Unit 5 Pre-Test Review – Trig Identities and Equations
MHF4U
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SOLUTIONS

## Section 1: Transformation and Co-function Identities

**1)** Given that $\sin\left(\frac{2\pi}{7}\right) \cong 0.7818$, use equivalent trigonometric expressions to evaluate the following,

**a)** $\cos\frac{3\pi}{14}$

$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)$

$= \sin\left(\frac{7\pi}{14} - \frac{3\pi}{14}\right)$

$= \sin\left(\frac{4\pi}{14}\right)$

$= \sin\left(\frac{2\pi}{7}\right)$

$= 0.7818$

**b)** $\cos\frac{11\pi}{14}$

$= \sin\left(\frac{\pi}{2} - \frac{11\pi}{14}\right)$

$= \sin\left(\frac{7\pi}{14} - \frac{11\pi}{14}\right)$

$= \sin\left(\frac{-4\pi}{14}\right)$

$= \sin\left(\frac{-2\pi}{7}\right)$

$= -\sin\left(\frac{2\pi}{7}\right)$

$= -0.7818$

## Section 2: Compound Angles

**2)** Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each.

**a)** $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{2\pi}{6} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{2}\right)$ ← use unit circle

$= 1$

**b)** $\cos\frac{\pi}{3}\cos\frac{5\pi}{12} - \sin\frac{\pi}{3}\sin\frac{5\pi}{12}$

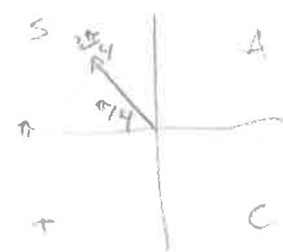
$= \cos\left(\frac{\pi}{3} + \frac{5\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12} + \frac{5\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$




**3)** Apply a compound angle formula, and then determine an exact value for each.

**a)** $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= -\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{-\sqrt{3}+1}{2\sqrt{2}}$

**b)** $\sin\left(\frac{5\pi}{4} - \frac{2\pi}{3}\right)$

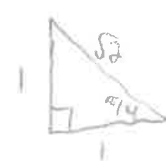
$= \sin\frac{5\pi}{4}\cos\frac{2\pi}{3} - \cos\frac{5\pi}{4}\sin\frac{2\pi}{3}$

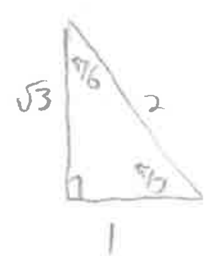
$= \left(-\sin\frac{\pi}{4}\right)\left(-\cos\frac{\pi}{3}\right) - \left(-\cos\frac{\pi}{4}\right)\left(\sin\frac{\pi}{3}\right)$

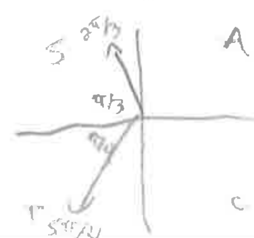
$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$



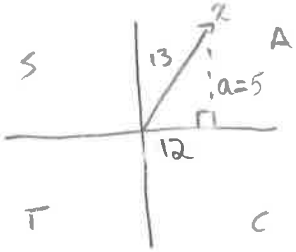
**4)** Angle $x$ is in the first quadrant and angle $y$ is in the second quadrant such that $\cos x = \frac{12}{13}$ and $\sin y = \frac{7}{25}$. Determine an exact value for...

**a)** $\sin x$

$a^2 + 12^2 = 13^2$

$a^2 = 25$

$a = 5$



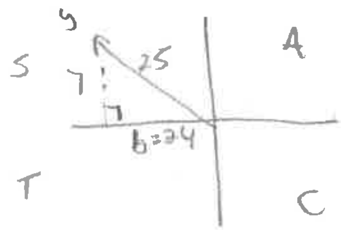

$\boxed{\sin x = \frac{5}{13}}$

**b)** $\cos y$

$b^2 + 7^2 = 25^2$

$b^2 = 576$

$b = 24$




$\boxed{\cos y = -\frac{24}{25}}$

**c)** $\sin(x + y)$

$= \sin x \cos y + \cos x \sin y$

$= \left(\frac{5}{13}\right)\left(\frac{-24}{25}\right) + \left(\frac{12}{13}\right)\left(\frac{7}{25}\right)$

$= \frac{-120}{325} + \frac{84}{325}$

$\boxed{= \frac{-36}{325}}$

**d)** $\sin(x - y)$

$= \sin x \cos y - \cos x \sin y$

$= -\frac{120}{325} - \frac{84}{325}$

$\boxed{= -\frac{204}{325}}$

**e)** $\cos(x + y)$

$= \cos x \cos y - \sin x \sin y$

$= \left(\frac{12}{13}\right)\left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right)\left(\frac{7}{25}\right)$

$= \frac{-288}{325} - \frac{35}{325}$

$\boxed{= \frac{-323}{325}}$

**f)** $\cos(x - y)$

$= \cos x \cos y + \sin x \sin y$

$= \frac{-288}{325} + \frac{35}{325}$

$\boxed{= -\frac{253}{325}}$

**5)** Use an appropriate compound angle formula to determine an exact value for each.

**a)** $\sin \frac{11\pi}{12}$

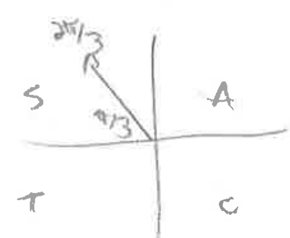
$= \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \left(-\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$




$\boxed{= \frac{\sqrt{3} - 1}{2\sqrt{2}}}$

**b)** $\cos\frac{25\pi}{12}$ $= \cos\left(\frac{25\pi}{12} - 2\pi\right)$

$= \cos\left(\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$

**c)** $\tan\frac{7\pi}{12}$ $= \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \dfrac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{3}}$

$= \dfrac{1+\sqrt{3}}{1-\sqrt{3}} \times \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

If you rationalize the denominator

$= \dfrac{1 + 2\sqrt{3} + 3}{1-3}$

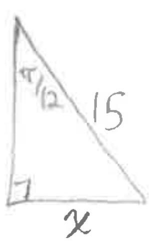
$= \dfrac{4+2\sqrt{3}}{-2}$

$= \dfrac{2(2+\sqrt{3})}{-2}$

$= -1(2+\sqrt{3})$

$= -2-\sqrt{3}$

**6)** A 15-m ladder leaning against a wall is in an unsafe position if it makes an angle of less than $\frac{\pi}{12}$ radians with the wall. Use a compound angle formula to determine an exact expression for the minimum distance that the foot of the ladder can be placed from the wall so that the ladder is standing safely.




$\sin\frac{\pi}{12} = \frac{x}{15}$

$\dfrac{\sqrt{3}-1}{2\sqrt{2}} = \dfrac{x}{15}$

$x = 15\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)$ meters

$\sin\frac{\pi}{12} = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

**Section 3: Double Angle Formulas**

**7)** Express each of the following as a single trig ratio and then evaluate

**a)** $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$= \sin\left[2\left(\frac{\pi}{12}\right)\right]$

$= \sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{2}$

**b)** $\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$

$= \cos\left[2\left(\frac{\pi}{12}\right)\right]$

$= \cos\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$

**c)** $1 - 2\sin^2\left(\frac{3\pi}{8}\right)$

$= \cos\left[2\left(\frac{3\pi}{8}\right)\right]$

$= \cos\frac{6\pi}{8}$

$= \cos\frac{3\pi}{4}$

$= -\cos\frac{\pi}{4}$

$\boxed{= -\frac{1}{\sqrt{2}}}$

**d)** $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\left(\frac{\pi}{6}\right)}$

$= \tan\left[2\left(\frac{\pi}{6}\right)\right]$

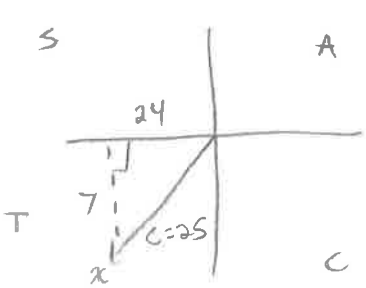
$= \tan\left(\frac{2\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$\boxed{= \sqrt{3}}$

**7)** Angle $x$ lies in the third quadrant, and $\tan x = \frac{7}{24}$.

**a)** Determine an exact value for $\cos(2x)$




$7^2 + 24^2 = c^2$

$c = 25$

$\cos(2x) = \cos^2 x - \sin^2 x$

$= \left(-\frac{24}{25}\right)^2 - \left(-\frac{7}{25}\right)^2$

$= \frac{576}{625} - \frac{49}{625}$

$\boxed{= \frac{527}{625}}$

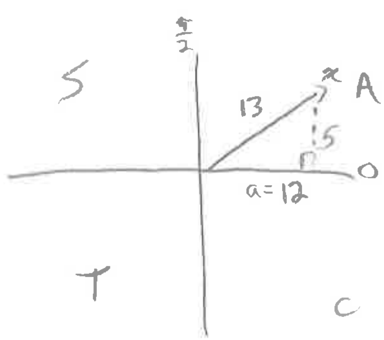
**b)** Determine an exact value for $\sin(2x)$

$\sin(2x) = 2\sin x\cos x$

$= 2\left(\frac{-7}{25}\right)\left(\frac{-24}{25}\right)$

$\boxed{= \frac{336}{625}}$

**8)** Given $\sin x = \frac{5}{13}$ and $0 \le x \le \frac{\pi}{2}$, find $\sin(2x)$




$a^2 + 5^2 = 13^2$

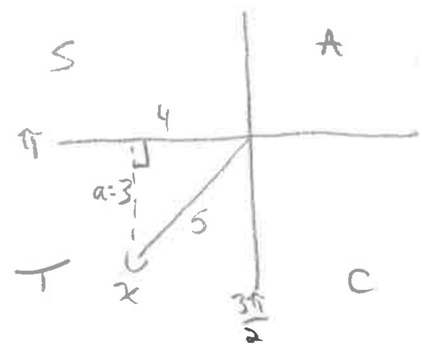
$a = 12$

$\sin(2x) = 2\sin x\cos x$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$\boxed{= \frac{120}{169}}$

**9)** Given $\cos x = -\frac{4}{5}$ and $\pi \leq x \leq \frac{3\pi}{2}$, find $\tan(2x)$




$a^2 + 4^2 = 5^2$

$a = 3$

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$= \frac{\left(\frac{3}{2}\right)}{1-\frac{9}{16}}$

$= \frac{\left(\frac{3}{2}\right)}{\left(\frac{7}{16}\right)}$

$= \frac{3}{2} \times \frac{16}{7}$

$\boxed{= \frac{24}{7}}$

**Section 4: Trig Identities**

**10)** Prove each of the following identities on a separate piece of paper

**a)** $\sin^2 x\,(1+\cot^2 x) = 1$

**b)** $1-\cos^2 x = \tan x \cos x \sin x$

**c)** $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

**d)** $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

**e)** $\sin(2\pi - x) = -\sin x$

**f)** $[\sin(2x)](\tan x + \cot x) = 2$

**g)** $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

**h)** $(\cos x - \sin x)^2 = 1 - \sin(2x)$

$\frac{-\sin^2 x}{\cot^2 x} = 1 - \cos^2 x$

**j)** $(\csc x - \cot x)^2 = \frac{1-\cos x}{1+\cos x}$

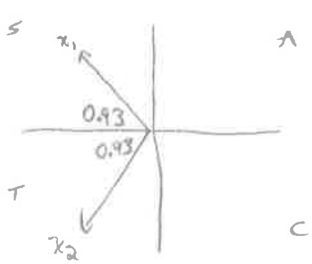
**k)** $\frac{1-2\sin^2 x}{\cos x + \sin x} + 2\sin\frac{x}{2}\cos\frac{x}{2} = \cos x$

**Section 5: Solving Linear Trigonometric Equations**

**11)** Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

**a)** $\cos x + 0.6 = 0$

$\cos x = -0.6$




$x_1 = \cos^{-1}(-0.6)$

$\boxed{x_1 = 2.21}$

$x_2 = \pi + 0.93$

$\boxed{x_2 = 4.07}$

**b)** $\sin x - 2.5 = 0$

$\sin x = 2.5$

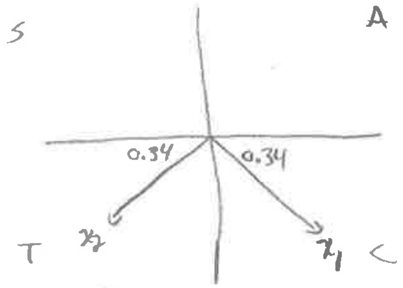
∴ No solutions

* all sine and cosine ratios are between -1 and 1 *

**c)** $\csc x + 3 = 0$

$\csc x = -3$

$\sin x = -\frac{1}{3}$




$x_1 = \sin^{-1}\left(\frac{-1}{3}\right)$

$x_1 = -0.3398369 + 2\pi$

$\boxed{x_1 = 5.94}$

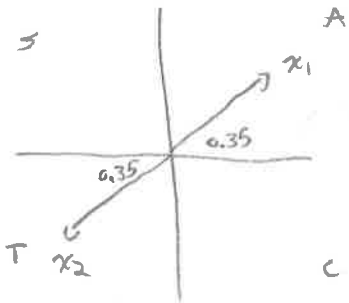
$x_2 = \pi + 0.34$

$\boxed{x_2 = 3.48}$

**d)** $\cot x - 2.75 = 0$

$\cot x = 2.75$

$\tan x = \frac{1}{2.75}$




$x_1 = \tan^{-1}\left(\frac{1}{2.75}\right)$

$\boxed{x_1 = 0.35}$

$x_2 = \pi + 0.35$

$\boxed{x_2 = 3.49}$

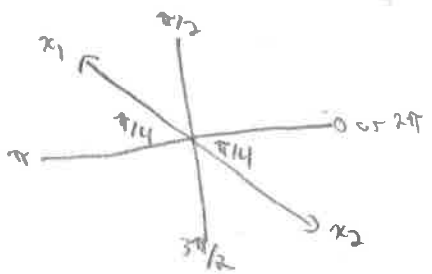
**12)** Determine exact solutions for each equation in the interval $0 \le x \le 2\pi$.

**a)** $\tan x + 1 = 0$

$\tan x = -1$

use special $\Delta$; $\tan\frac{\pi}{4} = 1$

place $\frac{\pi}{4}$ in Q2 + Q4




$x_1 = \pi - \frac{\pi}{4}$

$\boxed{x_1 = \frac{3\pi}{4}}$

$x_2 = 2\pi - \frac{\pi}{4}$

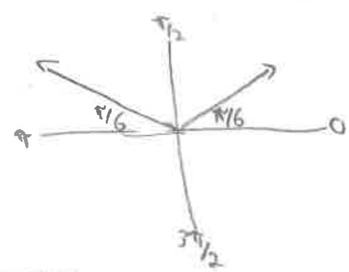
$\boxed{x_2 = \frac{7\pi}{4}}$

**b)** $\sin x - 0.5 = 0$

$\sin x = \frac{1}{2}$

use special $\Delta$; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place $\frac{\pi}{6}$ in Q1 + Q2




$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = \pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{5\pi}{6}}$

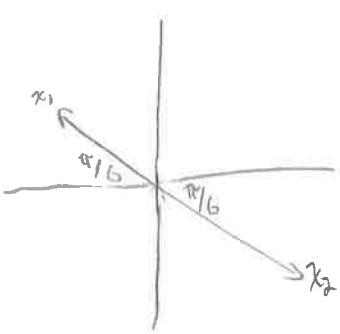
**c)** $\cot x + \sqrt{3} = 0$

$\cot x = -\sqrt{3}$

$\tan x = -\frac{1}{\sqrt{3}}$

use special $\Delta$; $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Place in Q2 + Q4




$x_1 = \pi - \frac{\pi}{6}$

$\boxed{x_1 = \frac{5\pi}{6}}$

$x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{11\pi}{6}}$

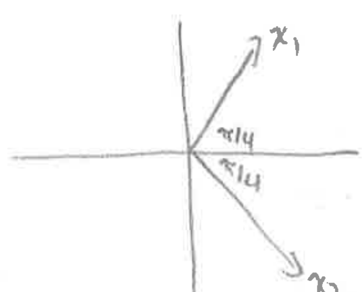
**d)** $\sec x - \sqrt{2} = 0$

$\sec x = \sqrt{2}$

$\cos x = \frac{1}{\sqrt{2}}$

use special $\Delta$; $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

place in Q1 + Q4




$\boxed{x_1 = \frac{\pi}{4}}$

$x_2 = 2\pi - \frac{\pi}{4}$

$\boxed{x_2 = \frac{7\pi}{4}}$

## Section 6: Solving Double Angle Trigonometric Equations

**13)** Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. /e exact answers where possible.

**a)** $\cos(2x) = -\frac{1}{4}$

Note: period $= \frac{2\pi}{2} = \pi$

let $\theta = 2x$

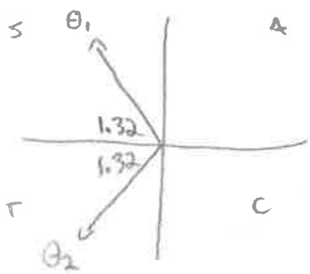
$\cos\theta = -\frac{1}{4}$

$\theta_1 = \cos^{-1}\left(-\frac{1}{4}\right)$

$\theta_1 = 1.82$

$\theta_2 = \pi + 1.32$

$= 4.46$




$2x = \theta$

$2x = 1.82$ $\quad$ $\boxed{x_1 = 0.91}$

$2x = 4.46$ $\quad$ $\boxed{x_2 = 2.23}$

Add period to find other solutions

$\boxed{x_3 = x_1 + \pi = 4.05}$ $\quad$ $\boxed{x_4 = x_2 + \pi = 5.37}$

**b)** $5\tan(3x) - 5 = 0$

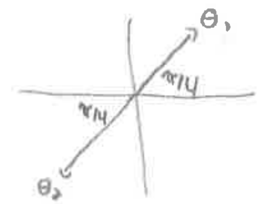
**Note: period =** $\frac{\pi}{3}$

$\tan 3x = 1$

let $\theta = 3x$

$\tan\theta = 1$

from $\Delta$; $\tan\frac{\pi}{4} = 1$




$\theta_1 = \frac{\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$3x = \theta$

$3x = \frac{\pi}{4}$ $\quad$ $\boxed{x_1 = \frac{\pi}{12}}$

$3x = \frac{5\pi}{4}$ $\quad$ $\boxed{x_2 = \frac{5\pi}{12}}$

Add period to find other solutions

$\boxed{x_3 = \frac{9\pi}{12} = \frac{3\pi}{4}}$ $\quad$ $\boxed{x_5 = \frac{13\pi}{12}}$

$\boxed{x_4 = \frac{17\pi}{12}}$ $\quad$ $\boxed{x_6 = 21\pi/12 = 7\pi/4}$

## Section 7: Solving Quadratic Trigonometric Equations

**14)** Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

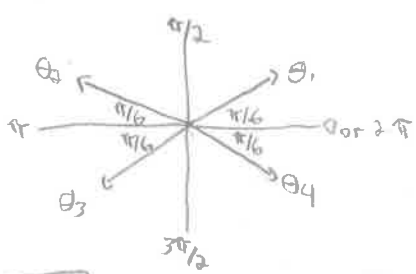
**a)** $\sin^2 x - 0.25 = 0$

$\sin^2 x = 0.25$

$\sin x = \pm\sqrt{0.25}$

$\sin x = \pm\frac{1}{2}$

from $\Delta$; $\sin\frac{\pi}{6} = \frac{1}{2}$; Place in all 4 Q's




$\boxed{\theta_1 = \frac{\pi}{6}}$

$\theta_2 = \pi - \frac{\pi}{6}$

$\boxed{\theta_2 = \frac{5\pi}{6}}$

$\theta_3 = \pi + \frac{\pi}{6}$

$\boxed{\theta_3 = \frac{7\pi}{6}}$

$\theta_4 = 2\pi - \frac{\pi}{6}$

$\boxed{\theta_4 = \frac{11\pi}{6}}$

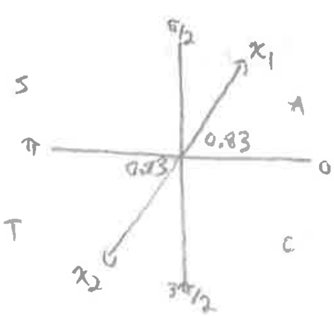
**b)** $\tan^2 x - 1.21 = 0$

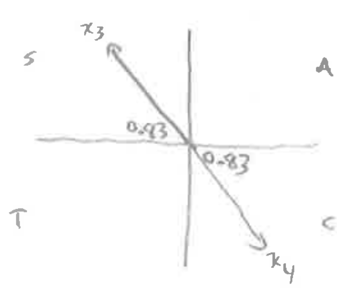
$\tan^2 x = 1.21$

$\tan x = \pm\sqrt{1.21}$

$\tan x = \pm 1.1$

$\tan x = 1.1$ $\quad$ $\tan x = -1.1$




$x_1 = \tan^{-1}(1.1)$

$\boxed{x_1 = 0.83}$

$x_2 = \pi + 0.83$

$\boxed{x_2 = 3.97}$

$x_3 = \pi - 0.83$

$\boxed{x_3 = 2.31}$

$x_4 = 2\pi - 0.83$

$\boxed{x_4 = 5.45}$

**15)** For each of the following questions, determine exact solution if possible. If exact solutions are not possible, determine approximate solutions rounded to the nearest hundredth.

**a)** $2\sin^2 x - \sin x - 1 = 0$

$\frac{-1}{1} = \frac{-2}{2}$ ; $\frac{1}{2}$

$(\sin x - 1)(2\sin x + 1) = 0$

$\sin x - 1 = 0$

$\sin x = 1$

from unit circle $\sin\frac{\pi}{2} = 1$

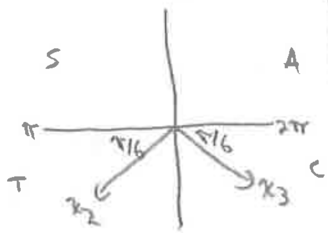
$\boxed{x_1 = \frac{\pi}{2}}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

from $\Delta$; $\sin\frac{\pi}{6} = \frac{1}{2}$

place in Q3 + Q4




$\boxed{x_2 = \frac{7\pi}{6}}$

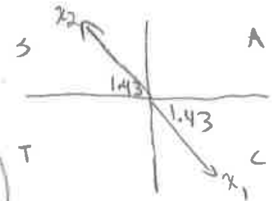
$\boxed{x_3 = \frac{11\pi}{6}}$

**b)** $\tan^2 x + 3\tan x - 28 = 0$

$(\tan x + 7)(\tan x - 4) = 0$

$\tan x + 7 = 0$

$\tan x = -7$




$x_1 = \tan^{-1}(-7)$

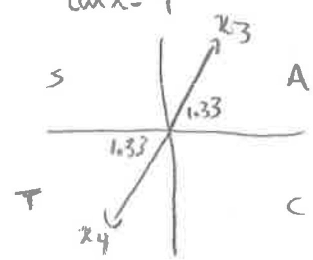
$x_1 = -1.43 + 2\pi$

$\boxed{x_1 = 4.85}$

$\boxed{x_2 = 1.71}$

$\tan x - 4 = 0$

$\tan x = 4$




$x_3 = \tan^{-1}(4)$

$\boxed{x_3 = 1.33}$

$\boxed{x_4 = 4.47}$

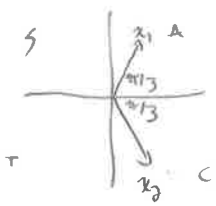
**c)** $\sec^2 x - 5\sec x + 6 = 0$

$(\sec x - 2)(\sec x - 3) = 0$

$\sec x - 2 = 0$

$\cos x = \frac{1}{2}$

from $\Delta$; $\cos\frac{\pi}{3} = \frac{1}{2}$

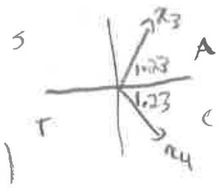



$\boxed{x_1 = \frac{\pi}{3}}$

$\boxed{x_2 = \frac{5\pi}{3}}$

$\sec x - 3 = 0$

$\cos x = \frac{1}{3}$




$x_3 = \cos^{-1}\left(\frac{1}{3}\right)$

$\boxed{x_3 = 1.23}$

$\boxed{x_4 = 5.05}$

**d)** $2\cos^2 x - 4\cos x = 0$

$2\cos x(\cos x - 2) = 0$

$\cos x = 0$

from unit circle;

$\cos\frac{\pi}{2}$ and $\cos\frac{3\pi}{2} = 0$

$\boxed{x_1 = \frac{\pi}{2}}$

$\boxed{x_2 = \frac{3\pi}{2}}$

$\cos x - 2 = 0$

$\cos x = 2$

$\therefore$ No solutions

**e)** $6\sin^2 x - \sin x - 1 = 0$

$\frac{-1}{2} = \frac{-3}{6}$ ; $\frac{2}{6} = \frac{1}{3}$

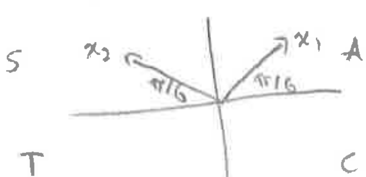
$(2\sin x - 1)(3\sin x + 1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

From $\Delta$; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2

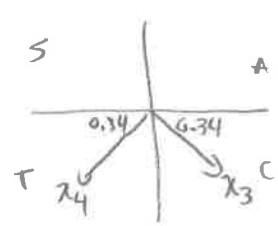



$\boxed{x_1 = \frac{\pi}{6}}$

$\boxed{x_2 = \frac{5\pi}{6}}$

$3\sin x + 1 = 0$

$\sin x = -\frac{1}{3}$




$x_3 = \sin^{-1}\left(-\frac{1}{3}\right)$

$x_3 = -0.34 + 2\pi$

$\boxed{x_3 = 5.94}$

$\boxed{x_4 = 3.48}$

**16)** The height, $h$, in meters, above the ground of a rider on a Ferris wheel can be modelled by the equation $h(t) = 10\sin\left[\frac{\pi}{15}(t - 7.5)\right] + 12$, where $t$ is the time in seconds. At $t = 0$, the rider is at the lowest point. Determine the first two times that the rider is 20 meters above the ground, to the nearest hundredth of a second.

$20 = 10\sin\left[\frac{\pi}{15}(t-7.5)\right] + 12$

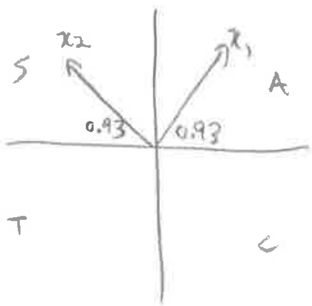
$0.8 = \sin\left[\frac{\pi}{15}(t-7.5)\right]$    let $x = \frac{\pi}{15}(t-7.5)$

$\sin x = 0.8$

$x_1 = \sin^{-1}(0.8)$

$\boxed{x_1 = 0.93}$

$\boxed{x_2 = 2.21}$




$x = \frac{\pi}{15}(t-7.5)$

$0.93 = \frac{\pi}{15}(t-7.5)$

$4.44 = t - 7.5$

$\boxed{t = 11.94 \text{ seconds}}$

$2.21 = \frac{\pi}{15}(t-7.5)$

$10.55 = t - 7.5$

$\boxed{t = 18.05 \text{ seconds}}$

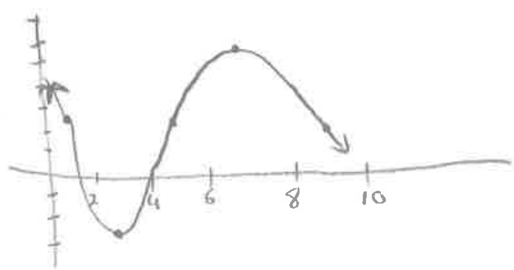
**17)** The height, in meters, of a nail in a water wheel above the surface of the water, as a function of time, can be modelled by the function $h(t) = -4\sin\left[\frac{\pi}{4}(t - 1)\right] + 2.5$, where $t$ is the time in seconds. During what periods of time is the nail below the water in the first 24 seconds that the wheel is rotating?

amp $= 4$

period $= \frac{2\pi}{\left(\frac{\pi}{4}\right)} = 8$ seconds

max $= 2.5 + 4 = 6.5$

min $= 2.5 - 4 = -1.5$




$0 = -4\sin\left[\frac{\pi}{4}(t-1)\right] + 2.5$

$0.625 = \sin\left[\frac{\pi}{4}(t-1)\right]$

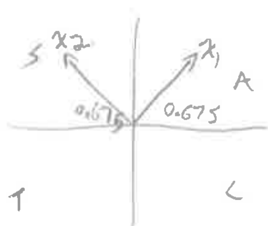
Let $x = \frac{\pi}{4}(t-1)$

$\sin x = 0.625$

$x_1 = \sin^{-1}(0.625)$

$x_1 = 0.675$

$x_2 = 2.47$




$0.675 = \frac{\pi}{4}(t-1)$

$0.8594366927 = t-1$

$t = 1.86$

$2.47 = \frac{\pi}{4}(t-1)$

$t = 4.14$

below water $1.86 < t < 4.14$ seconds

period of 8, so below water again at

$9.86 < t < 12.14$ and $17.86 < t < 20.14$

**ANSWER KEY**

**1)a)** $0.7818$ **b)** $-0.7818$

**2)a)** $\sin\frac{\pi}{2} = 1$ **b)** $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

**3)a)** $\frac{-\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{1+\sqrt{3}}{2\sqrt{2}}$

**4)a)** $\frac{5}{13}$ **b)** $-\frac{24}{25}$ **c)** $-\frac{36}{325}$ **d)** $-\frac{204}{325}$ **e)** $-\frac{323}{325}$ **f)** $-\frac{253}{325}$

**5)a)** $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **b)** $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **c)** $-2-\sqrt{3}$

**6)** $15\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ meters

**7)a)** $\sin\frac{\pi}{6} = \frac{1}{2}$ **b)** $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ **c)** $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ **d)** $\tan\frac{\pi}{3} = \sqrt{3}$

**8)a)** $\frac{527}{625}$ **b)** $\frac{336}{625}$

**9)** $\sin(2x) = \frac{120}{169}$

$\cancel{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}}$

**10)** $\tan(2x) = \frac{24}{7}$

**11)a)** 2.21, 4.07 **b)** no solutions **c)** 3.48, 5.94 **d)** 0.35, 3.49

**12)a)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}$ **c)** $\frac{5\pi}{6}, \frac{11\pi}{6}$ **d)** $\frac{\pi}{4}, \frac{7\pi}{4}$

**13)a)** 0.91, 2.23, 4.05, 5.31 **b)** $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$

**14)a)** 0.52, 2.62, 3.66, 5.76 **b)** 0.83, 2.31, 3.97, 5.45

**15)a)** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** 1.33, 1.71, 4.47, 4.85 **c)** $\frac{\pi}{3}, 1.23, 5.05, \frac{5\pi}{3}$ **d)** $\frac{\pi}{2}, \frac{3\pi}{2}$ **e)** $\frac{\pi}{6}, \frac{5\pi}{6}, 3.48, 5.94$

**16)** 11.93, 18.07

**17)** $1.86 < t < 4.14, 9.86 < t < 12.14, 17.86 < t < 20.14$

(10) a)

LS

$= \sin^2 x(1+\cot^2 x)$

$= \sin^2 x + \sin^2 x \cot^2 x$

$= \sin^2 x + \cancel{\sin^2 x}\left(\dfrac{\cos^2 x}{\cancel{\sin^2 x}}\right)$

$= \sin^2 x + \cos^2 x$

$= 1$

RS

$= 1$

LS=RS

b)

LS

$= 1 - \cos^2 x$

$= \sin^2 x$

RS

$= \tan x \cos x \sin x$

$= \left(\dfrac{\sin x}{\cancel{\cos x}}\right) \cancel{\cos x} \sin x$

$= \sin x \sin x$

$= \sin^2 x$

LS=RS

c)

LS

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$

$= \dfrac{(\cos x - \sin x)\cancel{(\cos x + \sin x)}}{\cos x\,\cancel{(\cos x + \sin x)}}$

$= \dfrac{\cos x - \sin x}{\cos x}$

RS

$= 1 - \tan x$

$= \dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos x - \sin x}{\cos x}$

LS=RS

d)

LS

$= \dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$

$= \dfrac{1(1-\cos x) + 1(1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \dfrac{1 - \cos x + 1 + \cos x}{1 - \cos^2 x}$

$= \dfrac{2}{\sin^2 x}$

RS

$= 2\csc^2 x$

$= \dfrac{2}{\sin^2 x}$

LS=RS

e) LS | RS

$= \sin(2\pi - x)$ | $= -\sin x$

$= \sin 2\pi \cos x - \cos 2\pi \sin x$

$= 0(\cos x) - 1 \sin x$

$= -\sin x$

LS = RS

f) LS | RS

$= [\sin(2x)](\tan x + \cot x)$ | $= 2$

$= 2\sin x\cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$

$= 2\sin x\cos x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$

$= 2\cancel{\sin x}\cancel{\cos x}\left(\frac{1}{\cancel{\cos x}\cancel{\sin x}}\right)$

$= 2$

LS = RS

g) LS | RS

$= 2\sin x \cos y$ | $= \sin(x+y) + \sin(x-y)$

$= \sin x\cos y + \cancel{\cos x \sin y} + \sin x \cos y - \cancel{\cos x \sin y}$

$= 2\sin x \cos y$

LS = RS

h) LS

$= (\cos x - \sin x)^2$

$= \cos^2 x - 2\cos x \sin x + \sin^2 x$

$= 1 - 2\cos x \sin x$

RS

$= 1 - \sin(2x)$

$= 1 - 2\sin x \cos x$

LS = RS

i) LS

$= \dfrac{1-\sin^2 x}{\cot^2 x}$

$= \dfrac{\cos^2 x}{\left(\dfrac{\cos^2 x}{\sin^2 x}\right)}$

$= \cancel{\cos^2 x} \times \dfrac{\sin^2 x}{\cancel{\cos^2 x}}$

$= \sin^2 x$

RS

$= 1 - \cos^2 x$

$= \sin^2 x$

LS = RS

j)

LS

$= (\csc x - \cot x)^2$

$= \csc^2 x - 2\csc x \cot x + \cot^2 x$

$= \frac{1}{\sin^2 x} - 2\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) + \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x}$

RS

$= \frac{1 - \cos x}{1 + \cos x} \frac{(1-\cos x)}{(1-\cos x)}$

$= \frac{1 - 2\cos x + \cos^2 x}{1 - \cos^2 x}$

$= \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x}$

k)

LS

$= \frac{1 - 2\sin^2 x}{\cos x + \sin x} + 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= \frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin\left[2\left(\frac{x}{2}\right)\right]$

$= \frac{1 - 2\sin^2 x}{\cos x + \sin x} + \frac{\sin x}{1} \frac{(\cos x + \sin x)}{(\cos x + \sin x)}$

$= \frac{1 - 2\sin^2 x}{\cos x + \sin x} + \frac{\sin x \cos x + \sin^2 x}{\cos x + \sin x}$

$= \frac{\cos^2 x + \sin^2 x - 2\sin^2 x + \sin x \cos x + \sin^2 x}{\cos x + \sin x}$

$= \frac{\cos^2 x + \sin x \cos x}{\cos x + \sin x}$

$= \frac{\cos x \cancel{(\cos x + \sin x)}}{\cancel{\cos x + \sin x}} = \cos x$

RS

$= \cos x$

LS = RS