Unit 5 Pre-Test Review – Cartesian Vectors Unit 5

MCV4U

Jensen

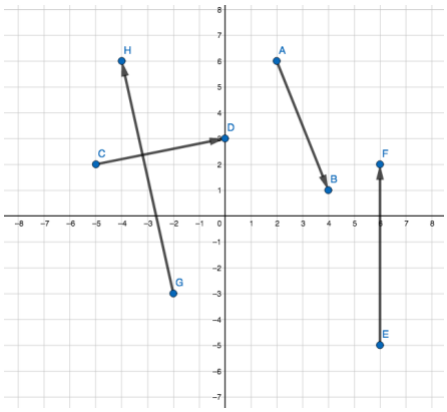
1) Express each vector in terms of vectors $\hat{\imath}$ and $\hat{\jmath}$

a) $[-3,-6]$ **b)** $[0,-8]$ **c)** $[-6,0]$

2) Express each vector in the form $[a,b]$

a) $-4\hat{\imath}$ **b)** $7\hat{\imath}-4\hat{\jmath}$ **c)** $2\hat{\jmath}$

3) Write the coordinates of each Cartesian Vector in the form $[a,b]$ and determine its magnitude.



4) You are given the vector $\vec{v}=[5,-1]$. An equivalent vector $\overrightarrow{PQ}$ has its initial point at $P(-2,-7)$. Determine the coordinates of Q.

5) Given the points $P(-6,1)$, $Q(-2,-1)$, and $R(-3,4)$, find

a) $\overrightarrow{QP}$
b) $\left|\overrightarrow{RP}\right|$
c) the perimeter of ΔPQR

6) If $\vec{u}=[4,-1]$ and $\vec{v}=[2,7]$, find

a) $8\vec{u}$
b) $-8\vec{u}$
c) $\vec{u}+\vec{v}$
d) $\vec{v}-\vec{u}$
e) $5\vec{u}-3\vec{v}$
f) $-4\vec{u}+7\vec{v}$

7) Which vector is collinear with $\vec{a}=[6,-4]$? Give proof.

$\vec{b}=[-9,6]$, $\vec{c}=[-6,-4]$

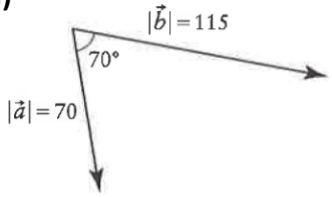
8) A person pulls a sleigh, exerting a force of 180 N along a rope that makes an angle of $30°$ to the horizontal. Write this force in component form as a Cartesian vector.

9) A person pushes a lawnmower with a force of 250 N. The handle makes an angle of $35°$ with the ground. Write this force in component form as a Cartesian vector.

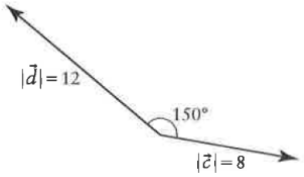
10) A ship's course is set at a heading of $192°$, with a speed of 30 knots. A current is flowing from a bearing of $112°$, at 14 knots. Use Cartesian vectors to determine the resultant velocity of the ship.

11) Calculate the dot product of each pair of vectors.

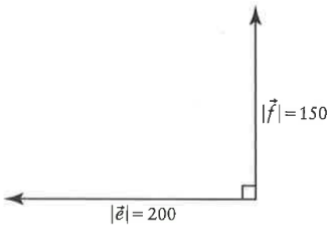
a)



b)



c)



d) $\vec{u} = [2,4], \vec{v} = [3,-1]$

e) $\vec{s} = [9,-3], \vec{t} = [3,-3]$

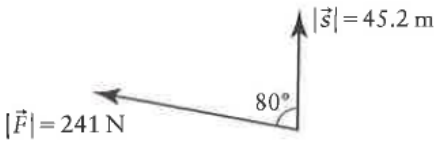
f) $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}, \vec{b} = 9\hat{\imath} - 7\hat{\jmath}$

12) Let $\vec{u} = [3,-5]$, $\vec{v} = [-6,1]$, and $\vec{w} = [4,7]$. Evaluate each of the following if possible. If it is not possible, explain why not.

a) $\vec{u} \cdot (\vec{v} + \vec{w})$
b) $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$
c) $\vec{u} + \vec{v} \cdot \vec{w}$
d) $-3\vec{v} \cdot (2\vec{w})$
e) $(\vec{u} + 2\vec{v}) \cdot (3\vec{w} - \vec{u})$
f) $\vec{v} \cdot \vec{v} + \vec{w} \cdot \vec{w}$

13) ΔABC has points $A(3,1)$, $B(-2,3)$, and $C(5,6)$. Is it a right-angled triangle? If it is, identify the right angle.
14) Find a vector that is perpendicular to $\vec{u} = [9,2]$. Verify that the vectors are perpendicular.
15) Determine the value of k so that $\vec{u} = [2,5]$ and $\vec{v} = [k,4]$ are perpendicular.
16) Determine the value of k so that $\vec{u} = [k,3]$ and $\vec{v} = [k,2k]$ are perpendicular.
17) Determine the work done by force $\vec{F}$, in Newtons, for an object moving along the vector $\vec{s}$, in meters.

a) $\vec{F} = [5,2], \vec{s} = [7,4]$
b) $\vec{F} = [100,400], \vec{s} = [12,27]$
c)



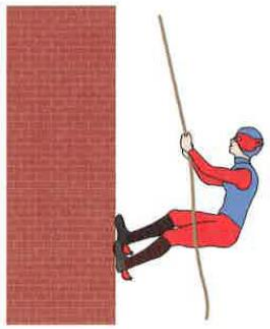
18) Calculate the angle between the vectors in each pair.

a) $\vec{p} = [7,8], \vec{q} = [4,3]$
b) $\vec{t} = [-7,2], \vec{u} = [6,11]$

19) Determine the projection of $\vec{u}$ on $\vec{v}$
a) $|\vec{u}| = 56$, $|\vec{v}| = 100$, angle θ between $\vec{u}$ and $\vec{v}$ is $125°$.
b) $\vec{u} = [7,1]$, $\vec{v} = [9,-3]$

20) Find the magnitude of the projection of $\vec{a}$ on $\vec{b}$ and also the vector projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} = [6,-1]$ and $\vec{b} = [11,5]$

21) A superhero pulls herself 15 m up the side of a wall with a with a force of 500 N, at an angle of $12°$ to the vertical. What is the work done?



22) A crate is dragged 3 meters along a smooth level floor by a 30 N force, applied at $25°$ to the floor. Then, it is pulled 4 meters up a ramp inclined at $20°$ to the horizontal, using the same force. Then, the crate is dragged a further 5 meters along a level platform using the same force again. Determine the total work done in moving the crate.

23) How much work is done against gravity by a worker who carries a 25-kg carton up a 6 meter long set of stairs, inclined at $30°$.

24) Draw each position vector. Then, determine its exact magnitude.

$\vec{a} = [-1,5,-2]$
$\vec{b} = [-2,0,-4]$

25) Are the vectors $\vec{u} = [6,-2,-5]$ and $\vec{v} = [-12,4,10]$ collinear? Explain.

26) Find a and b such that $\vec{u} = [a,3,6]$ and $\vec{v} = [-8,12,b]$ are collinear.

27) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Then, write the vector in the form $[x,y,z]$ and determine its magnitude.

a) $A(2,13)$, $B(5,7,1)$ **b)** $A(3,-4,1)$, $B(6,-1,5)$

28) Given the vectors $\vec{a} = [-4,1,7]$, $\vec{b} = [2,0,-3]$, and $\vec{c} = [1,-1,5]$, simplify each expression.

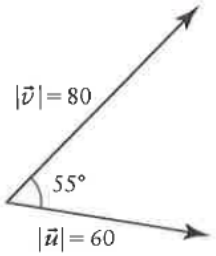
a) $7\vec{a}$
b) $3\vec{a} - 2\vec{b} + 4\vec{c}$
c) $\vec{a} \cdot \vec{c}$
d) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

29) Determine the angle between the vectors $\vec{g} = [6,1,2]$ and $\vec{h} = [-5,3,6]$

30) Determine a vector that is orthogonal to $\vec{e} = [3,-1,4]$

31) Identify the type of triangle with vertices $A(2,3,-5)$, $B(-4,8,1)$, and $C(6,-4,0)$.

32) Determine $\vec{u} \times \vec{v}$



33) Determine $\vec{a} \times \vec{b}$ for each pair of vectors

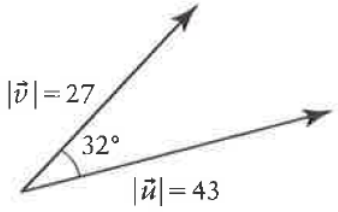
a) $\vec{a} = [3,-2,9]$, $\vec{b} = [1,1,6]$

b) $\vec{a} = [-8,10,3]$, $\vec{b} = [2,0,5]$

34) Determine the area of the parallelogram defined by each pair of vectors.

a) $\vec{p} = [6,3,8]$, $\vec{q} = [3,3,5]$

b)

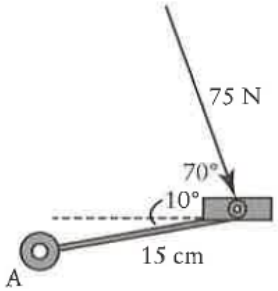


35) Given $\vec{a} = [2,-6,3]$, $\vec{b} = [-1,5,8]$, and $\vec{c} = [-4,5,6]$, evaluate each of the following.

a) $\vec{a} \times (\vec{b} + \vec{c})$

b) $\vec{a} \times \vec{b} - \vec{a} \times \vec{c}$

36) A bicycle pedal is pushed by a 75-N force, exerted as shown in the diagram. The shaft of the pedal is 15 cm long. Find the magnitude of the torque vector, in Newton-meters, about point A.



37) A force of $\vec{F} = [3,5,12]$, in Newtons, is applied to lift a box, with displacement $\vec{s} = [2,1,6]$. Calculate the work against gravity and compare it to the work in the direction of travel.

38) Determine the projection, and its magnitude, of $\vec{u} = [3,1,4]$ on $\vec{v} = [6,2,7]$.

39) Given $\vec{a} = [-2,3,5]$, $\vec{b} = [4,0,-1]$, and $\vec{c} = [2,-2,3]$, evaluate $\vec{a} \cdot \vec{b} \times \vec{c}$

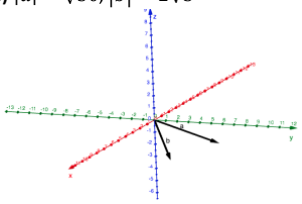
40) Find the volume of the parallelepiped, defined by the vectors $\vec{u} = [1,4,3]$, $\vec{v} = [2,5,6]$, and $\vec{w} = [1,2,7]$.

41) A triangle has vertices $A(-2,1,3)$, $B(7,8,-4)$, and $C(5,0,2)$. Determine the area of ΔABC.

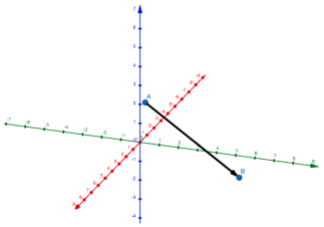
Answers:

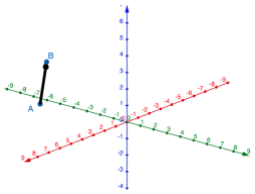
1)a) $-3\hat{\imath}-6\hat{\jmath}$ **b)** $-8\hat{\jmath}$ **c)** $-6\hat{\imath}$
2)a) $[-4,0]$ **b)** $[7,-4]$ **c)** $[0,2]$
3) $\overrightarrow{AB}=[2,-5]$, $\left|\overrightarrow{AB}\right|=\sqrt{29}$ units; $\overrightarrow{CD}=[5,1]$, $\left|\overrightarrow{CD}\right|=\sqrt{26}$ units; $\overrightarrow{EF}=[0,7]$, $\left|\overrightarrow{EF}\right|=7$ units; $\overrightarrow{GH}=[-2,9]$, $\left|\overrightarrow{GH}\right|=\sqrt{85}$ units
4) $Q(3,-8)$
5) $[-4,2]$ b) $3\sqrt{2}$ units c) 13.8 units
6)a) $[32,-8]$ **b)** $[-32,8]$ **c)** $[6,6]$ **d)** $[-2,8]$ **e)** $[14,-26]$ **f)** $[-2,53]$
7) $\vec{b}$ is collinear with $\vec{a}$; $\vec{b}=-\frac{3}{2}\vec{a}$
8) $[180\cos 30°, 180\sin 30°]$
9) $[250\cos(-35°), 250\sin(-35°)]$
10) 30.8 km/h on a bearing of 218.6°
11)a) 2753.3 **b)** -83.1 **c)** 0 **d)** 2 **e)** 36 **f)** -3
12)a) -46 **b)** -3 **c)** This is not possible. It is the sum of a vector and a scalar. **d)** 102 **e)** -159 **f)** 102
13) It is a right angled triangle. $\angle A$ is the right angle.
14) $[2,-9]$
15) $k=-10$
16) $k=0,-6$
17)a) 43 J **b)** 12 000 J **c)** 1891.6 J
18)a) 11.9° **b)** 102.7°
19)a) $-32.1\hat{v}$ **b)** $[6,-2]$
20) $\left|\text{proj}_{\vec{b}}\,\vec{a}\right|=\frac{61}{\sqrt{146}}$; $\text{proj}_{\vec{b}}\,\vec{a}=\left[\frac{671}{146},\frac{305}{146}\right]$
21) 7336.1 J
22) 337.1 J
23) 735 J

24) $|\vec{a}|=\sqrt{30}$; $\left|\vec{b}\right|=2\sqrt{5}$



25) Yes. $\vec{v}=-2\vec{u}$
26) $a=-2$, $b=24$
27)a) $\overrightarrow{AB}=[3,6,-2]$; $\left|\overrightarrow{AB}\right|=7$ **b)** $\overrightarrow{AB}=[3,3,4]$; $\left|\overrightarrow{AB}\right|=\sqrt{34}$





28)a) $[-28,7,49]$ **b)** $[-12,-1,47]$ **c)** 30 **d)** 53
29) 106.3°
30) Answers will vary. Ex. $[0,4,1]$. Use dot product to verify your answer
31) Scalene
32) $3931.9\hat{n}$
33)a) $[-21,-9,5]$ **b)** $[50,46,-20]$
34)a) $3\sqrt{22}$ units^2 **b)** 615.2 units^2
35)a) $[-114,-43,-10]$ **b)** $[-12,5,18]$
36) 11.1 $N\cdot m$
37) 72 J; 83 J
38) $\frac{48}{89}[6,2,7]$; $\frac{48}{\sqrt{89}}$
39) -78
40) 12 units^3
41) 35.9 units^2