:

$3 = k(12)$

$k = \frac{1}{4}$

z:

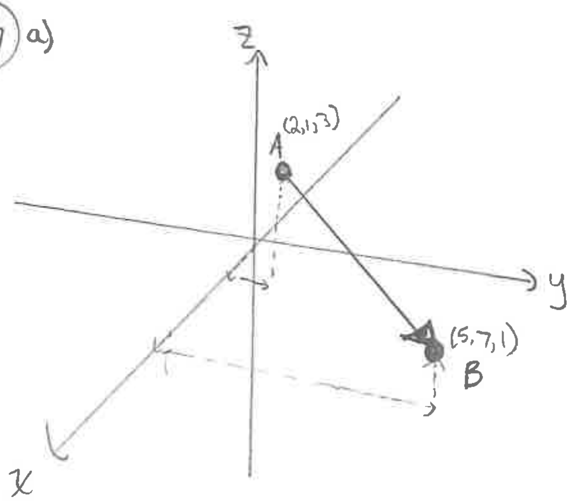
$6 = k(b)$

$6 = \frac{1}{4}(b)$

$24 = b$

$\vec{u} = \frac{1}{4}\vec{v}$. $\therefore$ $[-2, 3, 6]$ is collinear with $[-8, 12, 24]$

(27) a)



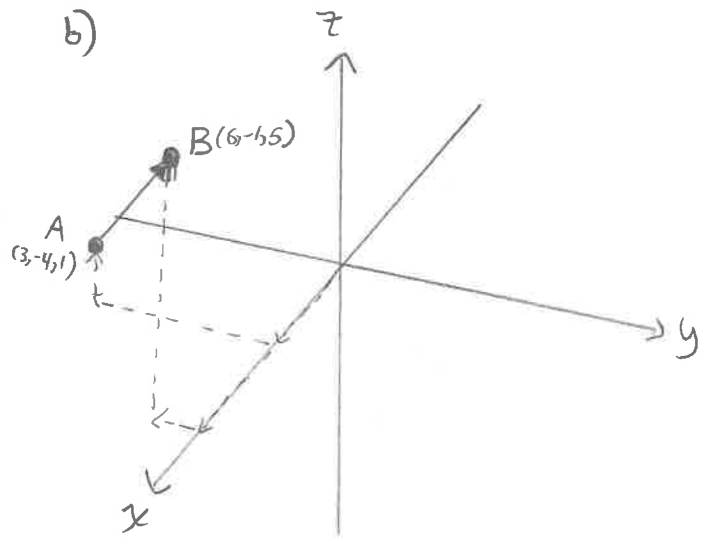
$\vec{AB} = [5-2, 7-1, 1-3]$

$\vec{AB} = [3, 6, -2]$

$|\vec{AB}| = \sqrt{(3)^2 + (6)^2 + (-2)^2}$

$|\vec{AB}| = 7$

b)



$\vec{AB} = [6-3, -1-(-4), 5-1]$

$\vec{AB} = [3, 3, 4]$

$|\vec{AB}| = \sqrt{(3)^2 + (3)^2 + (4)^2}$

$|\vec{AB}| = \sqrt{34}$

(28) $\vec{a} = [-4,1,7]$, $\vec{b} = [2,0,-3]$, $\vec{c} = [1,-1,5]$

a) $7\vec{a} = 7[-4,1,7]$
$= [-28,7,49]$

b) $3\vec{a} - 2\vec{b} + 4\vec{c} = 3[-4,1,7] - 2[2,0,-3] + 4[1,-1,5]$
$= [-12,3,21] - [4,0,-6] + [4,-4,20]$
$= [-12,-1,47]$

c) $\vec{a} \cdot \vec{c} = [-4,1,7] \cdot [1,-1,5]$
$= -4(1) + 1(-1) + 7(5)$
$= 30$

d) $(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b})$
$= ([-4,1,7]+[2,0,-3]) \cdot ([-4,1,7]-[2,0,-3])$
$= [-2,1,4] \cdot [-6,1,10]$
$= -2(-6) + 1(1) + 4(10)$
$= 53$

(29) $\cos\theta = \dfrac{\vec{g} \cdot \vec{h}}{|\vec{g}||\vec{h}|} = \dfrac{6(-5) + 1(3) + 2(6)}{(\sqrt{6^2+1^2+2^2})(\sqrt{(-5)^2+(3)^2+(6)^2})}$

$\cos\theta = \dfrac{-15}{(\sqrt{41})(\sqrt{70})}$

$\theta \approx 106.26°$

(30) Let $\vec{f}$ be the Orthogonal vector.

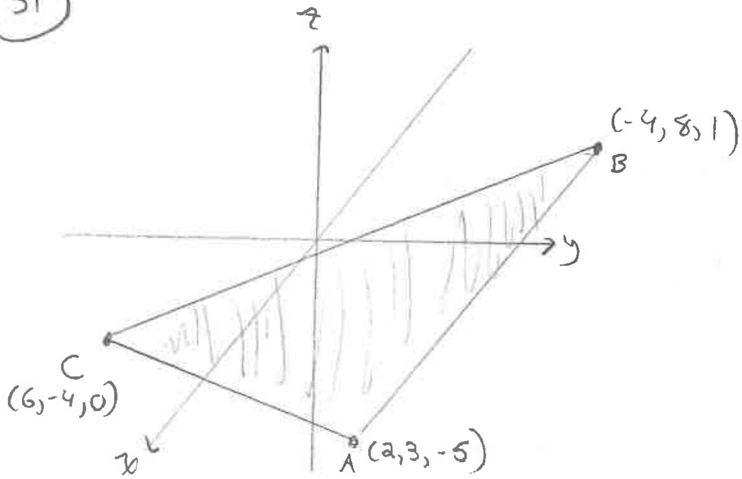
$\vec{f} = [0,4,1]$

verify $\vec{e} \cdot \vec{f} = 0$

$[3,-1,4] \cdot [0,4,1] = 0$
$3(0) + (-1)(4) + (4)(1) = 0$
$0 = 0$

$\therefore$ $\vec{e}$ and $\vec{f}$ are orthogonal.

(31)



$\vec{AB} = [-6, 5, 6]$

$|\vec{AB}| = \sqrt{97}$

$\vec{AC} = [4, -7, 5]$

$|\vec{AC}| = \sqrt{90}$

$\vec{BC} = [10, -12, -1]$

$|\vec{BC}| = \sqrt{245}$

$\therefore$ scalene

check if $a^2 + b^2 = c^2$

$$(\sqrt{97})^2 + (\sqrt{90})^2 = (\sqrt{245})^2$$

$$187 \neq 245$$

$\therefore$ not a right angle triangle.

(32)

$$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}| \sin\theta \, (\hat{n})$$
$$= (60)(80) \sin(55) \, (\hat{n})$$
$$\simeq 3931.93 \, \hat{n}$$

OR 3931.93 out of the page.

33 a) $\vec{a} \times \vec{b} = [-2(6) - 9(1),\ 9(1) - 3(6),\ 3(1) - (-2)(1)]$

$$\begin{array}{cc} -2 & 1 \\ 9 & 6 \\ 3 & 1 \\ -2 & 1 \end{array} \qquad = [-21, -9, 5]$$

b) $\vec{a} \times \vec{b} = [10(5) - 3(0),\ 3(2) - (-8)(5),\ -8(0) - 10(2)]$

$$\begin{array}{cc} 10 & 0 \\ 3 & 5 \\ -8 & 2 \\ 10 & 0 \end{array} \qquad = [50, 46, -20]$$

34 a) $\vec{p} \times \vec{q} = [3(5) - 8(3),\ 8(3) - 6(5),\ 6(3) - 3(3)]$

$$\begin{array}{cc} 3 & 3 \\ 8 & 5 \\ 6 & 3 \\ 3 & 3 \end{array} \qquad = [-9, -6, 9]$$

$$\text{Area} = |\vec{p} \times \vec{q}|$$
$$= \sqrt{(-9)^2 + (-6)^2 + (9)^2}$$
$$= \sqrt{198} \text{ units}^2$$
$$= 3\sqrt{22} \text{ units}^2$$

b) $\text{Area} = |\vec{u}||\vec{v}| \sin\theta$
$$= 43(27)\sin(32)$$
$$\doteq 615.2 \text{ units}^2$$

35 a) $\vec{a} \times (\vec{b} + \vec{c})$

$= [2, -6, 3] \times [-1+(-4), 5+5, 8+6]$

$= [2, -6, 3] \times [-5, 10, 14]$

$= [-6(14) - 3(10), 3(-5) - 2(14), 2(10) - (-6)(-5)]$

$= [-114, -43, -10]$

$$\begin{array}{cc} \vec{a} & \times \ (\vec{b}+\vec{c}) \\ -6 & 10 \\ 3 & 14 \\ 2 & -5 \\ -6 & 10 \end{array}$$

b) $\vec{a} \times \vec{b} - \vec{a} \times \vec{c}$

$= [-6(8) - 3(5), 3(-1) - 2(8), 2(5) - (-6)(-1)]$

$- [-6(6) - 3(5), 3(-4) - 2(6), 2(5) - (-6)(-4)]$

$= [-63, -19, 4] - [-51, -24, -14]$

$= [-12, 5, 18]$

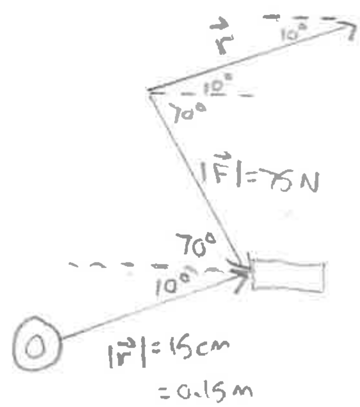
$$\begin{array}{cc} \vec{a} & \times \ \vec{b} \\ -6 & 5 \\ 3 & 8 \\ 2 & -1 \\ -6 & 5 \end{array} \qquad \begin{array}{cc} \vec{a} & \times \ \vec{c} \\ -6 & 5 \\ 3 & 6 \\ 2 & -4 \\ -6 & 5 \end{array}$$

36 $\tau = \vec{r} \times \vec{F}$

$|\tau| = |\vec{r}||\vec{F}| \sin \theta$

$= (0.15)(75) \sin(80°)$

$\simeq 11.08 \text{ J}$



(37) work against gravity:

$$W = [3,5,12] \cdot [0,0,6]$$

$$W = 3(0) + 5(0) + 12(6)$$

$$W = 72\ J$$

work in direction of travel:

$$W = [3,5,12] \cdot [2,1,6]$$

$$= 3(2) + 5(1) + 12(6)$$

$$= 83\ J$$

(38)

$$|\text{proj}_{\vec{v}} \vec{u}| = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

$$= \frac{3(6) + 1(2) + 4(7)}{\sqrt{(6)^2 + (2)^2 + (7)^2}}$$

$$= \frac{48}{\sqrt{89}}$$

$$\text{proj}_{\vec{v}} \vec{u} = |\text{proj}_{\vec{v}} \vec{u}|(\hat{v})$$

$$= \frac{48}{\sqrt{89}} \left(\frac{[6,2,7]}{\sqrt{89}} \right)$$

$$= \frac{48}{89} [6,2,7]$$

$$= \left[\frac{288}{89}, \frac{96}{89}, \frac{336}{89} \right]$$

(39)

$$\vec{a} \cdot \vec{b} \times \vec{c}$$

$$= [-2,3,5] \cdot [0(3) - (-1)(-2), -1(2) - 4(3), 4(-2) - 0(2)]$$

$$= [-2,3,5] \cdot [-2, -14, -8]$$

$$= -2(-2) + 3(-14) + 5(-8)$$

$$= -78$$

$\vec{b} \times \vec{c}$

$$\begin{array}{cc} 0 & -2 \\ -1 & 3 \\ 4 & 2 \\ 0 & -2 \end{array}$$

(40) $V = |\vec{u} \times \vec{v} \cdot \vec{w}|$

$= |[4(6)-3(5), 3(2)-1(6), 1(5)-4(2)] \cdot [1,2,7]|$

$= |[9, 0, -3] \cdot [1,2,7]|$

$= |9(1)+0(2)+(-3)(7)|$

$= |-12|$

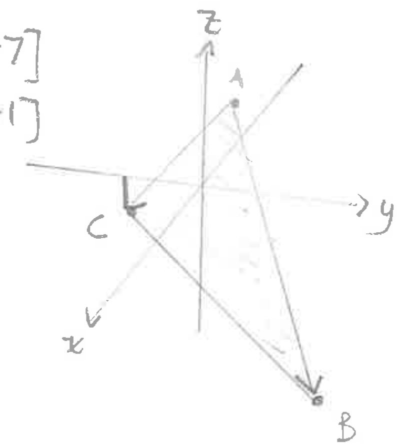
$= 12 \text{ units}^3$

$\vec{u}$	$\times$	$\vec{v}$
4		5
3		6
1		2
4		5

(41) $A = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$\vec{AB} = [9, 7, -7]$

$\vec{AC} = [7, -1, -1]$



$\vec{AB} \times \vec{AC} = [7(-1)-(-7)(-1), -7(7)-9(-1), 9(-1)-7(7)]$

$\vec{AB} \times \vec{AC} = [-14, -40, -58]$

$A = \frac{1}{2}\left(\sqrt{(-14)^2+(-40)^2+(-58)^2}\right.$

$A = \frac{1}{2}\sqrt{5160}$

$A = \frac{1}{2}(2)\sqrt{1290}$

$A = \sqrt{1290} \text{ units}^2$

OR $\approx 35.92 \text{ units}^2$

$\vec{AB}$	$\times$	$\vec{AC}$
7		-1
-7		-1
9		7
7		-1