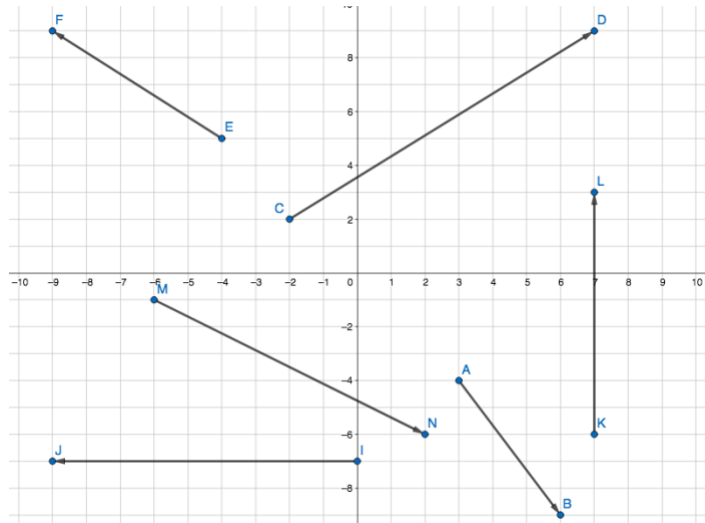## Unit 5 – Cartesian Vectors

1) Write the coordinates of each Cartesian vector.
Then find its magnitude.

2) Given $\vec{u}$ = [2, −1] and $\vec{v}$ = [5, −7], evaluate each expression.

a) $2\vec{v}$
b) $\vec{u} - \vec{v}$
c) $3\vec{u} + 5\vec{v}$
d) $4\vec{u} - 2\vec{v}$

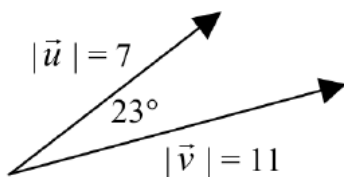**3)** Consider the points $A(2, -7)$, $B(-4, 5)$ and $C(6, 8)$.
a) Determine $\overrightarrow{AC}$
b) Determine $\left|\overrightarrow{AB}\right|$
c) Determine the perimeter of $\triangle ABC$

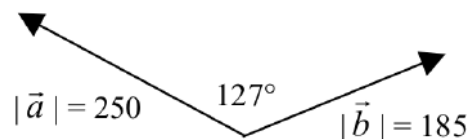**4)** Determine the value of k so that the vectors in each pair are collinear.
a) $\vec{u} = [2, k], \vec{v} = [-12, 30]$
b) $\vec{u} = [-4, 6], \vec{v} = [30, k]$

5) Calculate the dot product of each pair of vectors.

a)

b)

c) $\vec{u} = [-8, 9]$ and $\vec{v} = [1, -7]$

d) $\vec{u} = -5\hat{\imath} + 11\hat{\jmath}$ and $\vec{v} = 7\hat{\imath} - 2\hat{\jmath}$

6) Write a 250 N force applied at $60°$ to the horizontal as a Cartesian vector.

7) Evaluate, given vectors $\vec{u}$ = [2, –3] and $\vec{v}$ = [3, 1].
a) $3\vec{u} + 2\vec{v}$
b) $|\vec{u}|$
c) $\vec{u} \cdot \vec{v}$
d) $|\vec{u} + \vec{v}|$

8) Calculate the angle θ between the vectors.
a) $\vec{u}$ = [–1, 8] and $\vec{v}$ = [3, –5]
b) $\vec{u}$ = [3, –7] and $\vec{v}$ = [6, –1]

9) Determine the projection of $\vec{u}$ on $\vec{v}$.

a) $|\vec{u}|$ = 95 and $|\vec{v}|$ = 275, and the angle between $\vec{u}$ and $\vec{v}$ is 13°.
b) $\vec{u}$ = [6, –5] and $\vec{v}$ = [7, –11]

10) For vectors $\vec{c} = [2, -7]$ and $\vec{d} = [9,12]$
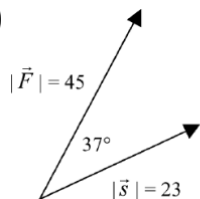**a)** determine the magnitude of the projection of $\vec{c}$ on $\vec{d}$
b) determine the vector projection of $\vec{c}$ on $\vec{d}$

11) Determine the work done by each $\vec{F}$, in Newtons, for an object moved along the vector, $\vec{d}$, in metres.
a) $\vec{F}$ = [18, 23] and $\vec{d}$ = [9, 12]
b)

$|\vec{F}| = 45$

$37°$

$|\vec{s}| = 23$

12) Determine if ΔPQR with vertices $P(-5, 3)$, $Q(-7, 8)$ and $R(3, 12)$ is a right triangle. If so, identify the right angle.
13) Determine the exact magnitude of the vector $\overrightarrow{AB}$ joining A(3, 8, –11) and B(7, –3, 15).

14) For each of the following calculate the angle between the two vectors.
a) [0, 1, –2] and [–3, 1, 4]
b) [1.5, 20, 0] and [–20, 1, 10]

15) Given $\vec{c}$ = [–6, 4, 0], $\vec{d}$ = [0, –5, –7], and $\vec{e}$ = [3, 1, 2].
a) $10\vec{c} - 10\vec{d}$
b) $2\vec{c} \bullet 3\vec{d} - 2\vec{c} \bullet 4\vec{e}$
c) $(3\vec{d} - 4\vec{e}) \bullet 2\vec{c}$
d) $\vec{c} \times \vec{d}$
e) $\vec{d} \times \vec{e}$
f) $\vec{c} \times \vec{c}$

16) If $\vec{u}$ = [5, 7, 9] is orthogonal to $\vec{v}$ = [1, t, 1], determine t.

17) Determine the area of the parallelogram bound by the vectors $\vec{a}$ = [7, 11, 4] and $\vec{b}$ = [3, –2, 5].

18) A triangle has vertices A(2, 5, –1), B(4, 1, –3), and C(–5, –6, 4). Find the area of the triangle.

19) Calculate the area of the parallelogram ABCD where A(4, 6), B(5, 6), C(11, 15), and D(–2, –3)

20) Find the volume of the parallelepiped defined by $\vec{a}$ = [1, 0, –4], $\vec{b}$ = [8, –7, 9], and $\vec{c}$ = [2, 3, –4]

21) A 525 N force is applied to the end of a 130 cm wrench and makes an angle of 25° with the handle of the wrench. What is the torque on the bolt at the other end of the wrench if the force is applied in a clockwise direction?

22) Find a unit vector parallel to $\vec{p} = [-8, 2, -3]$

23) Given $\vec{a} = [3, 4, -2]$, $\vec{b} = [2, -7, 1]$, and $\vec{c} = [-6, 5, 4]$, evaluate each expression

a) $\vec{a} + \vec{c} \times \vec{b}$
b) $(\vec{a} + \vec{c}) \times \vec{b}$
c) $\left| -2\vec{b} \times \vec{c} \right|$

24) A storage box is dragged 7 m along a smooth, level floor by a 43-N force, applied at 38° to the floor. Then, it is pulled 3.5 m up a ramp inclined at 24° to the horizontal, using the same force. The storage box is then dragged a further 6 m along a level platform using the same force again. Determine the total work done in moving the storage box.

25) A rock climber pulls himself 21 m up the side of a cliff with a force of 5.2 N, at an angle of 11.2° to the vertical. What is the work done?

Answers:

1) $\vec{AB} = [3, -5], |\vec{AB}| = \sqrt{34}; \vec{CD} = [9,7], |\vec{CD}| = \sqrt{130}; \vec{EF} = [-5,4], |\vec{EF}| = \sqrt{41}; \vec{IJ} = [-9,0], |\vec{IJ}| = 9; \vec{KL} = [0,9], |\vec{KL}| = 9; \vec{MN} = [8,-5], |\vec{MN}| = \sqrt{89}$

2)a) [10, −14] **b)** [−3, 6] **c)** [31, −38] **d)** [−2, 10]

3)a) [4, 15] **b)** $6\sqrt{5}\ units$ **c)** 39.4 units

4)a) −5   **b)** −45

5) a) 70.88 **b)** −27 833.94 **c)** −71 **d)** −57

6) [125, 216.5]

7) a) [12, −7] **b)** $\sqrt{13}$ **c)** 3 **d)** $\sqrt{29}$

8)a) θ = 156.2° **b)** θ = 57°

9)a) $92.57\hat{v}$ **b)** $\frac{97}{170}[7, -11]$

10)a) 4.4 **b)** $\left[-\frac{66}{25}, -\frac{88}{25}\right]$

11)a) 438 J **b)** 826.59 J

12) Yes; angle Q

13) $\sqrt{813}$

14)a) 127.9° **b)** 91.3°

15)a) [−60, 90, 70] **b)** −8 **c)** −8 **d)** [−28, −42, 30] **e)** [−3, −21, 15] **f)** $\vec{0}$

16) $t = -2$

17) $\sqrt{6707}\ units^2$

18) $\sqrt{1070}\ units^2$

19) 9 units²

20) 151 $units^3$

21) 288.4 Nm (in to the material; bolt is being tightened)

22) $\left[-\frac{8}{\sqrt{77}}, \frac{2}{\sqrt{77}}, -\frac{3}{\sqrt{77}}\right]$

23)a) [36,18,30] **b)** [23,7,3] **c)** $2\sqrt{2309}$

24) 586.5 J

25) 107 J