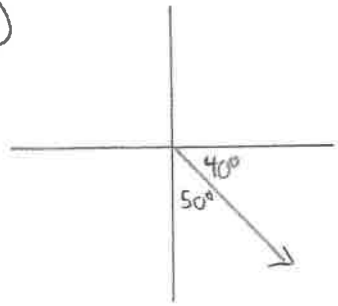
Exam Review

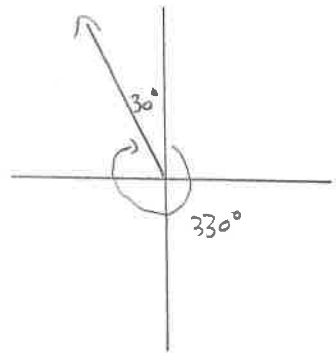
Unit 4 - Geometric Vectors

1) a)



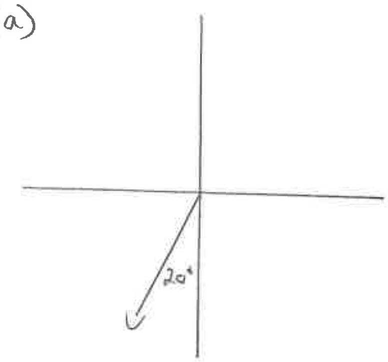
S 50° E

b)



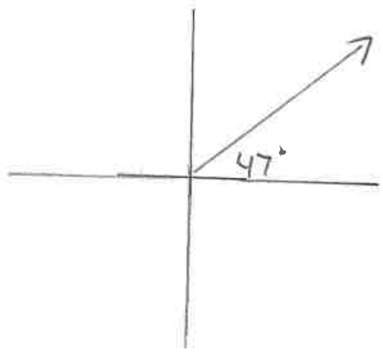
N 30° W

2) a)



200°

b)



043°

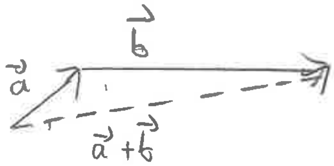
3) a) $\vec{EF}$, $\vec{GH}$, $\vec{IK}$

b) $\vec{CD}$, $\vec{EF}$, $\vec{GH}$, $\vec{IK}$

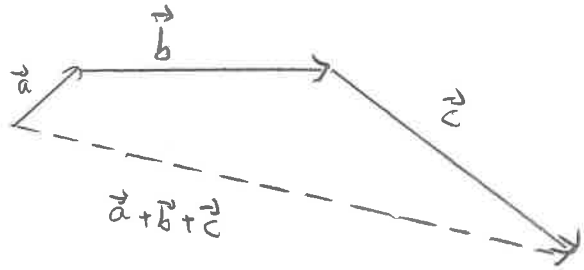
c) $\vec{EF}$, $\vec{IK}$

d) $\vec{CD}$

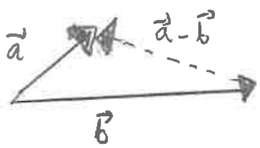
④ a)



b)



c)



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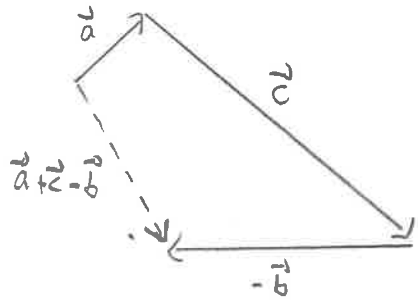
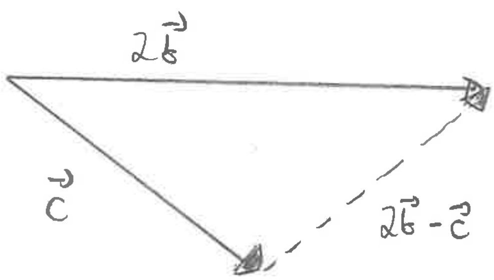


e)



f)

⑤ a) $\overrightarrow{FE} = \overrightarrow{BC} = \vec{w}$

b) $\overrightarrow{DE} = -\overrightarrow{AB} = -\vec{u}$

c) $\overrightarrow{DA} = -2\overrightarrow{BC} = -2\vec{w}$

d) $\overrightarrow{GE} = \overrightarrow{GD} + \overrightarrow{DE} = \vec{w} - \vec{u}$

e) $\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE} = 2\vec{w} - \vec{u}$

(6) a) $\vec{AB} - \vec{BF} = \vec{AB} - \vec{AE} = \vec{EB}$

b) $\vec{AB} + \vec{CG} = \vec{AB} + \vec{BF} = \vec{AF}$

c) $\vec{EF} + \vec{DH} = \vec{AB} + \vec{BF} = \vec{AF}$

d) $\vec{AB} + \vec{HD} + \vec{FG} = \vec{AB} - \vec{BF} + \vec{FG} = \vec{AB} - \vec{AE} + \vec{FG} = \vec{EB} + \vec{FG} = \vec{EB} + \vec{BC} = \vec{EC}$

e) $\vec{AB} - \vec{HD} + \vec{FG} = \vec{AB} - \vec{FB} + \vec{FG} = \vec{AB} + \vec{BF} + \vec{FG} = \vec{AF} + \vec{FG} = \vec{AG}$

f) $\vec{ED} + \vec{AB} - (\vec{HG} + \vec{FB}) = \vec{ED} + \vec{DC} - (\vec{HG} + \vec{GC}) = \vec{EC} - \vec{HC} = \vec{EC} - \vec{EB} = \vec{BC}$

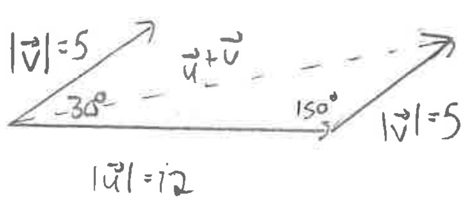
(7) a) $\vec{AB} = \frac{1}{3}\vec{v}$

b) $\vec{AZ} = \frac{2}{3}\vec{v} + \vec{u}$

c) $\vec{WB} = -\vec{u} + \frac{2}{3}\vec{v}$

d) $\vec{AC} = -\frac{1}{3}\vec{v} + \vec{u} + \frac{1}{2}\vec{v} = \frac{1}{6}\vec{v} + \vec{u}$

(8) a)



$|\vec{u}+\vec{v}|^2 = 12^2 + 5^2 - 2(12)(5)\cos(150°)$

$|\vec{u}+\vec{v}|^2 = 169 - 120\left(-\frac{\sqrt{3}}{2}\right)$

$|\vec{u}+\vec{v}|^2 = 169 + 60\sqrt{3}$

$|\vec{u}+\vec{v}| = \sqrt{169 + 60\sqrt{3}}$

unit vector in direction of $\vec{u}+\vec{v} = \frac{1}{|\vec{u}+\vec{v}|}(\vec{u}+\vec{v})$

$= \frac{1}{\sqrt{169+60\sqrt{3}}}(\vec{u}+\vec{v})$

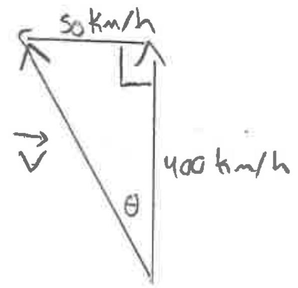
b) $|3\vec{u} + 2\vec{v}|^2 = 36^2 + 10^2 - 2(36)(10)\cos(150^\circ)$

$|3\vec{u} + 2\vec{v}|^2 = 1396 - 720\left(-\frac{\sqrt{3}}{2}\right)$

$|3\vec{u} + 2\vec{v}|^2 = 1396 + 360\sqrt{3}$

$|3\vec{u} + 2\vec{v}| = \sqrt{1396 + 360\sqrt{3}}$

(9)



$|\vec{v}|^2 = 50^2 + 400^2$

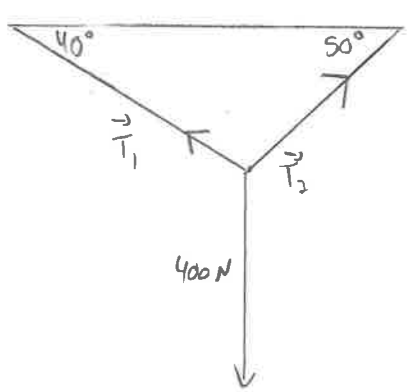
$|\vec{v}| \simeq 403.1$ km/h

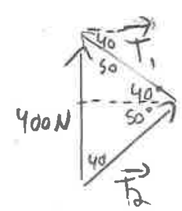
$\theta = \tan^{-1}\left(\frac{50}{400}\right)$

$\theta \simeq 7.1^\circ$

The ground velocity is 403.1 km/h at N 7.1° W

(10)





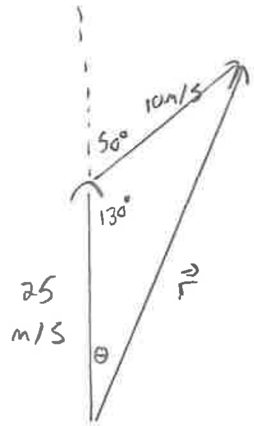
$\sin 40^\circ = \frac{|\vec{T}_1|}{400}$

$|\vec{T}_1| \simeq 257.1$ N

$\cos 40^\circ = \frac{|\vec{T}_2|}{400}$

$|\vec{T}_2| \simeq 306.4$ N

(11)



$$|\vec{r}|^2 = 25^2 + 10^2 - 2(25)(10)\cos(130°)$$

$$|r|^2 \simeq 1046.39\ 3805$$

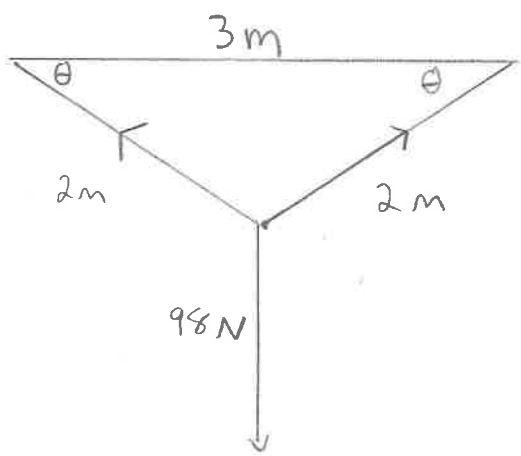
$$|r| \simeq 32.35 \text{ m/s}$$

$$\cos\theta = \frac{10^2 - 25^2 - 32.35^2}{-2(25)(32.35)}$$

$$\theta \simeq 13.7°$$

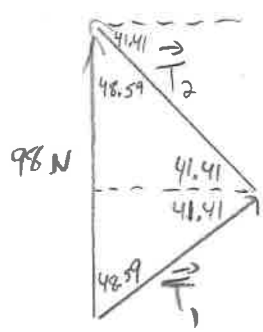
The toy's speed is 32.35 m/s with a direction of N 13.7° E

(12)



$$\cos\theta = \frac{2^2 - 3^2 - 2^2}{-2(3)(2)}$$

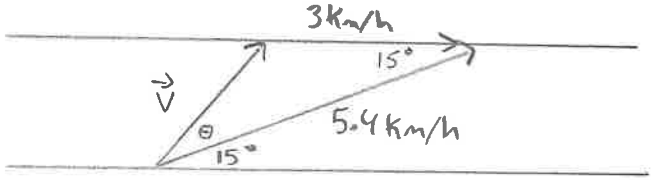
$$\theta \simeq 41.41°$$



$$\frac{|\vec{T}_2|}{\sin(48.59)} = \frac{|\vec{T}_1|}{\sin(48.59)} = \frac{98}{\sin(82.82)}$$

$$|\vec{T}_1| = |\vec{T}_2| \simeq 74.1 \text{ N}$$

(13)



$|\vec{v}|^2 = 3^2 + 5.4^2 - 2(3)(5.4)\cos(15°)$

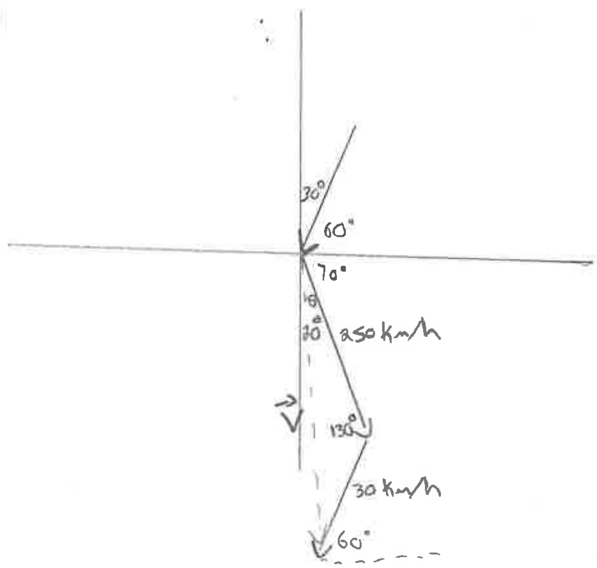
$|\vec{v}| \simeq 2.62$ Km/h

$$\cos\theta = \frac{3^2 - 5.4^2 - 2.62^2}{-2(5.4)(2.62)}$$

$\theta \simeq 17.24$

The velocity relative to the water is 2.62 Km/h at an of 32.24° relative to the shore.

(14)



$|\vec{v}|^2 = 250^2 + 30^2 - 2(250)(30)\cos(130°)$

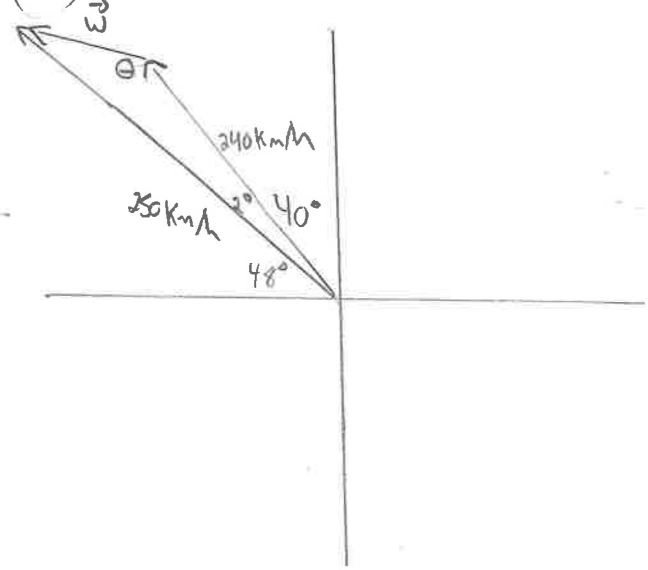
$|\vec{v}| \simeq 270.26$

$$\cos\theta = \frac{30^2 - 250^2 - 270.26^2}{-2(250)(270.26)}$$

$\theta \simeq 4.88°$

The ground velocity is 270.26 Km/h at a heading of S 15.12° E

(15)

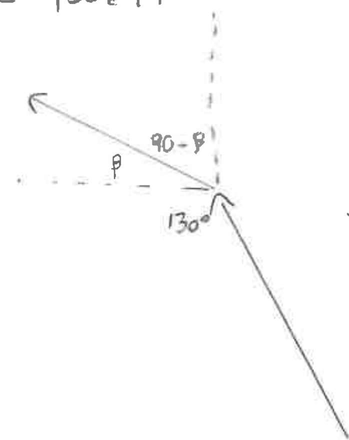


$$|\vec{w}|^2 = 250^2 + 240^2 - 2(250)(240)\cos(2°)$$

$$|\vec{w}| \simeq 13.16 \text{ Km/h}$$

$$\cos\theta = \frac{250^2 - 240^2 - 13.16^2}{-2(240)(13.16)}$$

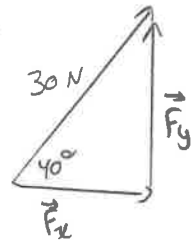
$$\theta \simeq 138.94°$$



$$\beta = 138.94 - 130$$
$$\simeq 8.94°$$

The wind has a velocity of 13.16 Km/h at a heading of N 81.1° W

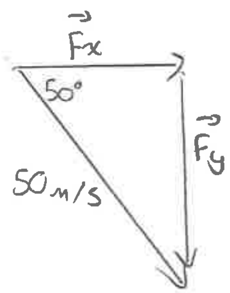
(16) a)



$|\vec{F_x}| = 30\cos(40°) \simeq 22.98\ N$

$|\vec{F_y}| = 30\sin(40°) \simeq 19.28\ N$

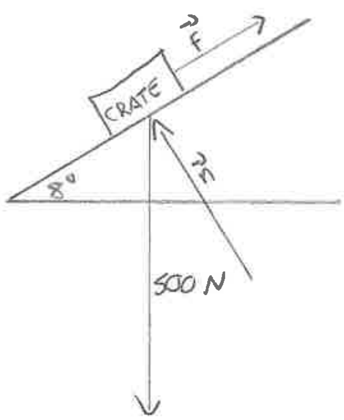
b)

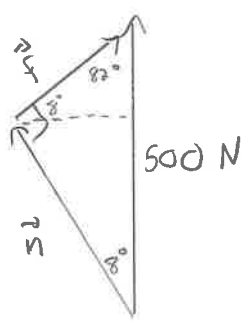


$|\vec{F_x}| = 50\cos(50°) \simeq 32.14\ m/s$

$|\vec{F_y}| = 50\sin(50°) \simeq 38.3\ m/s$

(17)

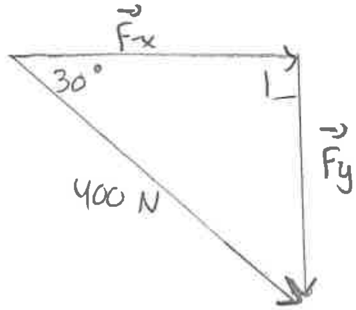




$|\vec{f}| = 500\sin(8°) \simeq 69.59\ N$

$|\vec{n}| = 500\cos(8°) \simeq 495.13\ N$

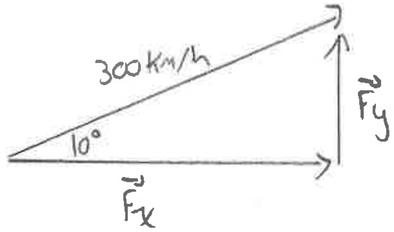
(18)



$|\vec{F_x}| = 400\cos(30°) \simeq 346.41\ N$ (horizontal)

$|\vec{F_y}| = 400\sin(30°) \simeq 200\ N$ (down)

(19)



$|\vec{F_x}|$ = horizontal speed $= 300\cos(10) \simeq 295.4$ Km/h

$|\vec{F_y}|$ = rate of climbing $= 300\sin(10) \simeq 52.1$ Km/h

UNIT 5 - Cartesian Vectors

① A(3, -4)
B(6, -9)

$\vec{AB} = [3, -5]$

$|\vec{AB}| = \sqrt{(3)^2 + (-5)^2}$

$|\vec{AB}| = \sqrt{34}$

C(-2, 2)
D(7, 9)

$\vec{CD} = [9, 7]$

$|\vec{CB}| = \sqrt{(9)^2 + (7)^2}$

$|\vec{CB}| = \sqrt{130}$

E(-4, 5)
F(-9, 9)

$\vec{EF} = [-5, 4]$

$|\vec{EF}| \; \sqrt{(-5)^2 + (4)^2}$

$|\vec{EF}| = \sqrt{41}$

I(0, -7)
J(-9, -7)

$\vec{IJ} = [-9, 0]$

$|\vec{IJ}| = \sqrt{(-9)^2}$

$|\vec{IJ}| = 9$

K(7, -6)
L(7, 3)

$\vec{KL} = [0, 9]$

$|\vec{KL}| = \sqrt{(9)^2}$

$|\vec{KL}| = 9$

M(-6, -1)
N(2, -6)

$\vec{MN} = [8, -5]$

$|\vec{MN}| = \sqrt{(8)^2 + (-5)^2}$

$|\vec{MN}| = \sqrt{89}$

② $\vec{u} = [2, -1]$
$\vec{v} = [5, -7]$

a) $2\vec{v} = 2[5, -7]$
$= [10, -14]$

b) $\vec{u} - \vec{v} = [2, -1] - [5, -7]$
$= [-3, 6]$

c) $3\vec{u} + 5\vec{v} = 3[2, -1] + 5[5, -7]$
$= [6, -3] + [25, -35]$
$= [31, -38]$

d) $4\vec{u} - 2\vec{v} = 4[2, -1] - 2[5, -7]$
$= [8, -4] - [10, -14]$
$= [-2, 10]$

③ a) $\vec{AC} = [6-2, 8-(-7)]$
$\vec{AC} = [4, 15]$

b) $\vec{AB} = [-6, 12]$
$|\vec{AB}| = \sqrt{(-6)^2 + (12)^2}$
$|\vec{AB}| = \sqrt{180} = 6\sqrt{5}$

c) $|\vec{AC}| = \sqrt{(4)^2 + (15)^2}$
$= \sqrt{241}$

$\vec{BC} = [10, 3]$
$|\vec{BC}| = \sqrt{(10)^2 + (3)^2}$
$|\vec{BC}| = \sqrt{109}$

Perimeter $= 6\sqrt{5} + \sqrt{241} + \sqrt{109} \simeq 39.38$ units

4 a) If collinear, $\vec{u} = a\vec{v}$

x

$2 = -12a$

$a = \frac{-1}{6}$

y

$K = 30a$

$K = 30\left(\frac{-1}{6}\right)$

$\boxed{K = -5}$

b) If collinear, $\vec{u} = a\vec{v}$

x

$-4 = 30a$

$a = \frac{-2}{15}$

y

$6 = Ka$

$6 = K\left(\frac{-2}{15}\right)$

$\boxed{K = -45}$

5 a) $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$= 7(11)\cos(23^\circ)$

$\simeq 70.88$

b) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 250(185)\cos(127^\circ)$

$\simeq -27\,833.94$

c) $\vec{u} \cdot \vec{v} = -8(1) + 9(-7)$

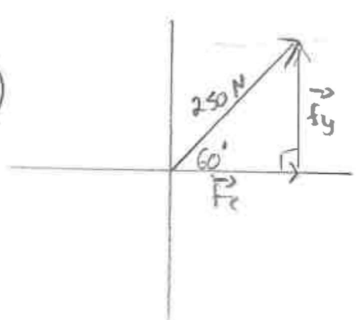
$= -71$

d) $\vec{u} = [-5, 11]$ $\vec{v} = [7, -2]$

$\vec{u} \cdot \vec{v} = -5(7) + 11(-2)$

$= -57$

6



$= [250\cos(60^\circ), 250\sin(60^\circ)]$

$= [125, 125\sqrt{3}]$

7) a) $\vec{u} = [2,-3]$ $\quad \vec{v} = [3,1]$

a) $3\vec{u} + 2\vec{v} = 3[2,-3] + 2[3,1]$

$= [6,-9] + [6,2]$

$= [12,-7]$

b) $|\vec{u}| = \sqrt{(2)^2 + (-3)^2}$

$= \sqrt{13}$

c) $\vec{u} \cdot \vec{v} = 2(3) + (-3)(1)$

$= 3$

d) $\vec{u} + \vec{v} = [2+3, -3+1]$

$= [5,-2]$

$|\vec{u} + \vec{v}| = \sqrt{(5)^2 + (-2)^2}$

$= \sqrt{29}$

8) $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

a) $\cos\theta = \dfrac{-1(3) + 8(-5)}{(\sqrt{65})(\sqrt{34})}$

$\theta \simeq 156.16°$

b) $\cos\theta = \dfrac{3(6) + (-7)(-1)}{(\sqrt{58})(\sqrt{37})}$

$\theta \simeq 57.34°$

9) a) $proj_{\vec{v}}\vec{u} = |\vec{u}| \cos\theta \; \hat{v}$

$= 95 \cos(13°) \; \hat{v}$

$\simeq 92.6 \hat{v}$

b) $proj_{\vec{v}}\vec{u} = \dfrac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} (\vec{v})$

$= \dfrac{6(7) + (-5)(-11)}{7^2 + (-11)^2} [7,-11]$

$= \dfrac{97}{170} [7,-11]$

(10) a) $|\text{proj}_{\vec{d}}\vec{c}| = \dfrac{|\vec{c}\cdot\vec{d}|}{|\vec{d}|}$

$= \dfrac{|2(9)+(-7)(12)|}{15}$

$= \dfrac{|-66|}{15}$

$= \dfrac{66}{15}$

$= \dfrac{22}{5}$

b) $\text{proj}_{\vec{d}}\vec{c} = \dfrac{-66}{15}\hat{d}$

$= \dfrac{-66}{15}\left(\dfrac{\vec{d}}{|\vec{d}|}\right)$

$= \dfrac{-66}{15}\left(\dfrac{1}{15}\right)[9,12]$

$= \dfrac{-22}{75}[9,12]$

$= \left[\dfrac{-66}{25}, \dfrac{-88}{25}\right]$

(11) a) $W = \vec{F}\cdot\vec{d}$

$= 18(9) + 23(12)$

$= 438 \text{ N}\cdot\text{m}$

b) $W = \vec{F}\cdot\vec{d}$

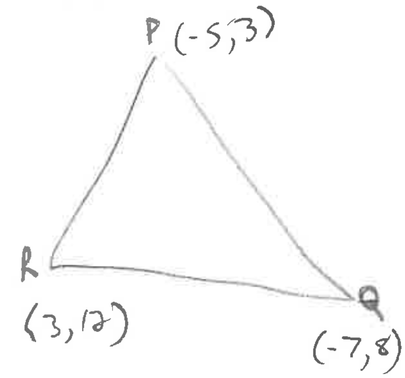
$= 45(23)\cos(37°)$

$\approx 826.59 \text{ N}\cdot\text{m}$

(12) $\vec{PQ} = [-2, 5]$ $\quad |\vec{PQ}| = \sqrt{29}$

$\vec{PR} = [8, 9]$ $\quad |\vec{PR}| = \sqrt{145}$

$\vec{RQ} = [-10, -4]$ $\quad |\vec{RQ}| = \sqrt{116}$



Check pythag. theorem: $\sqrt{29}^2 + \sqrt{116}^2 \stackrel{?}{=} \sqrt{145}^2$

$29 + 116 \stackrel{?}{=} 145$

$145 = 145$

$\therefore$ ΔPQR is a right triangle. $\angle Q$ is the right angle.

(13) $\vec{AB} = [4, -11, 26]$

$|\vec{AB}| = \sqrt{(4)^2 + (-11)^2 + (26)^2}$

$|\vec{AB}| = \sqrt{813}$

(14) $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

a) $\cos\theta = \dfrac{0(-3) + 1(1) + (-2)(4)}{(\sqrt{5})(\sqrt{26})}$

$\theta \simeq 127.87°$

b) $\cos\theta = \dfrac{1.5(-20) + 20(1) + 0(10)}{(\sqrt{402.25})(\sqrt{501})}$

$\theta \simeq 91.28°$

(15) $\vec{c} = [-6, 4, 0]$, $\vec{d} = [0, -5, -7]$, $\vec{e} = [3, 1, 2]$

a) $10\vec{c} - 10\vec{d} = 10[-6, 4, 0] - 10[0, -5, -7]$

$= [-60, 40, 0] - [0, -50, -70]$

$= [-60, 90, 70]$

b) $2\vec{c} \cdot 3\vec{d} - 2\vec{c} \cdot 4\vec{e}$

$= [-12, 8, 0] \cdot [0, -15, -21] - [-12, 8, 0] \cdot [12, 4, 8]$

$= -120 - (-112)$

$= -8$

$c = [-6, 4, 0]$ $\quad d = [0, -5, -7]$ $\quad \vec{e} = [3, 1, 2]$

c) using distributive property of dot product :

$$(3\vec{d} - 4\vec{e}) \cdot 2\vec{c} = 3\vec{d} \cdot 2\vec{c} - 4\vec{e} \cdot 2\vec{c}$$

$$= -8 \quad (\text{From part b})$$

d) $\vec{c} \times \vec{d} = [4(-7) - 0(-5), 0(0) - (-6)(-7), -6(-5) - 4(0)]$

$= [-28, -42, 30]$

4	-5
0	-7
-6	0
4	-5

e) $\vec{d} \times \vec{e} = [-5(2) - (-7)(1), -7(3) - 0(2), 0(1) - (-5)(3)]$

$= [-3, -21, 15]$

-5	1
-7	2
0	3
-5	1

f) All collinear vectors have a cross product of 0.

$\therefore \vec{c} \times \vec{c} = 0$

you can verify algebraically if you want...

(16)

$\vec{u} \cdot \vec{v} = 0$

$5(1) + 7t + 9(1) = 0$

$7t = -14$

$t = -2$

(17)

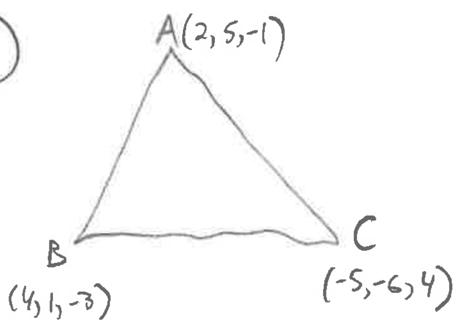
Area of parallelogram $= |\vec{a} \times \vec{b}| = \sqrt{(63)^2 + (-23)^2 + (-47)^2}$ $\boxed{= \sqrt{6707} \text{ units}^2}$

$\vec{a} \times \vec{b} = [55 - (-8), 12 - 35, -14 - 33]$

$$\begin{array}{cc} 11 & -2 \\ 4 & 5 \\ 7 & 3 \\ 11 & -2 \end{array}$$

$= [63, -23, -47]$

(18)



$\vec{AB} = [2, -4, -2]$

$\vec{AC} = [-7, -11, 5]$

$\vec{AB} \times \vec{AC} = [-20 - 22, 14 - 10, -22 - 28]$

$$\begin{array}{cc} -4 & -11 \\ -2 & 5 \\ 2 & -7 \\ -4 & -11 \end{array}$$

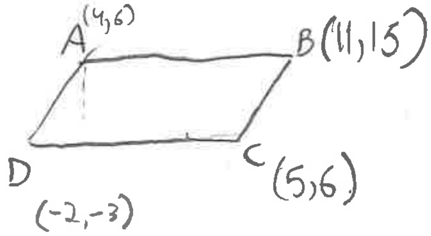
$= [-42, 4, -50]$

$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$= \frac{1}{2} \sqrt{4280}$

$= \sqrt{1070} \text{ units}^2$

(19)



$\vec{AB} = [7, 9, 0]$

$\vec{AD} = [-6, -9, 0]$

$\vec{AB} \times \vec{AD} = [0, 0, -9]$

$$\begin{array}{cc} 9 & -9 \\ 0 & 0 \\ 7 & -6 \\ 9 & -9 \end{array}$$

$\text{Area} = |\vec{AB} \times \vec{AD}|$

$= 9 \text{ units}^2$

(20)

$V = |\vec{a} \cdot \vec{b} \times \vec{c}|$

$\vec{b} \times \vec{c} = [28-27, 18-(-32), 24-(-14)]$

$$\begin{array}{cc} -7 & 3 \\ 9 & -4 \\ 8 & 2 \\ -7 & 3 \end{array}$$

$= [1, 50, 38]$

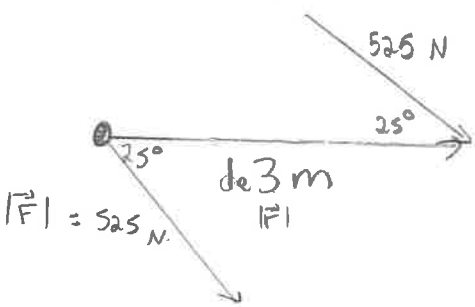
$V = |[1, 0, -4] \cdot [1, 50, 38]|$

$V = |1(1) + 0(50) + (-4)(38)|$

$V = |-151|$

$V = 151 \text{ units}^3$

(21)



$T = \vec{r} \times \vec{F}$

$T = |\vec{r}||\vec{F}|\sin(25°)(-\hat{n})$

$T = 1.3(525)\sin(25°)(-\hat{n})$

$T = -288.44\hat{n} \text{ N}\cdot\text{m}$

288.44 N·m into the material.

22

$\hat{p} = \frac{1}{|\vec{p}|}\vec{p}$

$= \frac{1}{\sqrt{77}}[-8, 2, -3]$

$= \left[\frac{-8}{\sqrt{77}}, \frac{2}{\sqrt{77}}, \frac{-3}{\sqrt{77}}\right]$

23 $\vec{a} = [3, 4, -2]$, $\vec{b} = [2, -7, 1]$, $\vec{c} = [-6, 5, 4]$

a) $\vec{a} + \vec{c} \times \vec{b}$

$= [3, 4, -2] + [33, 14, 32]$

$= [36, 18, 30]$

$\vec{c} \times \vec{b} = [5-(-28), 8-(-6), 42-10]$

$\begin{matrix} 5 & -7 \\ 4 & 1 \\ -6 & 2 \\ 5 & -7 \end{matrix}$

$= [33, 14, 32]$

b) $(\vec{a} + \vec{c}) \times \vec{b} = [9-(-14), 4-(-3), 21-18]$

$\begin{matrix} 9 & -7 \\ 2 & 1 \\ -3 & 2 \\ 9 & -7 \end{matrix}$

$= [23, 7, 3]$

$\vec{a} + \vec{c} = [-3, 9, 2]$

c) $-2\vec{b} = [-4, 14, -2]$

$-2\vec{b} \times \vec{c} = [56-(-10), 12-(-16), -20-(-84)]$

$\begin{matrix} 14 & 5 \\ -2 & 4 \\ -4 & -6 \\ 14 & 5 \end{matrix}$

$= [66, 28, 64]$

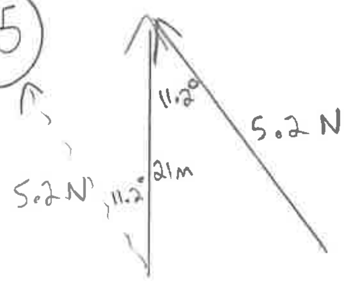
$|-2\vec{b} \times \vec{c}| = \sqrt{9236} = 2\sqrt{2309}$

(24)

$W = 7(43)\cos(38) + 3.5(43)\cos(14°) + 6(43)\cos(38°)$

$W \approx 586.53\ \text{J}$

(25)



$W = \vec{F} \cdot \vec{d}$

$= 5.2(21)\cos(11.2°)$

$\approx 107.12\ \text{N}\cdot\text{m}$

UNIT 6 - Lines & Planes

① a) $[x, y] = [4, -5] + t[3, 5]$ $l: \begin{cases} x = 4 + 3t \\ y = -5 + 5t \end{cases}$

b) $[x, y, z] = [-6, 2, 1] + t[3, 7, -2]$ $l: \begin{cases} x = -6 + 3t \\ y = 2 + 7t \\ z = 1 - 2t \end{cases}$

c) $[x, y, z] = [0, 1, 2] + t[1, 0, 0]$ $l: \begin{cases} x = t \\ y = 1 \\ z = 2 \end{cases}$

d) $\vec{m} = \vec{AB} = [-1, -2, -2] = -1[1, 2, 2]$

$[x, y, z] = [4, -5, 3] + t[1, 2, 2]$ $l: \begin{cases} x = 4 + t \\ y = -5 + 2t \\ z = 3 + 2t \end{cases}$

e) $\vec{m} = \vec{AB} = [8, 5, -2]$

$[x, y, z] = [1, -3, 2] + t[8, 5, -2]$ $l: \begin{cases} x = 1 + 8t \\ y = -3 + 5t \\ z = 2 - 2t \end{cases}$

f) $[x, y, z] = [1, 3, 5] + t[0, 1, 0]$ $l: \begin{cases} x = 1 \\ y = 3 + t \\ z = 5 \end{cases}$

(2)

$$l: \begin{cases} x = -2+3t \\ y = 3-2t \\ z = 7+5t \end{cases}$$

x	y	z
$10 = -2+3t$	$-5 = 3-2t$	$22 = 7+5t$
$t=4$	$t=4$	$t=3$

$\therefore$ the point is NOT on the line.

(3) B & D are both parallel

B

check if $(4,-1)$ is on l_2

$$l_2: \begin{cases} x = -2-3t \\ y = 9+5t \end{cases}$$

$4 = -2-3t$
$t=-2$

$-1 = 9+5t$
$t = -2$

$\therefore (4,-1)$ is on l_2

$\therefore$ the lines are coincident

(4)

a) $l: \begin{cases} x = 4+t \\ y = -3+5t \end{cases}$

b) when $t=2$:

$x = 4+2 = 6$

$y = -3+5(2) = 7$

$(6,7)$

c)

x	y
$3 = 4+t$	$-7 = -3+5t$
$t=-1$	$t = -\frac{4}{5}$

$\therefore$ Not on line.

(5) a) parallel if $\vec{m}_1 = k\vec{m}_2$

$-7 = ka$
$-7 = \frac{2}{9}a$
$\boxed{a = \frac{-63}{2}}$

$2 = 9k$
$k = \frac{2}{9}$

b) perpendicular if $\vec{m}_1 \cdot \vec{m}_2 = 0$

$-7a + 2(9) = 0$

$\boxed{a = \frac{18}{7}}$

6 $\vec{n} = [6,-7]$

$\vec{m} = [7,6]$

$P(9,-1)$

$$[x,y] = [9,-1] + t[7,6]$$

7 a) $\vec{m}_1 = \vec{DE} = [3,-7,-3]$

$\vec{m}_2 = \vec{DF} = [0,-5,1]$

V:

$$[x,y,z] = [1,7,2] + s[3,-7,-3] + t[0,-5,1]$$

P: $\pi : \begin{cases} x = 1 + 3s \\ y = 7 - 7s - 5t \\ z = 2 - 3s + t \end{cases}$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-22,-3,-15]$

$$\begin{array}{cc} -7 & -5 \\ -3 & 1 \\ 3 & 0 \\ -7 & -5 \end{array}$$

$= -1[22,3,15]$

$Ax + By + Cz + D = 0$

$22x + 3y + 15z + D = 0$

$22(1) + 3(7) + 15(2) + D = 0$

$D = -73$

S: $22x + 3y + 15z - 73 = 0$

b) $\vec{m}_1 = [1,0,0]$

$\vec{m}_2 = [0,0,1]$

V: $[x,y,z] = [2,-3,4] + s[1,0,0] + t[0,0,1]$

P: $\pi : \begin{cases} x = 2 + s \\ y = -3 \\ z = 4 + t \end{cases}$

S: $y = -3$

c) v: $[x,y,z] = [3,-5,1] + s[3,-1,1] + t[1,1,1]$

p:

$$\pi : \begin{cases} x = 3 - s + t \\ y = -5 - s + t \\ z = 1 + s + t \end{cases}$$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-2,-2,4]$

$= -2[1,1,-2]$

-1	1
1	1
3	1
-1	1

$Ax + By + Cz + D = 0$

$1x + 1y - 2z + D = 0$

$1(3) + 1(-5) - 2(1) + D = 0$

$D = 4$

s: $x + y - 2z + 4 = 0$

8 a) $2x + 4y - 6z = 24$

x-int

$2x = 24$

$x = 12$

$(12,0,0)$

y-int

$4y = 24$

$y = 6$

$(0,6,0)$

z-int

$-6z = 24$

$z = -4$

$(0,0,-4)$

b) $\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-18,-30,-3]$

$= -3[6,10,1]$

-1	-3
-2	12
2	3
-1	-3

$6x + 10y + z + D = 0$

$6(12) + 10(-9) + 4 + D = 0$

$D = 14$

$6x + 10y + z + 14 = 0$

x-int

$6x + 14 = 0$

$x = \frac{-7}{3}$

$(\frac{-7}{3},0,0)$

y-int

$10y + 14 = 0$

$y = \frac{-7}{5}$

$(0,-7/5,0)$

z-int

$z + 14 = 0$

$z = -14$

$(0,0,-14)$

c) $7x - 3z + 42 = 0$

$\underline{x\text{-int}}$	$\underline{y\text{-int}}$	$\underline{z\text{-int}}$
$7x + 42 = 0$	none	$-3z + 42 = 0$
$x = -6$		$z = 14$
$(-6, 0, 0)$		$(0, 0, 14)$

9) $\vec{n} = \vec{m}_1 \times \vec{m}_2 = [13, -9, -38]$

$$\begin{matrix} 3 & -4 \\ 1 & 3 \\ 5 & 6 \\ 3 & -4 \end{matrix}$$

$13x - 9y - 38z + D = 0$

$13(2) - 9(1) - 38(-3) + D = 0$

$D = -131$

$13x - 9y - 38z - 131 = 0$

$\underline{LS}$

$= 13(3) - 9(-6) - 38(-1) - 131$

$= 0$

$\underline{RS}$

$= 0$

$\therefore P(3, -6, -1)$ IS on the plane.

10) $3x - 2y + z = 4$

$\underline{LS}$

$= 3(1) - 2(3) + 7$

$= 4$

$\underline{RS}$

$= 4$

$\therefore A(1, 3, 7)$ IS on the plane

(11)

$2x - 4y + 3z + D = 0$

$2(3) - 4(-5) + 3(1) + D = 0$

$D = -29$

$\boxed{2x - 4y + 3z - 29 = 0}$

(12)

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-25, -7, -5]$

5	0
3	-5
-2	1
5	0

$= -1[25, 7, 5]$

$25x + 7y + 5z + D = 0$

$25(2) + 7(1) + 5(4) + D = 0$

$D = -77$

$\boxed{25x + 7y + 5z - 77 = 0}$

(13)

$\vec{m}_1 = \vec{AB} = [1,1,1]$

$\vec{m}_2 = \vec{AC} = [3,3,2]$

$\vec{n} = \vec{m}_1 \times \vec{m}_2 = [-1, 1, 0]$

1	3
1	2
1	3
1	3

$= -1[1, -1, 0]$

$x - y + 0z + D = 0$

$1 - 2 + 0(3) + D = 0$

$D = 1$

$\boxed{x - y + 1 = 0}$

14 a) $l_1: \begin{cases} x = -1+t \\ y = -4-t \end{cases}$ $l_2: \begin{cases} x = 3-2s \\ y = -1+3s \end{cases}$

Not parallel ($\vec{m}_1 \neq k\vec{m}_2$)

$\therefore$ they intersect

$-1+t = 3-2s$ $-4-t = -1+3s$

① $t+2s = 4$ ② $-t-3s = 3$

① $t+2s = 4$
② $-t-3s = 3$ +

$-s = 7$
$s = -7$

POI:

$x = 3-2(-7) = 17$

$y = -1+3(-7) = -22$

$(17, -22)$

b) $l_1: \begin{cases} x = 3-t \\ y = -2+t \\ z = 3+2t \end{cases}$ $l_2: \begin{cases} x = 1+s \\ y = -1+s \\ z = 4+4s \end{cases}$

Not parallel ($\vec{m}_1 \neq k\vec{m}_2$)
& intersect at a point
OR are skewed.

$3-t = 1+s$ $-2+t = -1+s$ $3+2t = 4+4s$

① $2 = s+t$ ② $-1 = s-t$ ③ $-1 = 4s-2t$

① $2 = s+t$
② $-1 = s-t$ −

$3 = 2t$

$t = \frac{3}{2}$

Check if $t = \frac{3}{2}$ and $s = \frac{1}{2}$ satisfy ③

$2 = s + \frac{3}{2}$

$s = \frac{1}{2}$

LS $= -1$

RS $= 4(\frac{1}{2}) - 2(\frac{3}{2})$
$= 2-3$
$= -1$

$\therefore$ intersect at a point.

POI:

$x = 1+\frac{1}{2} = \frac{3}{2}$

$y = -1+\frac{1}{2} = -\frac{1}{2}$

$z = 4+4(\frac{1}{2}) = 6$

$(\frac{3}{2}, \frac{-1}{2}, 6)$

c) $l_1: \begin{cases} x = 3+3t \\ y = -3-t \\ z = t \end{cases}$ $l_2: \begin{cases} x = 4-s \\ y = s \\ z = 4+s \end{cases}$

Not parallel; either intersect at a point OR are skewed.

$3+3t = 4-s$ $\quad -3-t = s \quad$ $t = 4+s$

① $3t+s = 1$ $\quad$ ② $-t-s=3$ $\quad$ ③ $t-s=4$

$$\begin{array}{rl} ① & 3t+s=1 \\ ② & -t-s=3 \quad + \\ \hline & 2t = 4 \\ & t = 2 \end{array}$$

Check if $t=2$ and $s=-5$ satisfy ③

$-2-s=3$

$s=-5$

$\underline{LS}$
$= 2-(-5)$
$= 7$

$\underline{RS}$
$= 4$

$LS \neq RS$

$\therefore$ The lines are skewed; no intersection

d) $l_1: \begin{cases} x = 1+3t \\ y = 5t \\ z = 4t-3 \end{cases}$ $l_2: \begin{cases} x = -s \\ y = -9+2s \\ z = -1-3s \end{cases}$

$1+3t = -s$ $\quad 5t = -9+2s \quad$ $4t-3 = -1-3s$

① $3t+s=-1$ $\quad$ ② $5t-2s=-9$ $\quad$ ③ $4t+3s=2$

$$\begin{array}{rl} 2\times① & 6t+2s=-2 \\ ② & 5t-2s=-9 \quad + \\ \hline & 11t = -11 \\ & t = -1 \end{array}$$

Check if $t=-1$ and $s=2$ satisfy ③

$3(-1)+s=-1$

$s=2$

$\underline{LS}$
$= 4(-1)+3(2)$
$= 2$

$\underline{RS}$
$= 2$

$\therefore$ intersect at a point

$\underline{\text{POI:}}$

$x=-2$

$y=-9+2(2)=-5$

$z=-1-3(2)=-7$

$(-2,-5,-7)$

e) $2\vec{m}_1 = \vec{m}_2$; $\therefore$ parallel. Check if $(3,-3,0)$ is on l_2

$x = 6+6s$ $\quad$ $y = -4-2s$ $\quad$ $z = 1+2s$

$3 = 6+6s$ $\quad$ $-3 = -4-2s$ $\quad$ $0 = 1+2s$

$s = \frac{-1}{2}$ $\quad$ $s = -\frac{1}{2}$ $\quad$ $s = -\frac{1}{2}$

15

$l: \begin{cases} x = 8+3t \\ y = -1 \\ z = 4-t \end{cases}$ $\qquad \pi: x+y-z=6$

$8+3t+(-1)-(4-t) = 6$

$3+4t = 6$

$t = \frac{3}{4}$

POI:

$x = 8+3\left(\frac{3}{4}\right) = \frac{41}{4}$

$y = -1$

$z = 4 - \frac{3}{4} = \frac{13}{4}$

$\left(\frac{41}{4}, -1, \frac{13}{4}\right)$

b) $l: \begin{cases} x = 4-t \\ y = 2-2t \\ z = 5-2t \end{cases}$ $\qquad \pi: 2x+3y-4z=-7$

$2(4-t)+3(2-2t)-4(5-2t) = -7$

$8-2t+6-6t-20+8t = -7$

$-6+0t = -7$

$0t = -1$

$\therefore$ no intersection ; parallel and distinct.

c) $l: \begin{cases} x = 1+t \\ y = 13+t \\ z = 2-t \end{cases}$ $\pi: 3x - y + 2z + 6 = 0$

$3(1+t) - (13+t) + 2(2-t) + 6 = 0$

$3 + 3t - 13 - t + 4 - 2t + 6 = 0$

$0t = 0$

$\therefore$ infinite solutions; the line lies in the plane.

d) $l: \begin{cases} x = 3+2t \\ y = -3+3t \\ z = -3+2t \end{cases}$ $\pi: 4x - 2y + z = 19$

$4(3+2t) - 2(-3+3t) + (-3+2t) = 19$

$12 + 8t + 6 - 6t - 3 + 2t = 19$

$15 + 4t = 19$

$4t = 4$

$t = 1$

POI:

$x = 3 + 2(1) = 5$

$y = -3 + 3(1) = 0$

$z = -3 + 2(1) = -1$

$(5, 0, -1)$

16 a) $\pi_1: 3x + 2y + 5z = 4$

$\pi_2: 4x - 3y + z = -1$

$\vec{n}_1 \neq k\vec{n}_2$; $\therefore$ intersect in a line.

① $3x + 2y + 5z = 4$

$5 \times$ ② $20x - 15y + 5z = -5$ −

$-17x + 17y = 9$

Let $x = t$

$-17t + 17y = 9$

$17y = 9 + 17t$

$y = \frac{9}{17} + t$

$3t + 2(\frac{9}{17} + t) + 5z = 4$

$3t + \frac{18}{17} + 2t + 5z = 4$

$5t + 5z = \frac{50}{17}$

$5z = \frac{50}{17} - 5t$

$z = \frac{10}{17} - t$

Line of intersection:

$[x, y, z] = [0, \frac{9}{17}, \frac{10}{17}] + t[1, 1, -1]$

b) $2\vec{n}_1 = \vec{n}_2$; $\therefore$ parallel

$\pi_1 \neq k\pi_2$; $\therefore$ parallel and distinct.

No solutions.

Algebraic proof:

$$\begin{array}{rl} 2\times\pi_1: & 6x+4y+10z=8 \\ \pi_2: & 6x+4y+10z=3 \quad - \\ \hline & 0=5 \end{array}$$

c) $\pi_1: 4x+2y+3z=5$

$\pi_2: x-6y+z=14$

$\pi_3: -5x-14y+2z=11$

$$\begin{array}{rl} (1) & 4x+2y+3z=5 \\ 3\times(2) & 3x-18y+3z=42 \quad - \\ \hline (4) & x+20y=-37 \end{array}$$

$$\begin{array}{rl} 2\times(2) & 2x-12y+2z=28 \\ (3) & -5x-14y+2z=11 \quad - \\ \hline (5) & 7x+2y=17 \end{array}$$

$$\begin{array}{rl} 7\times(4) & 7x+140y=-259 \\ (5) & 7x+2y=17 \quad - \\ \hline & 138y=-276 \\ & y=-2 \end{array}$$

$4(3)+2(-2)+3z=5$

$3z=-3$

$z=-1$

$7x+2(-2)=17$

$7x=21$

$x=3$

POI is $(3,-2,-1)$

d) $\pi_1: 2x+y-z=3$

$\pi_2: x+y=2$

$\pi_3: x-z=1$

$$\begin{array}{rl} ① & 2x+y-z=3 \\ 2\times② & 2x+2y \quad\;\; =4 \quad - \\ \hline & -y-z=-1 \end{array}$$

$$\begin{array}{rl} ② & x+y \quad\; =2 \\ ③ & x \quad\; -z=1 \quad - \\ \hline ⑤ & y+z=1 \end{array}$$

$$\begin{array}{rl} ④ & y+z=1 \\ ⑤ & y+z=1 \quad - \\ \hline & 0=0 \end{array}$$

$\therefore$ intersect in a line.

Let $y=t$

$t+z=1$

$z=1-t$

$x+t=2$

$x=2-t$

Line of intersection:

$$[x,y,z]=[2,0,1]+t[-1,1,-1]$$

e) $\pi_1: x+3y+3z=8$

$\pi_2: x-y+3z=4$

$\pi_3: 2x+6y+6z=16$

* Notice $2\pi_1=\pi_3$

$\therefore$ could solve by find intersection of π_1 and π_2 OR π_3 and π_2

$$\begin{array}{rl} ① & x+3y+3z=8 \\ ② & x-y+3z=4 \quad - \\ \hline & 4y=4 \\ ④ & y=1 \end{array}$$

$$\begin{array}{rl} 2\times② & 2x-2y+6z=8 \\ ③ & 2x+6y+6z=16 \quad - \\ \hline & -8y=-8 \\ ⑤ & y=1 \end{array}$$

$$\begin{array}{rl} ④ & y=1 \\ ⑤ & y=1 \quad - \\ \hline & 0=0 \end{array}$$

$\therefore$ intersect in a line.

Let $x=t$

$t+3(1)+3z=8$

$t+3z=5$

$3z=5-t$

$z=\frac{5}{3}-\frac{1}{3}t$

Line of intersection:

$$[x,y,z]=[0,1,\tfrac{5}{3}]+t[1,0,-\tfrac{1}{3}]$$

(18) $\vec{n} = [2, 3, -4]$

$P(3, -4, 6)$

$Q(0, 0, -2)$

$\vec{PQ} = [-3, 4, -8]$

point on plane: $2(0) + 3(0) - 4z = 8$

$z = -2$

$(0, 0, -2)$

$$d = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|} = \frac{|-3(2) + 4(3) + (-8)(-4)|}{\sqrt{29}} = \frac{38}{\sqrt{29}} \text{ units.}$$

f) $\pi_1: 3x + 5y - 2z = 8$

$\pi_2: -2x - 3y + 4z = 2$

$\pi_3: 7x + 11y - 10z = 21$

$$\begin{array}{lrl} 2\times① & 6x + 10y - 4z &= 16 \\ ② & -2x - 3y + 4z &= 2 \quad + \\ \hline ④ & 4x + 7y &= 18 \end{array}$$

$$\begin{array}{lrl} 5\times① & 15x + 25y - 10z &= 40 \\ ③ & 7x + 11y - 10z &= 21 \quad - \\ \hline ⑤ & 8x + 14y &= 19 \end{array}$$

$$\begin{array}{lrl} 2\times④ & 8x + 14y &= 36 \\ ⑤ & 8x + 14y &= 19 \quad - \\ \hline & 0 &= 17 \end{array}$$

$\therefore$ inconsistent system; intersects in pairs.

g) $\pi_1: x + 3y - 5z = -1$

$\pi_2: 3x + 9y - 15z = -3$

$\pi_3: 5x + 15y - 25z = 5$

Notice all normals are parallel.

$3\pi_1 = \pi_2$; $\therefore$ parallel & coincident

$\pi_1 \neq k\pi_3$; $\therefore$ parallel & distinct

$\therefore$ This is an inconsistent system.

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$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$\vec{n}_1 = [3, -6, -2]$

$\vec{n}_2 = [2, 1, -2]$

$$\cos\theta = \frac{3(2) + (-6)(1) + (-2)(-2)}{(\sqrt{49})(\sqrt{9})}$$

$\theta \simeq 79.02°$