<u>Unit 1 - Algebra</u>

Chapter 3 – Polynomials

LESSONS

MPM1D

Mark my words! You harness that negative power of yours, and you can make it to the top just like me!

<u>Chapter 3 Outline</u>

Unit Goal: By the end of this unit, you will be able to demonstrate an understanding of exponent rules and use them to simplify expressions. You will understand what a polynomial is and be able to simplify polynomial expressions using the distributive property and by collecting like terms.

Section	Subject	Learning Goals	Curriculum Expectations
3.2	Work With Exponents	- understand the components of a power - be able to substitute into and evaluate algebraic expressions involving exponents (including powers with a rational base)	A1.1
3.3a	Exponent Laws	- understand exponent laws through investigation - be able to apply the product, quotient, and power of a power laws	A1.3, A1.4
3.3b	Exponent Laws	- be able to simplify expressions containing powers with negative exponent	A1.3, A1.4
3.4	Communicate with Algebra	- understand what a term and a polynomial are - be able to classify polynomials by name and by degree	A1.2
3.5	Collecting Like Terms	- understand what 'like terms' are - be able to simplify expressions by collecting like terms	A2.4
3.6	Adding and Subtracting Polynomials	- be able to add and subtract polynomials and then simplify the expression by collecting like terms	A2.4
3.7	The Distributive Property	- be able to multiply a polynomial by a monomial using the distributive property - be able to simplify expressions involving the distributive property	A2.5, A2.6

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Skill Builder #1 – Exponent Laws	F/A		P/O	
Quiz – Exponent Laws	F		P	
Assignment - Polynomials	O	A1.2, A1.3, A1.4, A2.4, A2.5, A2.6	P	K(26%), T(9%), A(32.5%), C(32.5%)
Skill Builder #2 - Polynomials	F/A		P/O	
Group Problem Solving Task – Polynomial CSD	O	A1.3, A1.4, A2.4, A2.5, A2.6	P/O/C	K(30%), T(20%), A(30%), C(20%)
PreTest Review	F/A		P	
Test - Polynomials	O	A1.1, A1.2, A1.3, A1.4, A2.4, A2.5, A2.6	P	K(21%), T(34%), A(10%), C(34%)

Section 3.2 – Work With Exponents

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Part 1: Exponents Investigation

One day Sammy decided to try a new place for lunch. He went to a new restaurant called Barney's Burgers. He loved the food so much that when he got back to school he told two of his friends. Suppose that this trend continues and every day each new customer tells two new friends at school about Barney's Burgers. How many new customers will Barney get each day?

a) Complete the chart using your knowledge of exponents

Day	New Customers	Expanded Form	Power
1	**2**	**2**	2^1
2	**4**	**2 x 2**	2^2
3			
4			

b) Use this model to determine how many new customers Barney should expect on Day 7. Show your work.

c) Use this model to determine how many new customers Barney should expect on Day 14. Is this answer realistic? Why or why not?

d) Suppose that each new customer told three friends instead of two, and that this trend continued

i) **How many new customer should Barney expect after 2 days?**

ii) **How many new customer should Barney expect after 4 days?**

Brain Teaser: A rectangular sheet of paper measures 25 cm by 9 cm. The dimensions of a square sheet of paper with the same area are...

Part 2: Exponents

Repeated multiplication of the same number by itself can be expressed as a power. The number is said to be in exponential form.

$$2^3 = 2 \text{ x } 2 \text{ x } 2$$

Express each of the following in exponential form:

1) 3×3

2) $5 \times 5 \times 5 \times 5$

3) $(-2)(-2)(-2)(-2)(-2)$

4) $h \times h \times h$

Write each expression in expanded form and then evaluate:

5) 2^3

6) $(-3)^4$

7) -3^4

8) $\left(\frac{2}{3}\right)^3$

$Note: \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Therefore:

$\left(\frac{2}{3}\right)^3 =$

Find the Trend

Evaluate each of the following:

$(-2)^2$ $(-2)^3$ $(-2)^4$ $(-2)^5$

If the base of the power is negative:

- and the exponent is an even # , the answer will be________________________.

- and the exponent is an odd # , the answer will be________________________.

Part 3: Substitute and Evaluate

Evaluate the expression for the given values of the variables:

9) $6x^2$ for $x = 5$

10) $6x^2 - 2x - 24$ for $x = -6$

Section 3.3a – Exponent Laws

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Part 1: Exponent Laws Investigation

Product of Powers Rule: Complete the following table

Product	Expanded Form	Single Power
$3^2 \cdot 3^4$	(3 x 3) x (3 x 3 x 3 x 3) = 3 x 3 x 3 x 3 x 3 x 3	3^6
$4^3 \cdot 4^3$		
$2^3 \cdot 2^4 \cdot 2^2$		
$k^3 \cdot k^5$		
create your own example		

Describe any trends you see:

Quotient of Powers Rule: Complete the following table

Quotient	Expanded Form	Single Power
$5^5 \div 5^3$	$\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$	5^2
$7^4 \div 7^1$		
$10^6 \div 10^4$		
$x^8 \div x^5$		
create your own example		

Describe any trends you see:

Power of a Power Rule: Complete the following table

Power of a Power	Expanded Form	Single Power
$(2^2)^3$	$(2^2) \times (2^2) \times (2^2)$ $= (2 \times 2) \times (2 \times 2) \times (2 \times 2)$ $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6
$(5^3)^4$		
$(10^4)^2$		
Create your own example		

Describe any trends you see:

Summary of Exponent Laws:

Product Rule	$x^a \cdot x^b =$
Quoutient Rule	$x^a \div x^b =$
Power of a Power Rule	$(x^a)^b =$
Zero Exponent Rule	$x^0 =$

Part 2: Summary of Exponent Laws

Product of Powers Rule

When multiplying powers with the **same base,** keep the same ______________and ______________ the exponents.

General Rule:

$$x^a \cdot x^b =$$

Quotient of Powers Rule

When dividing powers with the **same base,** keep the same ______________and ______________ the exponents.

General Rule:

$$x^a \div x^b =$$

Power of a Power Rule

A power of a power can be written as a single power by ___________________ the exponents.

General Rule:

$$(x^a)^b =$$

Power of a Quotient

When you have a single power with a rational base, you can evaluate it by applying the exponent to the _________________________ and the _________________________.

Rule:

$$\left(\frac{a}{b}\right)^x =$$

Power of a Product

When you have a single power with a base that is a product, the exponent gets put on to each __________________________ in the brackets. Please notice that this only works when inside the brackets is a single term (no + or – signs separating terms)

Rule:

$(ab)^x =$

$x^a \cdot x^b = x^{a+b}$

Part 3: Apply the Product Rule

Write each product as a single power. Then, evaluate the power where possible.

1) $3^2 \times 3^3$

2) $5^2 \times 5 \times 5^2$

3) $(x^2)(x^7)$

4) $(a^4)(a^4)(a^5)$

5) $(-2)^4 \times (-2)^3$

6) $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

Part 4: Apply the Quotient Rule

$$x^{a} \div x^{b} = x^{a-b}$$

Write each quotient as a single power. Then, evaluate the power where possible.

7) $8^7 \div 8^5$

8) $4^7 \div 4 \div 4^3$

9) $x^{70} \div x^{40} \div x^{29}$

10) $\frac{x^7}{x^3}$

11) $\frac{(-0.5)^6}{(-0.5)^3}$

12) $\frac{\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^2}{\left(\frac{3}{4}\right)^5}$

13) $\frac{a^5 a^2}{a^6 a^1}$

Note: An exponent of zero always gives the answer of __________

Part 5: Apply the Power of a Power Rule

$$(x^{a})^{b} = x^{a \times b}$$

Write each power of a power as a single power. Then, evaluate the power where possible.

14) $(3^2)^4$

15) $[(-2)^3]^4$

16) $\left[\left(\frac{2}{3}\right)^2\right]^2$

17) $(3ab^7)^2$

Note: *for #16 you will need the power of a quotient rule and #17 you will need the power of a product rule.*

Product of Powers Rule	$x^a \cdot x^b = x^{a+b}$
Quotient of Powers Rule	$x^a \div x^b = x^{a-b}$
Power of a Power Rule	$(x^a)^b = x^{a \times b}$
Power of a Quotient	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Power of a Product	$(ab)^x = a^x \cdot b^x$
Zero Exponent Rule	$x^0 = 1$
Negative Exponent Rule	$x^{-a} = \frac{1}{x^a}$

Section 3.3b – Exponent Laws

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Part 1: Do It Now

Simplify and evaluate each of the following expressions:

1) $(3^2)(3^5)$

2) $\frac{y^{10}}{y}$

3) $(y^3)^4$

4) $3xy - 2xy$

Note: *#4 is a subtraction question. Simplifying this is called 'collecting like terms'.*

Complete the following table:

Product of Powers Rule	$x^a \cdot x^b =$
Quotient of Powers Rule	$x^a \div x^b =$
Power of a Power Rule	$(x^a)^b =$
Power of a Quotient	$\left(\frac{a}{b}\right)^x =$
Power of a Product	$(ab)^x =$
Zero Exponent Rule	$x^0 =$
Negative Exponent Rule	$x^{-a} =$

Part 2: Negative Exponents

Any non-zero number raised to a negative exponent is equal to its ____________________ raised to the opposite positive power

1) x^{-3} **2)** 5^{-2} **3)** $\frac{x^3}{x^5}$

You Try:

a) $x^5 \div x^9$

b) $\frac{6x^3}{3x^7}$

Part 3: Simplify Expressions Using Exponent Laws

4) $5x^2 \cdot 2x^7$

5) $2a^2b^3 \cdot 3a^6b^4$

Hint: *start by multiplying coefficients together. Then looks for powers with the same base and simplify by writing them as a single power by following the proper exponent laws.*

6) $(5x^3)^2$

7) $(x^4y^3)^2$

Hint: Use the power of a product rule. T*he exponent outside of the brackets must be applied to all coefficients and variables inside the brackets using the proper exponent laws.*

8) $\frac{12k^2m^8}{4k^5m^5}$

9) $\frac{-2uv^3 \cdot 8u^2v^2}{(4uv^2)^2}$

Hint: *start by simplifying the numerator and denominator separately as much as possible using exponent laws. Then reduce the coefficients if possible and use the quotient rule to simplify powers with the same base.*

10) $\frac{(3m^2n)^2}{(2mn)(3m^2n)}$

You try:

a) $\frac{5c^3d \cdot 4c^2d^2}{(2c^2d)^2}$

b) $\frac{(3xy)^3}{9x^4y^4}$

Section 3.4 – Communicate with Algebra

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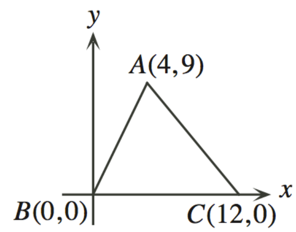
Brain Teaser:

In the diagram, what is the area of the triangle?



Part 1: Do It Now

A hockey team gets 2 points for a win, 1 point for a tie, and 0 points for a loss.

a) Write an equation for determining the amount of points a team has.

b) If the Penguins win 54 games, tie 8, and lose 20;   how many points will they get?

Part 2: Terms

Term: an expression formed by the **product** of ________________ and or ________________.

Example of a term:

$$4x^2$$

The number in front of the variable is called the_____________________.

Identify the coefficient and the variable for the expression $4x^2$:

Coefficient: ______

Variable: _______

Practice with Terms

Identify the coefficient and the variable of each term:

a) Jim earns $7 per hour at his part-time job. If he works for x hours, his earnings, in dollars, are $7x$.
b) The depth, in meters, of a falling stone in a well after t seconds is $-4.9t^2$
c) The area of a triangle with base b and height h is $\frac{1}{2}bh$
d) The area of a square with side length k is k^2

Expression	Coefficient	Variable	Comments
			The negative sign is included with the coefficient
			The variable can consist of more than one letter or symbol
			When the coefficient is not shown, it is 1.

Part 3: Polynomials

Polynomial: an algebraic expression consisting of one or more terms connected by ____________________ or ____________________ operators.

Example of a polynomial:

$$3x^2 + 2x$$

A polynomial can be classified by the number of terms it has:

A ______________________________ is a polynomial with only **one term**.

A ______________________________ is a polynomial with **two terms.**

A ______________________________ is a polynomial with **three terms**.

A ______________________________ is a polynomial with **four terms**.

Classify each polynomial by the number of terms it has:

Polynomial	Number of Terms	Type of Polynomial
$3x^2 + 2x$		
$-2m$		
$4x^2 - 3xy + y^2$		
$a - 2b + c - 3$		

Hint: *You can find the number of terms by looking for the addition and subtraction operators that separate the terms.*

Part 4: Degree of a Term

Degree of a term: the sum of the ______________________ on the variables in a term.

Example of determining the degree of a term:

Term: $5x^2y^3$

Sum of exponents on variables:

Degree of term:

Find the degree of each term by adding the exponents of the variables:

Term	Sum of Exponents	Degree of Term
x^2		
$3y^4$		
$0.7uv$		
$-2a^2b$		
-5		

Note:

- a variable that appears to have no exponent actually has an exponent of _____
- a constant has a degree of _____

Part 5: Degree of a Polynomial

The ***degree of a polynomial*** is equal to the degree of the ____ in the polynomial.

Example:

Polynomial: $3x^2y^4 + 11x^2y^2 + y^5$

Highest degree term:

Degree of highest-degree term:

Degree of polynomial:

Find the degree of each polynomial:

Polynomial	Term with Highest Degree	Degree of Term with Highest Degree	Degree of Polynomial
$x + 3$			
$5x^2 - 2x$			
$3y^3 + 0.2y - 1$			
$7x^2y^4 + x^6y$			

Part 6: Apply Our Knowledge

Mr. Jensen works part time as a golf instructor. He earns $125 for the season, plus $20 for each children`s lesson and $30 for each adult lesson that he gives.

a) Write an expression that describes Mr. Jensen's total earnings for the season. Identify the variables and what they stand for.

b) If Mr. Jensen gave **8 children's** lessons and **6 adult lessons**, what were his total earnings?

Review of Terms

______________________________ : an expression formed by the product of numbers and/or variables

______________________________ : an algebraic expression consisting of one or more terms connected by addition or subtraction signs.

______________________________ : the sum of the exponents on the variables in a term

______________________________: equal to the degree of the highest-degree term in a polynomial

Section 3.5 – Collecting Like Terms

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Brain Teaser:

At King's, the ratio of males to females writing the Pascal Contest is 3 : 7. If there are 21 males writing the Contest, what is the total number of students writing?

Part 1: Do It Now

1) What is the degree of the term: $3x^2yz$

2) What is the degree of this polynomial: $3a^2b^3c + 2ab^4c^2 - 7abc^2$

3) Classify the polynomial from question **2)** by name:

Part 2: Like Terms

Like Terms are terms that have the EXACT same ______________________________ with the EXACT same ______________________________.

These are like terms:
$3x^2y$ and $15x^2y$

These are NOT like terms:
$3x^2y$ and $3x^2y^2$

Identify the like terms in this polynomial:

$3x^3 - 5x + 2x^3 + 3 - 1 + 4x + 12x^3 - 120$

Identify the like terms in this polynomial:

$5x^2y - 9xy + 6x^2y + 17.3x - 2xy + 4x^2y + 92x - 133xy$

Part 3: Collecting Like Terms

When adding/subtracting like terms, keep the variables the same, and add/subtract only the coefficients.

Example:

$6x + 4 + 8x + 3$

$=$ **Step 1:** Rearrange like terms into groups

$=$ **Step 2:** Add/Subtract the like terms

Practice Collecting Like Terms

1) $3x + 4x$

2) $3x^2 + 5x^2 + 3$

3) $2b - b + 7 - 8 + 3b$

4) $3x^2 + 2 - 6x + 9x - 3x^2$

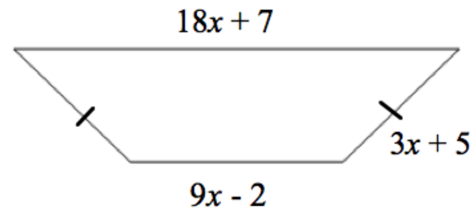
5) $2x^2 - 3y^2 + xy + 2y^2 - 8x^3$

6) $a^2b + 2ab - ab^2 + 2ab^2 - 3ab + a^2b$

Note: degree of terms should be in descending order (highest degree terms on the left).

Part 4: Apply our Knowledge

a) Write an expression in simplest form for the perimeter of the given shape



b) Evaluate the expression if $x = 5$. (What is the perimeter?)

Section 3.6 – Add and Subtract Polynomials

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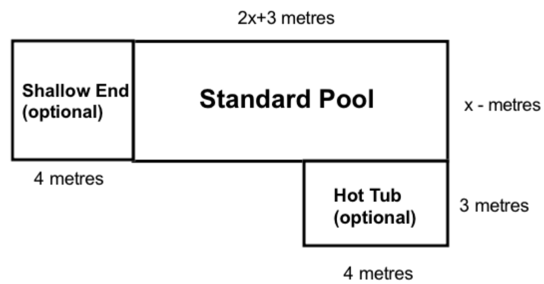
Part 1: Do It Now

The Cool Pool Company makes pools and uses the following diagram to calculate the perimeter of pool with different options:

Option 1 – Bronze package (standard pool only)
Option 2 – Silver package (standard pool and shallow pool)
Option 3 – Gold package (all sections of the pool are included)



a) In your group choose either the Bronze, Silver, or Gold package and create a simplified expression for the perimeter of your pool.

b) What is the perimeter of your pool if $x = 4$

Part 2: Adding Polynomials

Polynomial: an algebraic expression consisting of one or more terms connected by addition or subtraction operators

When **adding** polynomials you can simply ________________ the brackets and collect the like terms

Example:

$(4x + 3) + (7x + 2)$

$=$ **Step 1:** Remove the Brackets

$=$ **Step 2:** Rearrange like terms into groups

$=$ **Step 3:** Collect the like terms

Practice Adding Polynomials

1) $(3y + 5) + (7y - 4)$

2) $(2p - 2) + (4p - 7)$

3) $(6x - 12) + (-9x - 4) + (x + 14)$

4) $(5x - 4y - 1) + (-2x + 5y + 13)$

Part 3: Subtracting Polynomials

To **subtract** polynomials, add the ______________________________ (switch the signs of the terms of the polynomial being subtracted)

Example:

$(3y + 5) - (7y - 4)$

The opposite of $(7y - 4)$ is: ________________

$\therefore (3y + 5) - (7y - 4) =$

OR

To **subtract** polynomials, subtract ______________________________________ in the second polynomial.

$$(3y + 5) - (7y - 4)$$

$\therefore (3y + 5) - (7y - 4) =$

5) $(4x + 3) - (7x + 2)$

6) $(a^2 - 2a + 1) - (-a^2 - 2a - 5)$

7) $(3x + y - 4z) - (7x + 3y - 2z)$

8) $(6x - 12) - (-9x - 4) - (x + 14)$

Part 4: Apply Our Knowledge

The Burgh Birds players get a $\$x$ bonus added to their base salary for every goal that they score during the playoffs. Here are the salaries and goals scored for the 3 highest scoring players on the Burg Birds during the playoffs last season.

Player	Base salary	Goals in playoffs last season
Bardown Jensen	$1200	18
Wayne Goal	$1300	22
Timmy Toe Drag	$900	5

a) Write and simplify an expression for each player's year end salary.

Bardown Jensen:

Wayne-G:

Timmy:

b) Write and simplify an expression for the total amount of money that the owner of the team will need to pay the three players at the end of the season.

c) If $x = 25$, what is the total amount of money that the owner will need to pay the three players at the end of the season?

Section 3.7 – The Distributive Property

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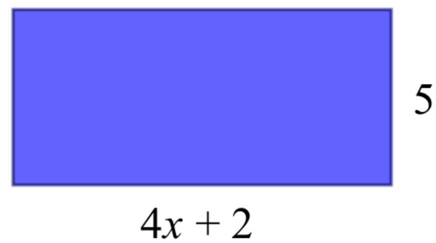
Part 1: Do It Now

Write a simplified expression for the area of the rectangle:

$Area\ of\ rectangle =$



Remember: $Area = length \times width$

Before we can simplify the expression we need to learn the distributive property!

Distributive Property

$$a(x + y) = ax + ay$$

When you apply the distributive property, you are getting rid of the brackets by multiplying everything in the brackets by the term in front of the brackets.

Example:

$5(4x + 2)$

$= 20x + 10$

To apply the distributive property, I must multiply both terms in the bracket by 5.

Part 2: Multiply a Constant by a Polynomial

Expand and simplify the following:

1) $2(5x + 3)$

2) $-2(7x - 4)$

Note: Make sure to include the negative sign when distributing the -2. Follow integer rules for multiplication.

3) $-3(2x^2 - 5x + 4)$

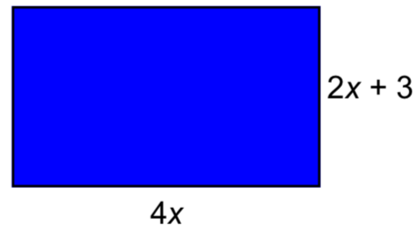
Note: You can also apply the distributive property to trinomials.

4) $2(6m - 3) + 3(16 + 4m)$

Remember: You can collect like terms! Like terms have identical variables (same letters and exponents)

Part 3: Apply Our Knowledge

Write an expression for the area of the rectangle in expanded form:



What is the area of the rectangle if $x = 5$ cm?

Part 4: Distribute Variables

Example:

$x(x^2 - 3)$

$= x^3 - 3x$

Remember the exponent laws:

$x(x^2) = x^{1+2} = x^3$

6) $x(x - 3)$

7) $-x(7x - 4)$

8) $-3x(2x^2 - 5x + 4)$

9) $3m(m - 5) - (2m^2 - m)$

For this question you can multiply the second polynomial by -1 or use the properties for subtracting polynomials; both give the same result!

10) $\frac{1}{2}(2w-6)-\frac{2}{3}(9w-6)$

<u>Part 5: Nested Brackets</u>

If there is a bracket inside of a bracket, simplify the inner most brackets first and then work your way out.

11) $3[2+5(2k-1)]$