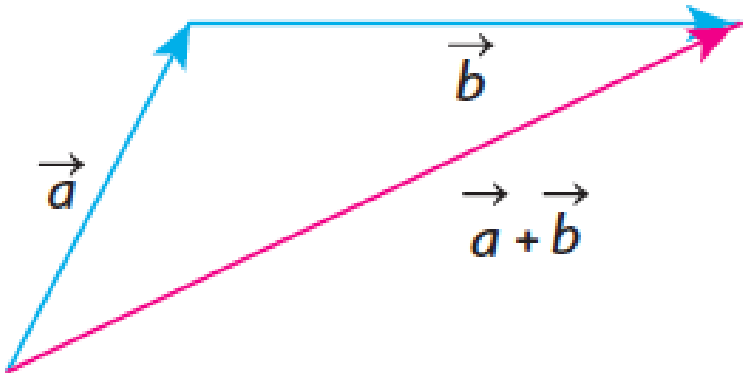
Name:

Unit 4- Geometric Vectors

WORKBOOK

MCV4U



W1 – Introduction to Vectors Unit 4

MCV4U

Jensen

1) Convert each true bearing to its equivalent quadrant bearing.

a) 065° **b)** 120° **c)** 235°

d) 270° **e)** 310° **f)** 017°

2) Convert each quadrant bearing to its equivalent true bearing.

a) N40°E **b)** S65°E **c)** S32°W

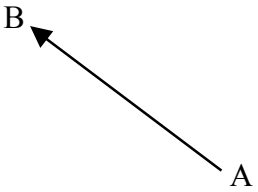
d) N27°W **e)** S54°W **f)** N11°W

3) In the space to the right, draw and name…

a) a vector parallel to $\overrightarrow{AB}$

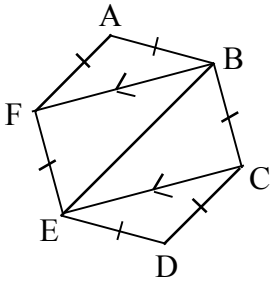
b) a vector opposite to $\overrightarrow{AB}$

c) a vector equivalent to $\overrightarrow{AB}$



4) Name all the equivalent vectors in each diagram.

a)



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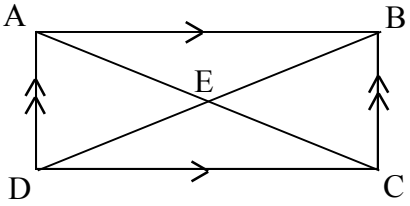


5) State the opposite of each vector.

a) 300 km north

b) 700 N on a bearing of 120°

c) 180 km/h on a quadrant bearing of S25°W

6) Use an appropriate scale to draw each vector. Label magnitude, direction

a) displacement of 50 km west

b) velocity of 8 m/s on a true bearing of 140°

c) force of 1200 N downward

d) acceleration of 240 m/s^2 on a quadrant bearing of N40°W

e) velocity of 7 km/h at 25° to the horizontal

7) State whether the following are vectors or scalars:

a) A table weighs 80 N

b) A woman's age is 60 years old

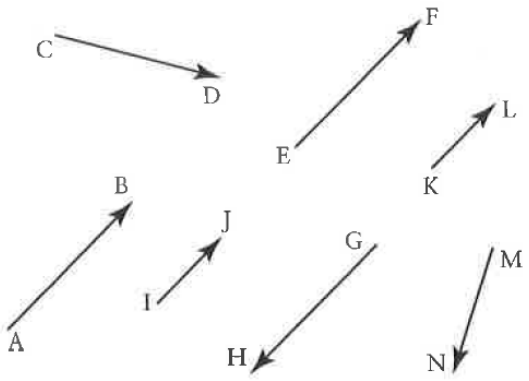
c) 300 km/h

d) An elevator lifts a person 20 m

8)a) Which vectors are parallel to $\overrightarrow{AB}$?

b) Which vectors are equivalent to $\overrightarrow{AB}$?

c) Which vectors are opposite to $\overrightarrow{AB}$?

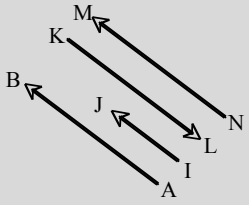


Answer Key:

1) a) N65°E **b)** S60°E **c)** S55°W **d)** W **e)** N50°W **f)** N17°E

2) a) 40° **b)** 115° **c)** 212° **d)** 333° **e)** 234° **f)** 349°

3) Diagrams may vary. For example, in the diagram shown, $\overrightarrow{IJ}$ is parallel to $\overrightarrow{AB}$, $\overrightarrow{KL}$ is opposite to $\overrightarrow{AB}$, and $\overrightarrow{MN}$ is equivalent to $\overrightarrow{AB}$.



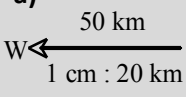
4) a) $\overrightarrow{AB} = \overrightarrow{ED},\ \overrightarrow{BC} = \overrightarrow{FE},\ \overrightarrow{CD} = \overrightarrow{AF},\ \overrightarrow{DE} = \overrightarrow{BA},\ \overrightarrow{EF} = \overrightarrow{CB}\ ,\overrightarrow{FA} = \overrightarrow{DC},\ \overrightarrow{FB} = \overrightarrow{EC},\ \overrightarrow{BF} = \overrightarrow{CE}$

b) $\overrightarrow{AB} = \overrightarrow{DC},\ \overrightarrow{BA} = \overrightarrow{CD},\ \overrightarrow{AD} = \overrightarrow{BC},\ \overrightarrow{DA} = \overrightarrow{CB},\ \overrightarrow{DE} = \overrightarrow{EB},\ \overrightarrow{BE} = \overrightarrow{ED},\ \overrightarrow{AE} = \overrightarrow{EC},\ \overrightarrow{CE} = \overrightarrow{EA}$

5) a) 300 km south **b)** 700 N on a bearing of 300° **c)** 180 km/h on a quadrant bearing of N25°E

6) Diagrams may vary.

a)



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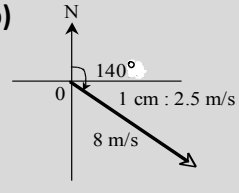

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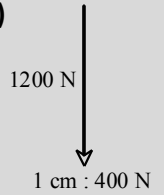

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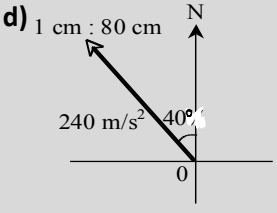
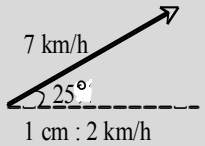


e)



7) a) Vector – weight is due to the force of gravity and therefore has a direction

b) Scalar – this has no direction **c)** Scalar – there is no direction so this is just speed

d) Vector – there is magnitude and direction (up)

8)a) $\overrightarrow{EF}, \overrightarrow{IJ}, \overrightarrow{KL}, \overrightarrow{GH}$ **b)** $\overrightarrow{EF}$ **c)** $\overrightarrow{GH}$

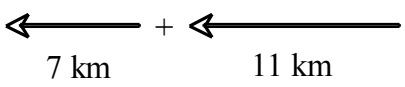
W2 – Vector Addition Unit 4

MCV4U

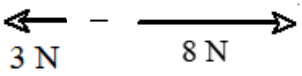
Jensen

1) Draw a diagram to illustrate each vector sum or difference.

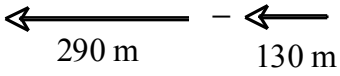
a)



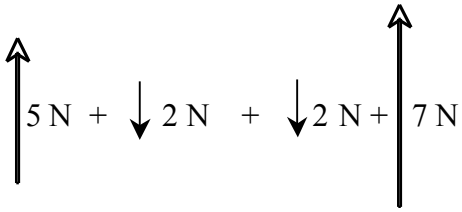
b)



c)

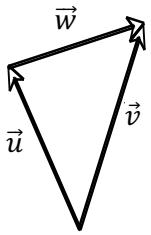


d)

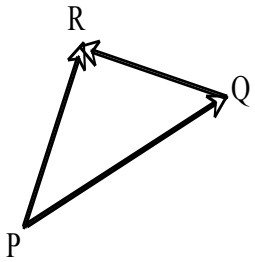


2) Express the shortest vector in each diagram as the sum or difference of the other two vectors.

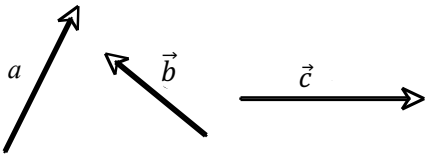
a)



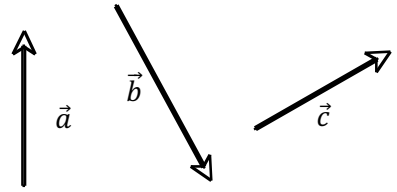
b)



3) Given the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, construct $\vec{a} + \vec{b}$ and $(\vec{a} + \vec{b}) + \vec{c}$.



4) Given the vectors $\vec{a}, \vec{b}$ and $\vec{c}$, draw a diagram of each expression.



a) $\vec{a} + \vec{b} + \vec{c}$

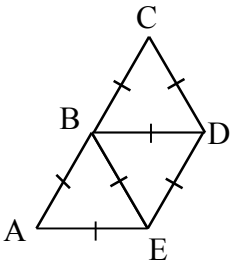
b) $\vec{a} + \vec{b} - \vec{c}$

c) $\vec{a} - \vec{b} - \vec{c}$

d) $-\vec{a} - \vec{b} + \vec{c}$

5) The diagram shows three congruent equilateral triangles.

Express each difference as a single vector. Show your simplification steps.



a) $\overrightarrow{AB} - \overrightarrow{AE}$

b) $-\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{ED}$

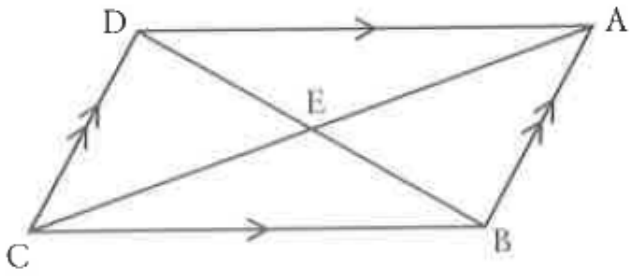
c) $\overrightarrow{BD} - \overrightarrow{BE}$

d) $\overrightarrow{CD} - \overrightarrow{BD}$

6) An airplane leaves the airport travelling N30°W at 720 km/h. After 1 h, the airplane then turns north and travels another 1.5 h at 850 km/h. What is the displacement of the airplane after 2.5 h?

7) ABCD is a parallelogram, and E is the intersection point of the diagonal AC and BD. Name a single vector equivalent to each expression.

a) $\overrightarrow{AE} + \overrightarrow{EB}$ **b)** $\overrightarrow{BC} + \overrightarrow{BA}$

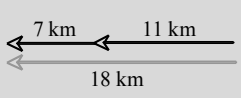


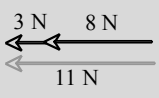
c) $\overrightarrow{AE} + \overrightarrow{AE}$ **d)** $\overrightarrow{AD} + \overrightarrow{AB}$

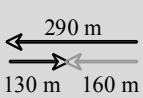
e) $\overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC}$ **f)** $\overrightarrow{AB} - \overrightarrow{DB}$

g) $\overrightarrow{AB} - \overrightarrow{CB} - \overrightarrow{DC}$ **h)** $\overrightarrow{AE} - \overrightarrow{EB} - \overrightarrow{BC}$

Answer Key:

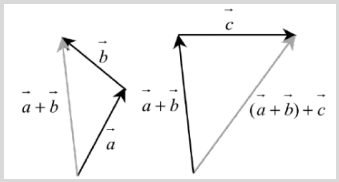
1)a) 

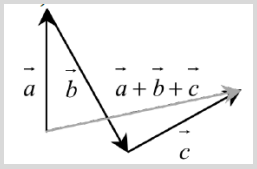
b) 

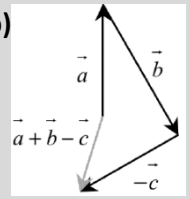
c) 

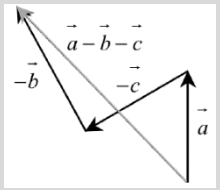
d) 

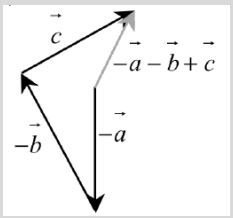
2) **a)** $\vec{w} = \vec{v} - \vec{u}$ **b)** $\overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ}$

3) 

4)a) 

b) 

c) 

d) 

5a) $\overrightarrow{EB}$ **b)** $\overrightarrow{BD}$ **c)** $\overrightarrow{ED}$ **d)** $\overrightarrow{CB}$

6) 1932.4 km $N10.7°W$

7)a) $\overrightarrow{AB}$ **b)** $\overrightarrow{BD}$ **c)** $\overrightarrow{AC}$ **d)** $\overrightarrow{AC}$ **e)** $\overrightarrow{BC}$ **f)** $\overrightarrow{AD}$ **g)** $\overrightarrow{AD}$ **h)** $\vec{0}$

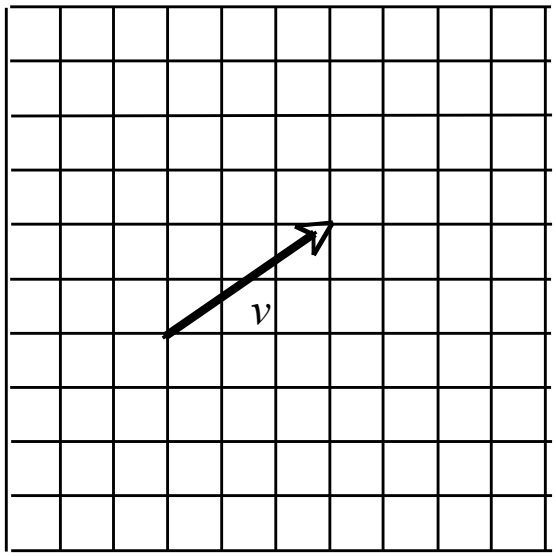
W3 – Multiplication of a Vector by a Scalar Unit 4

MCV4U

Jensen

1) Draw representatives of the following vectors on the grid provided.

a) $3\vec{v}$
b) $0.5\vec{v}$
c) $-2\vec{v}$
d) $-\vec{v}$



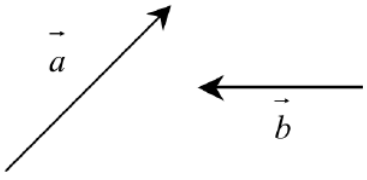
2) Simplify each of the following algebraically.

a) $\vec{a} + 2\vec{a} + 4\vec{a}$ **b)** $7\vec{u} + 5\vec{v} - 2\vec{u} + 8\vec{v}$ **c)** $2(\vec{u} + \vec{v}) - 3(\vec{u} - 2\vec{v})$

d) $7\vec{u} + 5\vec{v} - 2(\vec{u} - \vec{v}) + 2\vec{u}$ **e)** $-3(\vec{u} + \vec{v}) + 2(\vec{u} - \vec{v})$ **f)** $6(\vec{u} + 2\vec{v}) - 5(\vec{u} - 3\vec{v})$

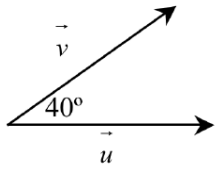
3) For the vectors $\vec{a}$ and $\vec{b}$ shown, draw and label…

a) $2\vec{a}$ **b)** $-3\vec{b}$



c) $\vec{a} + 2\vec{b}$ **d)** $-\vec{a} - 3\vec{b}$

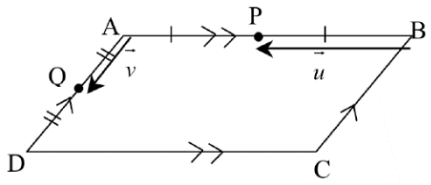
4) Two vectors $\vec{u}$ and $\vec{v}$ make an angle of 40° with each other. Construct each vector sum or difference.



a) $\vec{u} + 2\vec{v}$

b) $2\vec{u} - 2\vec{v}$ **c)** $-2\vec{u} + \vec{v}$

5) In parallelogram ABCD, opposite sides are parallel and equal, $\overrightarrow{BP} = \overrightarrow{PA}$, and $\overrightarrow{AQ} = \overrightarrow{QD}$. Let $\overrightarrow{BP} = \vec{u}$ and $\overrightarrow{AQ} = \vec{v}$. Express the following vectors in terms of $\vec{u}$ and $\vec{v}$.



a) $\overrightarrow{AD}$ **b)** $\overrightarrow{PA}$ **c)** $\overrightarrow{CD}$

d) $\overrightarrow{PQ}$ **e)** $\overrightarrow{BD}$ **f)** $\overrightarrow{PD}$ **g)** $\overrightarrow{AC}$

6) Given that $|\vec{u}| = 8$ and $|\vec{v}| = 10$ and the angle between $\vec{u}$ and $\vec{v}$ is $60°$ determine:

a) $|\vec{u} - \vec{v}|$

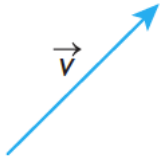
b) the direction of $\vec{u} - \vec{v}$ relative to $\vec{u}$

c) the unit vector in the direction of $\vec{u} + \vec{v}$

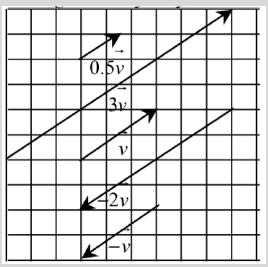
d) $|5\vec{u} + 2\vec{v}|$

7) $|\vec{v}| = 2$. Draw the following factors and express each of them as a scalar multiple of $\vec{v}$.

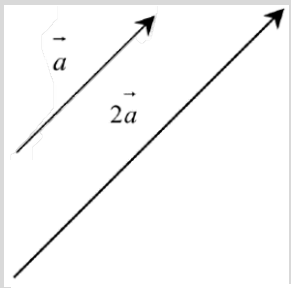
a) A vector in the same direction as $\vec{v}$ with tice its magnitude
b) a vector in the same direction as $\vec{v}$ with one half its magnitude
c) a vector in the opposite direction as $\vec{v}$ with two-thirds its magnitude
d) a vector in the opposite direction as $\vec{v}$ with twice its magnitude
e) a unit vector in the same direction as $\vec{v}$

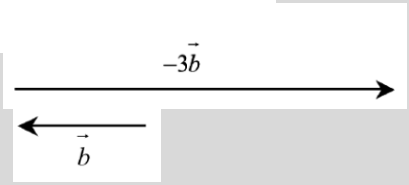


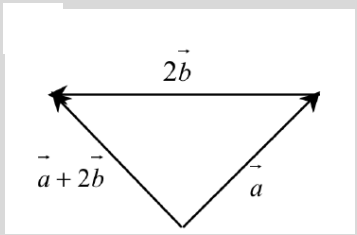
Answers:

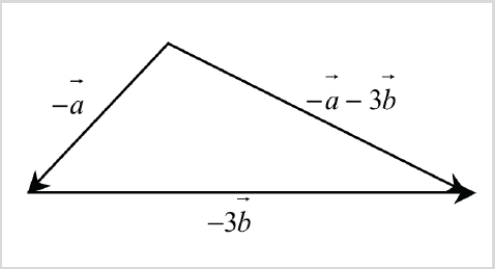
1) 

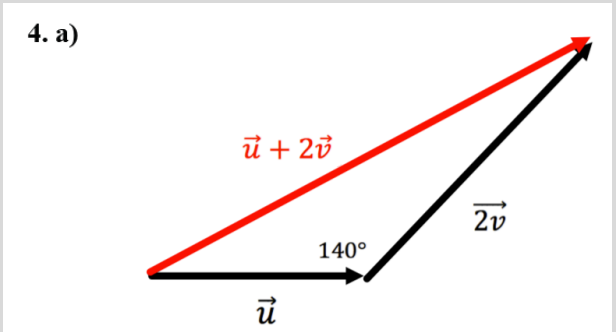
2)a) $7\vec{a}$ **b)** $5\vec{u}+13\vec{v}$ **c)** $-\vec{u}+8\vec{v}$ **d)** $7\vec{u}+7\vec{v}$ **e)** $-\vec{u}-5\vec{v}$ **f)** $\vec{u}+27\vec{v}$

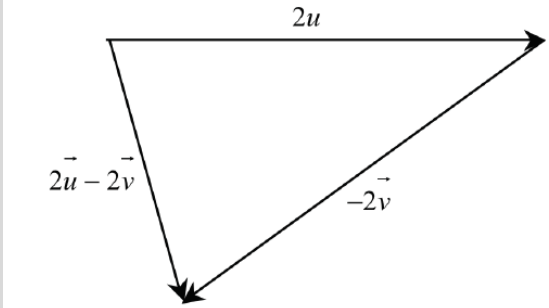
3)a) 

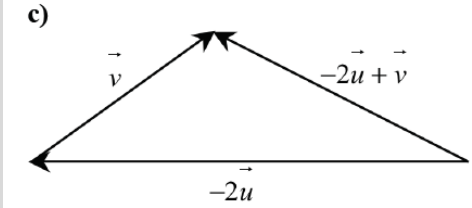
b) 

c) 

d) 

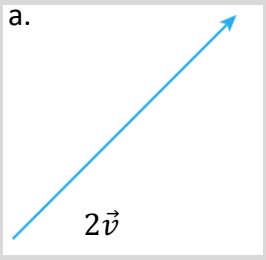
4. a) 

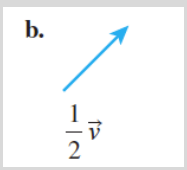
b) 

c) 

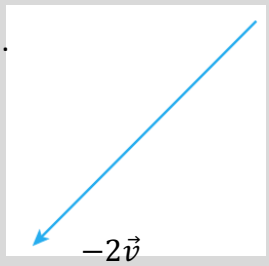
5)a) $2\vec{v}$ **b)** $\vec{u}$ **c)** $2\vec{u}$ **d)** $\vec{u}+\vec{v}$ **e)** $2\vec{u}+2\vec{v}$ **f)** $\vec{u}+2\vec{v}$ **g)** $2\vec{v}-2\vec{u}$

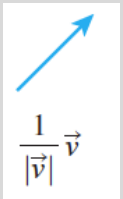
6)a) $2\sqrt{21}$ **b)** $71°$ **c)** $\frac{1}{2\sqrt{61}}(\vec{u}+\vec{v})$ **d)** $20\sqrt{7}$

7) a. 

b. 

c. 

d. 

e. 

W4 – Applications of Vector Addition Unit 4

MCV4U

Jensen

1) The velocity of an airplane is $800\ km/h$ north. A wind is blowing due east at $100\ km/h$. Determine the velocity of the airplane relative to the ground.

2) A particle is displaced 5 units to the east and then displaced 12 units in a direction $N45°E$. Find the magnitude and direction of the resultant displacement.

3) A mass of 5 kg is suspended by two strings, 24 cm and 32 cm long, from two points that are 40 cm apart and at the same level. Determine the tension in each of the strings.

4) A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of $30°$ and $45°$ with the ceiling. Determine the tension in each rope.

5) A river is 2 km wide and flows at 6 km/h. Anna is driving a motorboat, which has a speed of 20 km/h in still water and she heads out from one bank in a direction perpendicular to the current. A marina lies directly across the river from the starting point on the opposite bank.

a) How far downstream from the marine will the current push the boat?

b) How long will it take for the boat to cross the river?

c) If Anna decides that she wants to end up directly across the river at the marina, in what direction should she head? What is the resultant velocity of the boat?

6) Adam can swim at the rate of 2 km/h in still water. At what angle to the bank of a river must he head if he wants to swim directly across the river and the current in the river move at the rate of 1 km/h.

7) An airplane is travelling $N60°E$ with a resultant ground speed of 205 km/h. The nose of the plane is actually pointing east with an airspeed of 212 km/h. Find the wind speed and direction.

ANSWER KEY:

1. 806 km/h $N\ 7.1°\ E$ **2.** 15.93 units $N\ 57.8\ \mathrm{E}$ **3.** 24 cm string: 39.2 N, 32 cm string: 29.4 N
4. $45°$ rope: 175.73 N $30°$ rope: 143.48 N **5. a)** 0.6 km downstream from the marina **b)** 6 minutes (0.1 hours)
c) upstream $17.5°$, resultant velocity: 19.08 km/h **6.** $60°$ **7.** 108 km/h $N18.6°W$

W5 – Resolution of Vectors in to Rectangular Components Unit 4

MCV4U

Jensen

1) Determine the magnitudes of the horizontal and vertical components of each force.

a) magnitude of 570 N, $\theta = 37°$ counterclockwise from the horizontal

b) magnitude of 29 N, $\theta = 52°$ clockwise from the horizontal

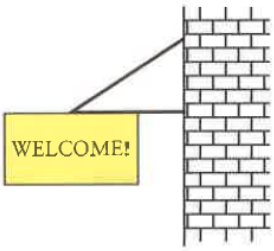
2) A woman is pulling on a rope attached to a toboggan with a 370 N force at an angle of 35° to the horizontal. Find the magnitude of the force pulling the sled forward and the magnitude of the force pulling the sled upward.

3) A 10 kg block lies on a smooth ramp that is inclined at $30°$. What force, parallel to the ramp, would prevent the block from moving. (Assume that 1 kg exerts a force of 9.8 N)

4) A 20 kg box rests on a ramp that is inclined $18°$. Resolve the weight into rectangular vector components that keep the box at rest.

5) Resolve a 200 N force into two rectangular vector components such that the ratio of their magnitudes is 3:1. Calculate the angle between the greater component and the 200 N force.

6) A sign is supported as shown in the diagram. The tension in the slanted rod supporting the sign is 110 N at an angle of $25°$ to the horizontal.



a) Draw a vector diagram showing the vector components of the tension vector.

b) What are the vertical and horizontal vector components of the tension?

ANSWER KEY:

1)a) $\left|\overrightarrow{F_x}\right| = 455.2\ N, \left|\overrightarrow{F_y}\right| = 343.0\ N$ **b)** $\left|\overrightarrow{F_x}\right| = 17.9\ N, \left|\overrightarrow{F_y}\right| = 22.9\ N$

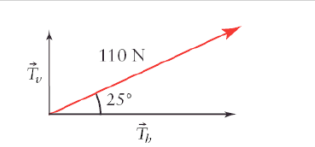
2) forward: 303.1 N; upward: 212.2 N

3) 49 N

4) $|\vec{n}| = 186.41\ N\ \left|\vec{f}\right| = 60.57\ N$

5) $18.4°$

6)a)



b) 99.7 N **c)** 46.5 N