Unit 6 – Lines and Planes

1) Write the vector and parametric equations of each line.

a) $\vec{m} = [3,5]$, $P(4,-5)$
b) $\vec{m} = [3,7,-2], P(-6,2,1)$
c) perpendicular to the yz-plane and through $(0,1,2)$
d) through the points $A(4,-5,3)$ and $B(3,-7,1)$
e) through the points $A(1,-3,2)$ and $B(9,2,0)$
f) parallel to the *y*-axis and containing the point $(1,3,5)$

2) A line is defined by the equation $[x,y,z] = [-2,3,7] + t[3,-2,5]$. Write the parametric equations for the line and determine if it contains the point $(10,-5,22)$.

3) Which pair of lines represented by vector equations are coincident?

A $[x,y] = [-2,5] + s[2,-1]$
$[x,y] = [12,-30] + t[5,-7]$

B $[x,y] = [4,-1] + s[-3,5]$
$[x,y] = [-2,9] + t[-3,5]$

C $[x,y] = [0,0] + s[1,1]$
$[x,y] = [-1,1] + t[1,-1]$

D $[x,y] = [5,4] + s[2,-4]$
$[x,y] = [9,-8] + t[2,-4]$

4) A line passes through the point $(4,-3)$ with direction vector $\vec{m} = [1,5]$

a) Determine the parametric equations of the line.
b) What point on the line corresponds to the parameter value $t = 2$?
c) Does the line contain the point P(3, –7)?

5) Consider the lines

$\ell_1: [x,y] = [1,4] + t[-7,2]$
$\ell_2: [x,y] = [-2,6] + t[a,9]$

a) For what value of a are the lines parallel?
b) For what value of a are the lines perpendicular?

6) Write a vector equation of the line through $P(9,-1)$ and perpendicular to $[x,y] = [1,-2] + t[6,-7]$

7) Write the vector, parametric, and scalar equations of each plane.

a) contains the points $D(1,7,2)$, $E(4,0,-1)$, and $F(1,2,3)$
b) parallel to the xz-plane and through the point $Q(2,-3,4)$
c) contains the point $(3,-5,1)$ and is parallel to $[x,y,z] = [-5,2,-5] + t[3,-1,1] + s[1,1,1]$.

8) Determine the x, y, and z-intercepts of each plane

a) $2x + 4y - 6z = 24$
b) $[x,y,z] = [12,-9,4] + s[2,-1,-2] + t[3,-3,12]$
c) $7x - 3z + 42 = 0$

9) Determine if the point $P(3,-6,-1)$ is contained in the plane $[x,y,z] = [2,1,-3] + s[5,3,1] + t[6,-4,3]$

10) Determine if the point $A(1,3,7)$ lies on the plane $3x - 2y + z = 4$.

11) Write the scalar equation of the plane with $\vec{n} = [2,-4,3]$ that contains the point $R(3,-5,1)$.

12) Write the scalar equation of this plane $[x,y,z] = [2,1,4] + t[-2,5,3] + s[1,0,-5]$

13) Write the scalar equation of the plane that contains the points $A(1,2,3), B(2,3,4)$, and $C(4,5,5)$

14) Determine if the lines in each pair intersect. If they intersect, find the intersection point. If they don't, explain why.

a) $\ell_1: [x,y] = [-1,-4] + t[1,-1]$ $\qquad \ell_2: \begin{cases} x = 3 - 2s \\ y = -1 + 3s \end{cases}$

b) $\ell_1: [x,y,z] = [3,-2,3] + t[-1,1,2]$ $\qquad \ell_2: [x,y,z] = [1,-1,4] + s[1,1,4]$

c) $\ell_1: [x,y,z] = [3,-3,0] + t[3,-1,1]$ $\qquad \ell_2: [x,y,z] = [4,0,4] + s[-1,1,1]$

d) $\ell_1: x = 1 + 3t,\ y = 5t,\ z = 4t - 3$ $\qquad \ell_2: [x,y,z] = [0,-9,-1] + s[-1,2,-3]$

e) $\ell_1: [x,y,z] = [3,-3,0] + t[3,-1,1]$ $\qquad \ell_2: [x,y,z] = [6,-4,1] + s[6,-2,2]$

15) Determine if each line intersects the plane. If so, state the solution.

a) $\ell: [x,y,z] = [8,-1,4] + t[3,0,-1]$ $\qquad \pi: x + y - z = 6$
b) $\ell: [x,y,z] = [4,2,5] + t[-1,-2,-2]$ $\qquad \pi: 2x + 3y - 4z = -7$
c) $\ell: [x,y,z] = [1,13,2] + t[1,1,-1]$ $\qquad \pi: 3x - y + 2z + 6 = 0$
d) $\ell: x = 3 + 2t, y = -3 + 3t, z = -3 + 2t$ $\qquad \pi: 4x - 2y + z = 19$

16) State the intersections of each of the following systems of equations. If there are no solutions, make sure you clearly indicate how you know and describe the scenario that is resulting in no solutions.

a) $\pi_1: 3x + 2y + 5z = 4$
$\pi_2: 4x - 3y + z = -1$

b) $\pi_1: 3x + 2y + 5z = 4$
$\pi_2: 6x + 4y + 10z = 3$

c) $\pi_1: 4x + 2y + 3z = 5$
$\pi_2: x - 6y + z = 14$
$\pi_3: -5x - 14y + 2z = 11$

d) $\pi_1: 2x + y - z = 3$
$\pi_2: x + y = 2$
$\pi_3: x - z = 1$

e) $\pi_1: x + 3y + 3z = 8$
$\pi_2: x - y + 3z = 4$
$\pi_3: 2x + 6y + 6z = 16$

f) $\pi_1: 3x + 5y - 2z = 8$
$\pi_2: -2x - 3y + 4z = 2$
$\pi_3: 7x + 11y - 10z = 21$

g) $\pi_1: x + 3y - 5z = -1$
$\pi_2: 3x + 9y - 15z = -3$
$\pi_3: 5x + 15y - 25z = 5$

17) Find the angle between

$\pi_1: 3x - 6y - 2z = 15$
$\pi_2: 2x + y - 2z = 5$

18) Determine the shortest distance between the point $P(3, -4, 6)$ and the plane $2x + 3y - 4z = 8$

Answers

1)a) $[x, y] = [4, -5] + t[3,5]$, $\ell\colon \begin{cases} 4+3t \\ y=-5+5t \end{cases}$

b) $[x, y, z] = [-6, 2, 1] + t[3, 7, -2]$, $\ell\colon \begin{cases} x=-6+3t \\ y=2+7t \\ z=1-2t \end{cases}$

c) $[x, y, z] = [0, 1, 2] + t[1, 0, 0]$, $\ell\colon \begin{cases} x=t \\ y=1 \\ z=2 \end{cases}$

d) $[x, y, z] = [4, -5, 3] + t[1, 2, 2]$, $\ell\colon \begin{cases} x=4+t \\ y=-5+2t \\ z=3+2t \end{cases}$

e) $[x, y, z] = [1, -3, 2] + t[8, 5, -2]$, $\ell\colon \begin{cases} x=1+8t \\ y=-3+5t \\ z=2-2t \end{cases}$

f) $[x, y, z] = [1, 3, 5] + t[0, 1, 0]$, $\ell\colon \begin{cases} x=1 \\ y=3+t \\ z=5 \end{cases}$

2) $\ell\colon \begin{cases} x=-2+3t \\ y=3-2t \\ z=7+5t \end{cases}$; the point is NOT on the line

3) B

4)a) $\ell\colon \begin{cases} x=4+t \\ y=-3+5t \end{cases}$ **b)** (6, 7) **c)** no

5)a) $-\frac{63}{2}$ **b)** $\frac{18}{7}$

6) $[x, y] = [9, -1] + t[7,6]$

7)a) $[x, y, z] = [1, 7, 2] + t[3, -7, -3] + s[0, -5, 1]$, $\pi\colon \begin{cases} x=1+3t \\ y=7-7t-5s \\ z=2-3t+s \end{cases}$; $22x+3y+15z-73=0$

b) $[x, y, z] = [2, -3, 4] + t[1, 0, 0] + s[0, 0, 1]$, $\pi\colon \begin{cases} x=2+t \\ y=-3 \\ z=4+s \end{cases}$; $y=-3$

c) $[x, y, z] = [3, -5, 1] + t[3, -1, 1] + s[1, 1, 1]$, $\pi\colon \begin{cases} x=3+3t+s \\ y=-5-t+s \\ z=1+t+s \end{cases}$; $x+y-2z+4=0$

8)a) $x_i = 12, y_i = 6, z_i = -4$ **b)** $x_i = -\frac{7}{3}, y_i = -\frac{7}{5}, z_i = -14$ **c)** $x_i = -6, y_i = none, z_i = 14$

9) Yes

10) Yes

11) $2x - 4y + 3z - 29 = 0$

12) $25x + 7y + 5z - 77 = 0$

13) $x - y + 1 = 0$

14)a) (17, −22) **b)** $\left(\frac{3}{2}, -\frac{1}{2}, 6\right)$ **c)** skewed lines, no intersection **d)** (−2, −5, −7) **e)** infinite solutions; parallel and coincident

15)a) $(\frac{41}{4}, -1, \frac{13}{4})$ **b)** do not intersect; parallel and distinct **c)** infinite intersections; parallel and coincident **d)** $(5, 0, -1)$

16)a) $[x, y, z] = \left[\frac{10}{17}, \frac{19}{17}, 0\right] + t[-1, -1, 1]$ **b)** no solutions; parallel and distinct **c)** $(3, -2, -1)$
d) $[x, y, z] = [1,1,0] + t[1, -1, 1]$ **e)** $[x, y, z] = [5,1,0] + t[-3,0,1]$ **f)** inconsistent system; intersects in pairs **g)** inconsistent system; planes 1&2 are coincident but plane 3 is parallel and distinct

17) $79°$

18) $\frac{38}{\sqrt{29}}$ units