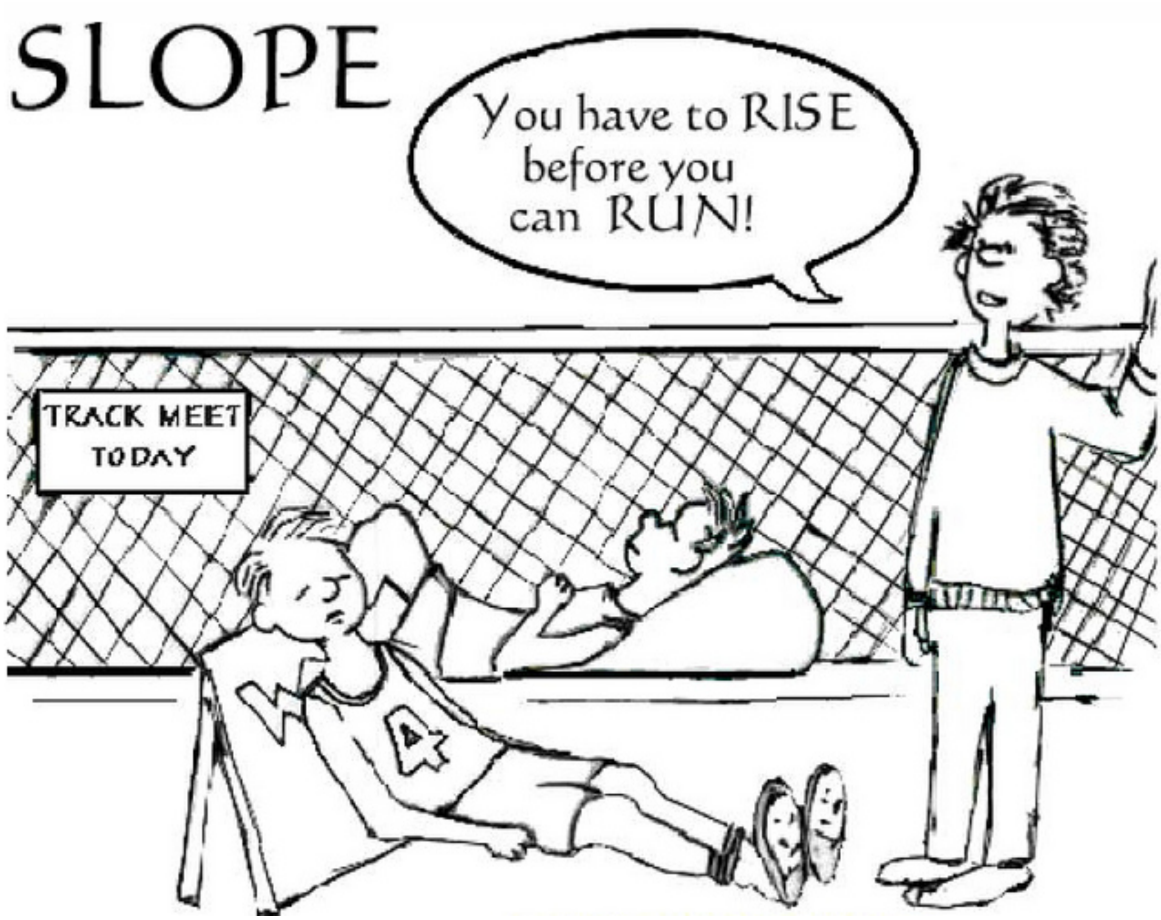
Unit 2 – Linear Relations

Chapter 5 – Modeling With Graphs

Lessons

MPM1D



Chapter 5 Outline

Section	Subject	Learning Goals	Curriculum Expectations
5.1	Direct Variation	- understand properties of a direct variation relationship - know how to represent a direct variation using an equation and graph	B2.4, B3.1, B3.3
5.2	Partial Variation	- understand properties of a partial variation relationship - know how to represent a partial variation using an equation and graph	B2.4, B3.1, B3.3
5.3a	Slope	- given the graph of a line, determine its slope	C2.1, C2.2
5.3b	Slope	- given the graph of a line, determine its slope	C2.1, C2.2
5.4	Slope as a Rate of Change	- understand the connection between slope and rate of change	B3.3, C2.3
5.5	First Differences	- be able to classify a relation as linear or non-linear using first differences	B2.3, C1.1
5.6	Connecting Everything	- be able to connect understanding of slope, rate of change, and first differences	B3.3, C3.3

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Direct vs. Partial Variation	F		P	
Quiz – Direct vs. Partial Variation	F		P	
Assignment – Cellphone Plans	O	B2.4, B3.1, B3.3, B3.4	P	K(28%), T(5%), A(19%), C(48%)
PreTest Review	F/A		P	
Test – Modelling With Graphs	O	B2.1, B2.2, B2.3, B2.4, B3.1, B3.4, C1.1, C2.1, C3.1, C3.3	P	K(45%), T(9%), A(28%), C(18%)

Section 5.1 – Direct Variation
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Jensen

Part 1: D0 IT NOW!!! "What is speed?"

You and a friend decide to have a friendly bicycle race on Saturday. How far ahead will you be after one hour, given that you travel at the following speeds?

a) Heading North your speed is 12km/h, your friend's speed is 10 km/h

b) Heading South you travel 5 km in 30 minutes, your friend travels 5 km in 20 minutes

Calculations:

a)

b)

Conclusion: Speed is an example of ______________________________

Part 2: Investigate Direct Variation using Tables

Example 1: Use a table to organize the information in the following problem.

a) How long does it take to fill a 3 000 litre hot tub if a water truck supplies water at a rate of 250 litres per hour and the tub is initially empty?

Time (h)	**Water (L)**

b) What if the water truck was able to double the flow rate?

Time (h)	**Water (L)**

Which variable is dependent? (If a variable is dependent its value cannot be known without knowing the other variable's value). **To determine the dependent variable ask yourself the following key questions:**

Does ___the amount of water___ depend upon ___time___ ?

OR

Does ___time___ depend upon ___the amount of water___ ?

$$\boldsymbol{Rate\ of\ Change} = \frac{\Delta \boldsymbol{dependent\ variable}}{\Delta \boldsymbol{independent\ variable}} = \frac{\Delta y}{\Delta x} = \frac{\Delta \boldsymbol{water}}{\Delta \boldsymbol{time}}$$

a) Calculate the rate of change of the amount of water in the hot tub between the following times, using the first table:

i) Between 0 hours and 1 hour: rate = ___________ =

ii) Between 0 hours and 2 hour: rate = ___________ =

iii) Between 2 hours and 5 hour: rate = _________________ =

What do you notice about these rates?

When the rate is ***constant,*** it is often referred to as the ***constant of variation***.

A **direct variation** is a relationship between two variables in which one variable is a constant multiple of the other. The rate of change is constant (constant of variation). A direct variation is a situation in which two quantities, such as hours and pay, increase or decrease at the same rate. That is, as one quantity doubles, the other quantity also doubles. The variables are said to be directly proportional.

For example: *if you get paid hourly, when you work twice as many hours, you will make twice as much money. How much you make is a constant multiple of how many hours you work.*

Example 2: Identify the dependent and independent variables in the following rates of change.

	Dependent variable (y)	**Independent variable (x)**
The car travelled at 75 km/h.		
In November the temperature drops 1.2 degrees Celsius per day.		

Example 3: Determine the constant of variation (rates of change) given the data in the following tables.

Recall: Rate of change (constant of variation) $= \dfrac{\Delta\text{dependent}}{\Delta\text{independent}}$

a)

Hours worked (h)	Money made ($)
0	**0**
1	35
2	70
3	105

Rate of change (m) =

b)

Mass of grain (kg)	Cost ($)
0	**0**
20	125
40	250
60	375

Rate of change (m)=

i) What is the same in the relationships given in parts a) and b)?

Both tables represent **direct variations**, the independent variable directly affects the dependent variable.

Direct variation will always have (0 , 0) as the initial value!

ii) Write an equation to model the data in the tables given in Example 3. Remember that 'y' varies directly with 'x'; **y = mx**

a)

b)

iii) Which is the independent variable (x)?

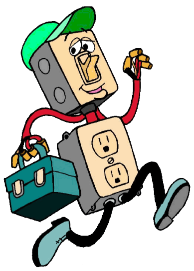
Which is the dependent variable (y)?

In general, the equation of a **direct variation** has the following properties:

- we say that the equation of a direct variation is always in the form $y = mx$, where m is the constant of variation

Part 2: Investigate Direct Variations using Graphs

Example 4: The cost to do electrical work varies directly with time. Electric company charges $25 per hour to do electrical work while AC-DC electrical charges $50 per hour.



a) Write equations to model each relationship.

Electric company:

AC-DC electrical:

b) Use the equations to complete the tables for 0 to 4 hours.

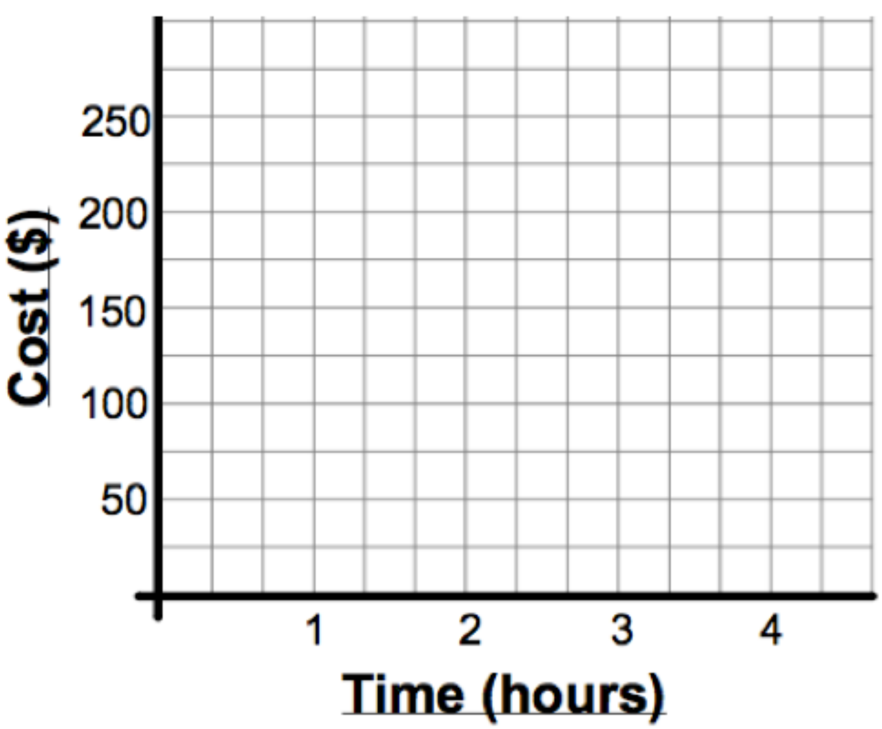
Electric company:

Hours (h)	Cost ($)
0	
1	
2	
3	
4	

AC-DC electrical:

Hours (h)	Cost ($)
0	
1	
2	
3	
4	

c) Graph the data for both companies on the same Cartesian coordinate grid.



d) Looking at the **graph or the table,** which company should I choose if I have a job that requires 3 hours of electrical work?

e) How can we show the rate of change on the graph? How does the steepness of the line relate to the rate of change?

f) What is the same for both graphs?

In general, the graph of a **direct variation** has the following properties:

- it is a straight line which always passes through the origin

Part 3: Interpreting Direct Variation Word Problems

Example 5:

The distance travelled by Mr. Jensen when he drives varies directly with time. His car travels 630 km in 3 hours.

i) What is the constant of variation?

ii) Write an equation to represent the relationship between the variables

iii) Which variable is the dependent/independent variable?

Example 6:

The total cost varies directly with the number of MP3's downloaded. 13 MP3 downloads costs $12.87

i) What is the constant of variation?

ii) Write an equation to represent the relationship between the variables

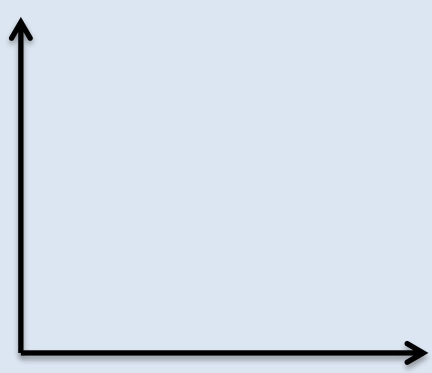
iii) Which variable is the dependent/independent variable?

<u>Consolidate</u>:

Direct variation occurs when the dependent variable ______________________________________ ________________ of the independent variable.

Direct variation can be defined algebraically as __________ where m is the constant of variation.

The graph of a direct variation is a straight line that passes through the origin (0, 0).



Remember:

Rate of change (constant of variation) $= m = \frac{\Delta dependant\ variable}{\Delta independent\ variable}$

Section 5.2 – Partial Variation

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Jensen

Part 1: DO IT NOW

The Keg Restaurant charges $100 to reserve a private dining room plus $40 per person.

a) Write an equation to show the relationship between the cost of the reservation and the number of people attending.

b) What is different about this equation and the equation of a direct variation ($y = mx$)?

c) How much will it cost to reserve the room if

i) An extended family of 25 want to have dinner to celebrate a recent birth of twins?

ii) The Pittsburgh Penguins want to celebrate their 2009 Cup Victory. There are 24 players and 6 coaches attending the celebration.

Part 2: Recall properties of direct variations

A direct variation is a relationship between two variables in which one variable is a constant multiple of the other.

Model a direct variation in an equation: $\boldsymbol{y = mx}$

Constant of variation is defined as: $m = rate\ of\ change = \frac{\Delta y}{\Delta x}$

Direct variations are linear relations that always pass through which point on the Cartesian coordinate grid?_______________

Part 3: Compare direct variations to partial variations

The Tesla electrical company charges $25 per hour to do electrical work plus a fee of $50 for the estimate on the proposed work. AC-DC electrical charges $50 per hour. Write equations to model each relationship. Let x represent the number of hours and let y represent the total cost.

Tesla Electric company:

AC-DC electrical:

Use the equations to create tables to organize the data for 0 to 4 hours.

Tesla electric company:

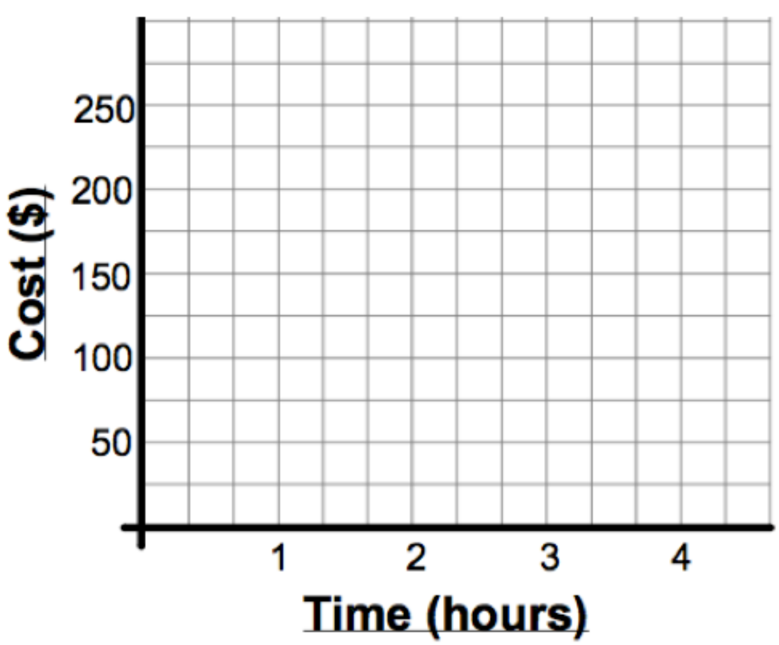
Hours (h)	Cost ($)
0	
1	
2	
3	
4	

AC-DC electrical:

Hours (h)	Cost ($)
0	
1	
2	
3	
4	

Which relation is a direct variation and how do you know?

Now graph the data for both companies on the same Cartesian coordinate grid.



Looking at the **graph or the table**, we should use ________________ for 3 hours of electrical work. Does this company always offer the best deal? Explain.

What is different about the two relations?

A **PARTIAL VARIATION** is a relationship between two variables in which the dependent variable is the sum of a constant number and a constant multiple of the independent variable.

In general, the graph of a **partial variation** has the following properties:

- it is a straight line which <u>does not</u> pass through the origin (0,0)
- the equation of a partial variation is always in the form $y = mx + b$
- 'b' is the initial value (y-intercept, fixed cost)
- 'm' is the constant of variation (rate of change, variable cost)

<u>Part 4: Working with Partial Variation</u>

a) Complete the following chart given that y varies partially with x (you may need to determine the constant of variation)

x	y
0	6
1	9
2	
3	15
4	
	27

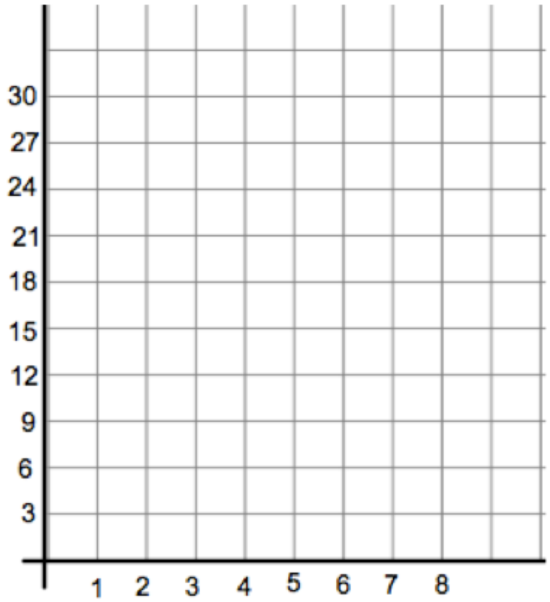
b) What is the initial value of 'y' (y-intercept)?

c) What is the constant of variation (rate of change)?

Remember: $m = \frac{\Delta y}{\Delta x}$

d) Write an equation relating y and x in the form $y = mx + b$

e) Graph the relation



Part 5: Application of Partial Variation

A school is planning an awards banquet. The cost of renting the banquet facility and hiring serving staff is $675. There is an additional cost of $12 per person for the meal.

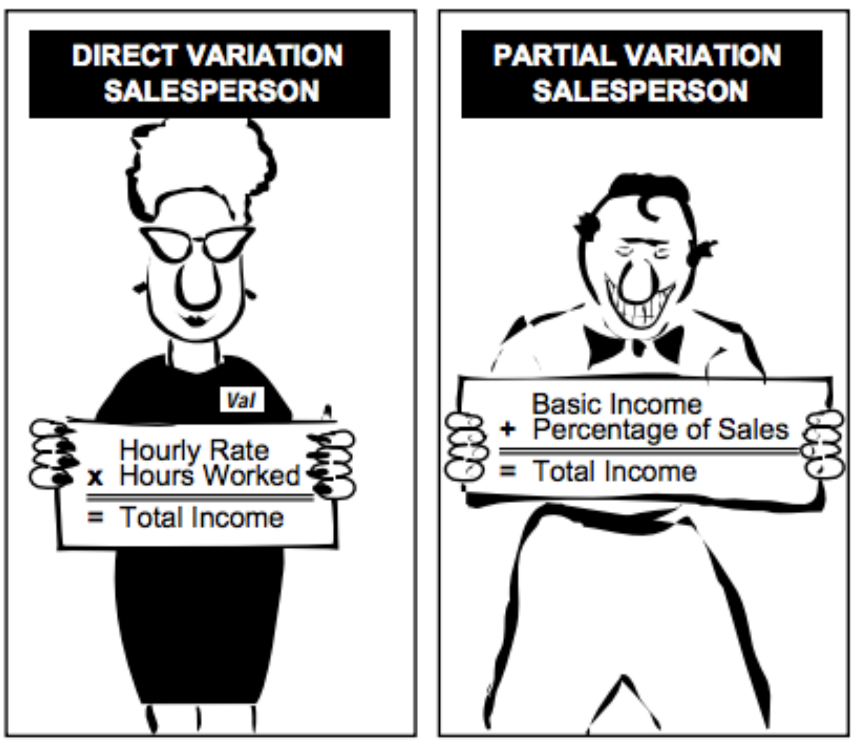
a) Identify the fixed cost (initial value; b) and the variable cost (constant of variation; m)

b) Write an equation to represent this relationship in the form $y = mx + b$

c) Use your equation to determine the total cost if 500 people attend the banquet.

Consolidate:

Direct variation			Partial variation		
Table	Graph	Equation	Table	Graph	Equation
Has (0,0) as the initial value	Passes through the origin	$y = mx$	Has an initial value other than zero	Crosses the dependent axis (y-axis) at an initial value other than 0	$y = mx + b$
Create an example:	**Create an example:**	**Create an example:**	**Create an example:**	**Create an example:**	**Create an example:**



In order to help you understand the content of this unit, Val and Sal have kindly volunteered to assist us by providing a simple but direct comparative illustration.

Section 5.3a – Slope

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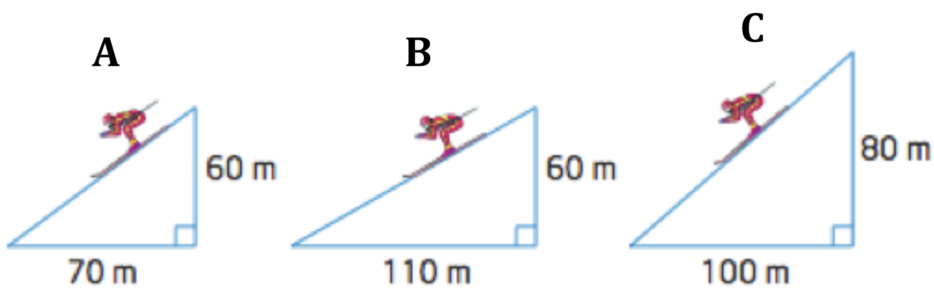
Jensen

Investigation

Slope: A measurement of the steepness of a line.

The following diagrams represent ski hills.



1. Rank the hills in order of their steepness, from least to greatest.

i. _______ **ii.** _______ **iii.** ________

2. A hill rises 2 meters over a horizontal run of 8 meters. A second hill rises 4 meters over a horizontal run of 10 meters. Which is the steeper hill?

3. Describe your method for determining steepness:

Part 1: How do we find the slope of a line?

The steepness of a line segment is measured by its _____________. The slope is the ratio of the _________________ to the _________________ and is often represented by the letter ______ .

You should maybe be starting to make a connection; what else did we use the letter m to represent?

______________: the vertical distance between two points (Δy)

______________: the horizontal distance between two points (Δx)

$$\boldsymbol{Slope = m = \frac{rise}{run} \quad or \quad \frac{\Delta y}{\Delta x}}$$

When determining the rise and run of a line from its graph you must know that:

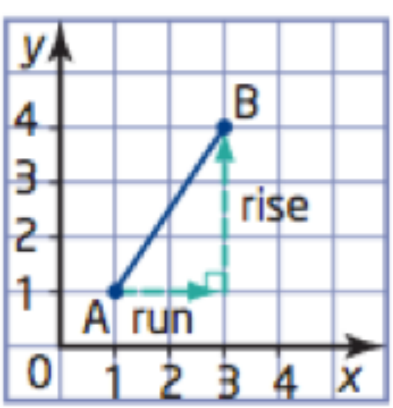
Counting units in the upward direction gives a ______________ rise

Counting units in the downward direction gives a _____________ rise

Counting units to the right gives a _____________________ run

Counting units to the left gives a ______________________ run

Example 1: Count the units on the grid to determine the rise and run.



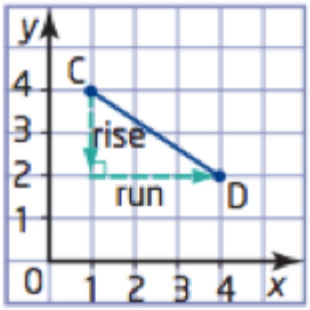
rise = _________

run = __________

What's the slope of this line?

Example 2:

Example 2: Count the units on the grid to determine the rise and run



rise = __________

run = __________

What's the slope of this line?

Looking at example 1:

Is the slope positive or negative? _________________________

What direction does the line go? _________________________

Looking at example 2:

Is the slope positive or negative? _________________________

What direction does the line go? _________________________

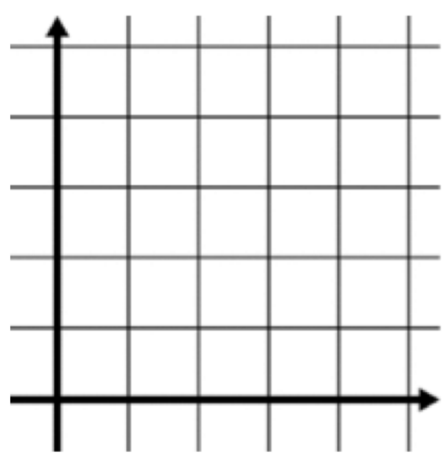
Conclusion about positive and negative slopes:

A line that ____ has a positive slope.

A line that ____ has a negative slope.

<u>Part 2: Finding the slope of vertical and horizontal lines</u>

Step 1: Plot the points A(1,1) and D(5,1) on the graph provided. Connect the points to form the line segment AD.



Step 2: Determine the rise and the run of line AD

rise = run = m =

The slope of any horizontal line is ____________________

Step 3: Plot the point E(1,5) on the same grid. Connect it to point A to form the line segment AE.

Step 4: Determine the rise and the run of line AE

rise = run = m =

The slope of any vertical line is ______________________________

Part 3: Practice Finding Slopes

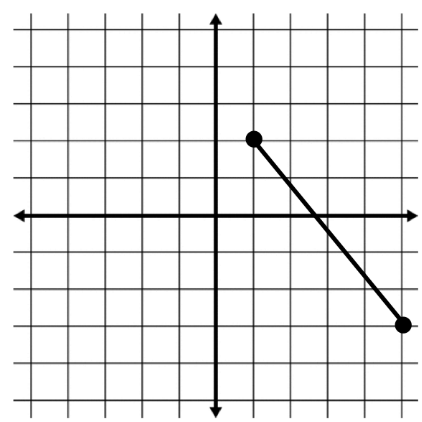
Calculate the slope of each line segments

Example 3:

rise is:

run is:

m =

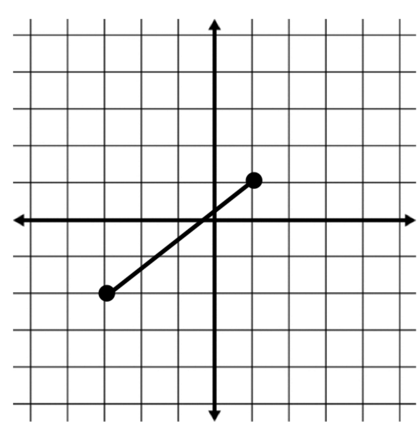


Example 4:

rise is:

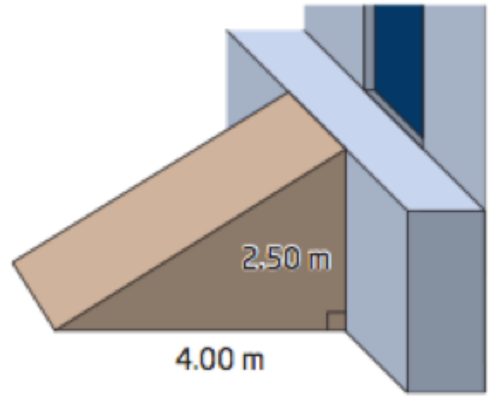
run is:

m =



Example 5: The ramp at a loading dock rises 2.5 meters over a run of 4 meters.

What is the slope of the ramp?



Section 5.3b – Slope

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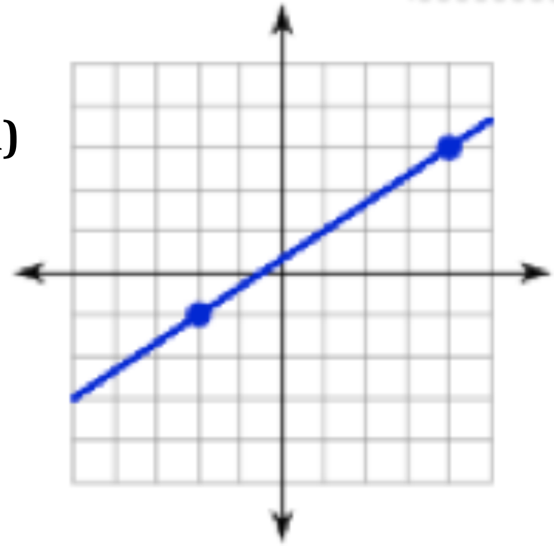
Part 1: Do It Now

Find the slope of each of the following lines by looking at the graph and determining the rise and the run.

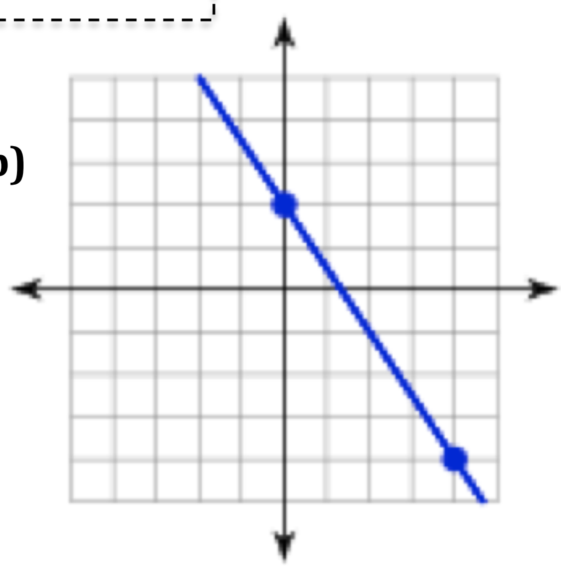
Remember: $slope = \frac{rise}{run}$

a)



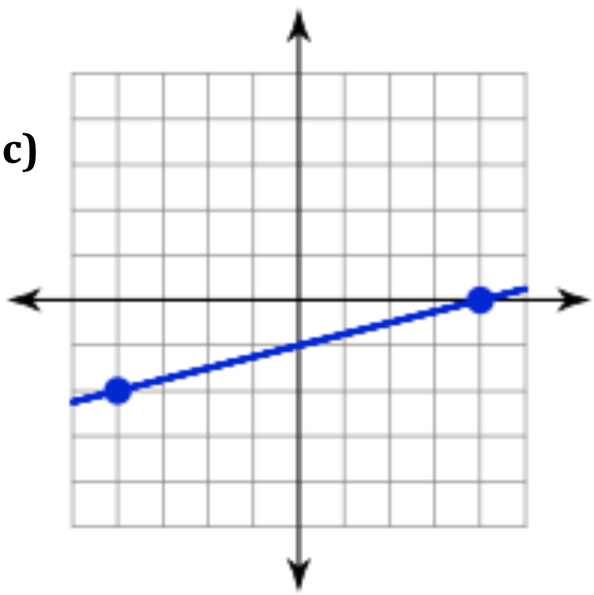
$m =$

b)



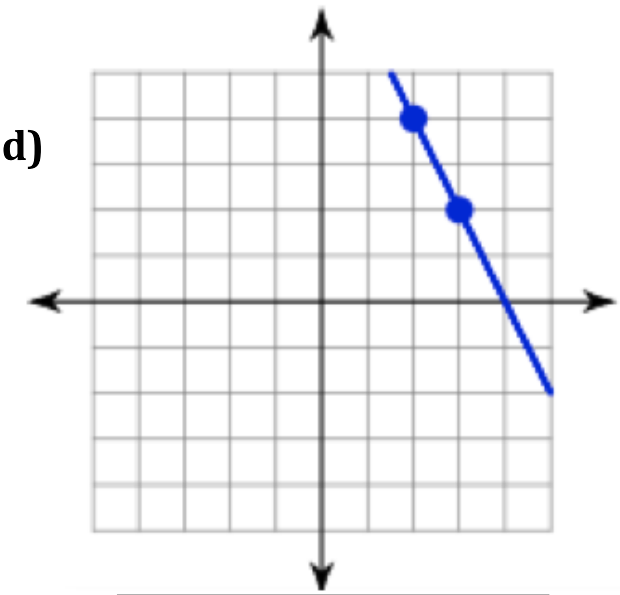
$m =$

c)



$m =$

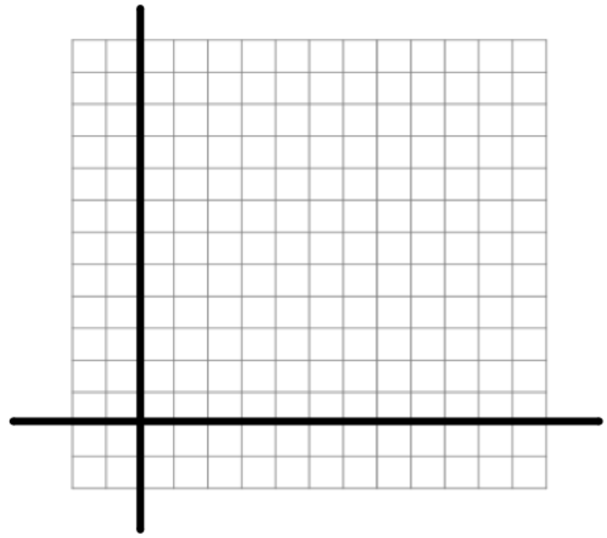
d)



$m =$

Part 2: Draw a graph to find another point on a line

Example 1: A line segment has one endpoint, A(4,7), and slope of $-\frac{3}{2}$. Find the coordinates of another possible endpoint, B.



Step 1: Plot the point A(4,7).

Step 2: Use the slope $-\frac{3}{2}$ to find another endpoint.

Note: $-\frac{3}{2} = \frac{-3}{2}$, therefore the line has a rise of ________ and run of ________

To plot another point, start at point A and use the slope of the line to plot another point.

The rise of -3 tells us we should go _________ 3 units.

The run of 2 tells us we should go _________ 2 units.

Another possible endpoint is: (,)

Note: There are an infinite number of solutions!!! What would have happen if you used a slope of $\frac{3}{-2}$? Why does this happen?

Example 2:

If a line has slope of $-\frac{1}{2}$, and the line passes through the point (4 , 5) determine the coordinates of two points to the left, and two points to the right on the same line.

Note: $-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

Graphical solution:
('move' to other points according to the slope)

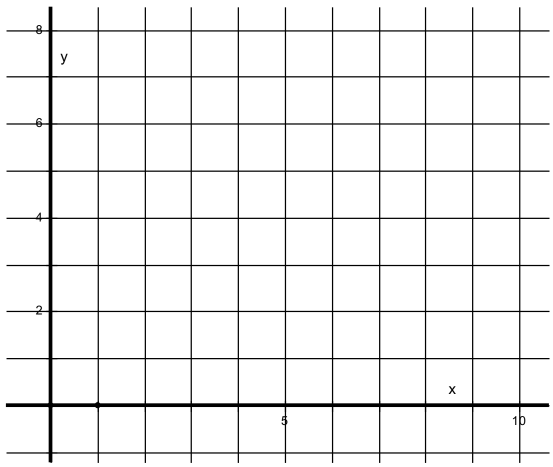


Table solution:
('move' to other points according to the slope)

x	*y*
4	*5*

Example 3:

If a line has slope of $m = \frac{1}{3}$, and the line passes through the point (-2 , -1) determine the coordinates of a point to the left and right on the same line.

Graphical solution:
('move' to other points according to the slope)

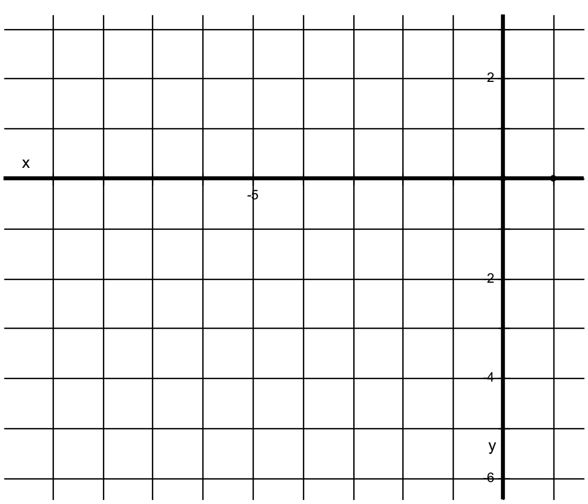


Table solution:
('move' to other points according to the slope)

x	y
-2	-1

Part 3: Use the coordinates to find another point on the line

Example 4: A line segment has one endpoint A(-2,7) and a slop of $-\frac{4}{3}$. Find the coordinates of another point on the line.

$-\frac{4}{3} = \frac{-4}{3}$ Therefore the line has a rise of ________ and a run of ________.

Add the rise to the y-coordinate and the run to the x-coordinate to find another possible point.

Other endpoint = (-2 + 3 , 7 + (-4)) =

Note: you could also subtract the rise and run to find a point to the other side on the line.

Example 5: A line segment has one endpoint A(3,-5) and a slope of $-\frac{7}{2}$. Find the coordinates of another point on the line.

Section 5.4 – Slope as a Rate of Change

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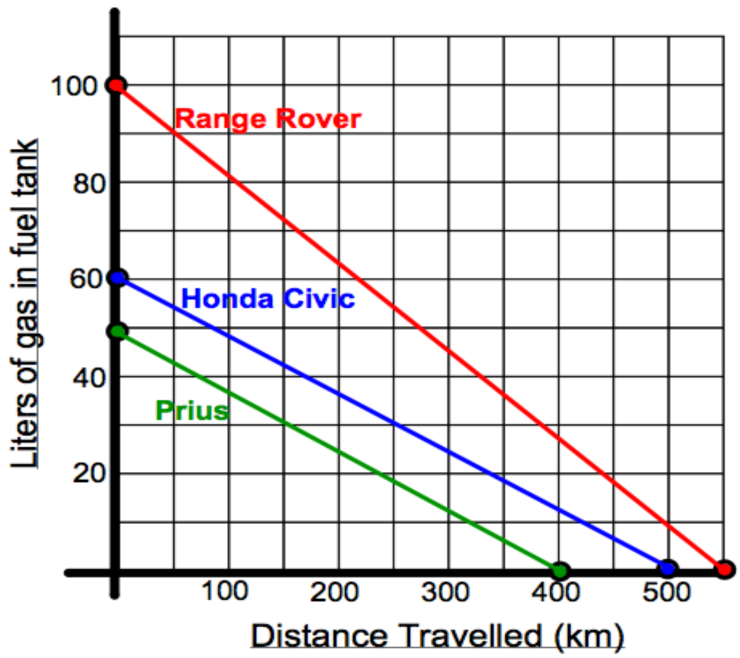
Jensen

Part 1: Do It Now



1. The independent variable is__________________________________

The dependent variable is ___________________________________

2. How can you determine the rate of fuel consumption using this data?

3. Determine the rate of gas consumption for each vehicle and then rank them in order of efficiency.

Vehicle	2010 Range Rover	2011 Prius	2007 Honda Civic
Rate of gas consumption			
Efficiency ranking			

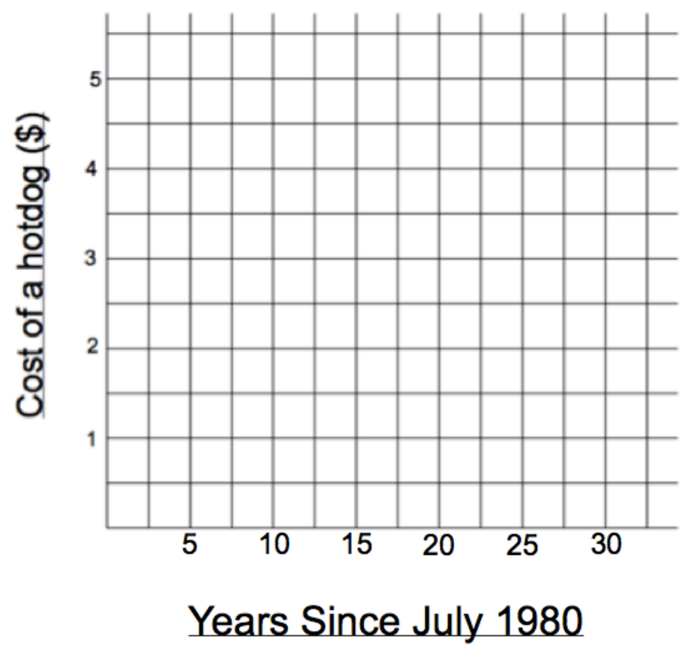
Part 2: Connecting Slope and Rate of Change

From the Do It Now question we have discovered that slope = rate of change. Look at the following table to see further how they are connected. How the linear equation is represented determines the terminology we use describe the slope.

Word problem	Table	Graph	Equation
m is the rate of change	$m = \frac{\Delta y}{\Delta x}$	$m = \frac{rise}{run}$	$m = slope$

Example 1:
The cost of a hot dog at the Rogers Centre has been going up for several years. Graph the data. Let x be the number of years since July 1980.

Years since July 1980	Cost of a hotdog ($)
0	2.50
5	2.75
10	3.00
15	3.25
20	3.50
25	3.75
30	4.00



a) Determine the slope using the graph

Rise = ________________ Run = ____________________ Slope = _____________________

b) Determine the rate of change of the cost of hot dogs using the table.

Remember:

Rate of change = $\frac{\Delta y}{\Delta x}$

c) Write an equation to represent the cost of a hot dog based on the number of years since July 1980. What part of the equation represents the slope?

Example 2: Mr. Jensen is training for a triple marathon and runs every day before school. This morning he ran 5 km in 20 minutes.

a) Calculate the rate of change of Mr. Jensen's distance from his starting point. (in this case rate of change is = average speed)

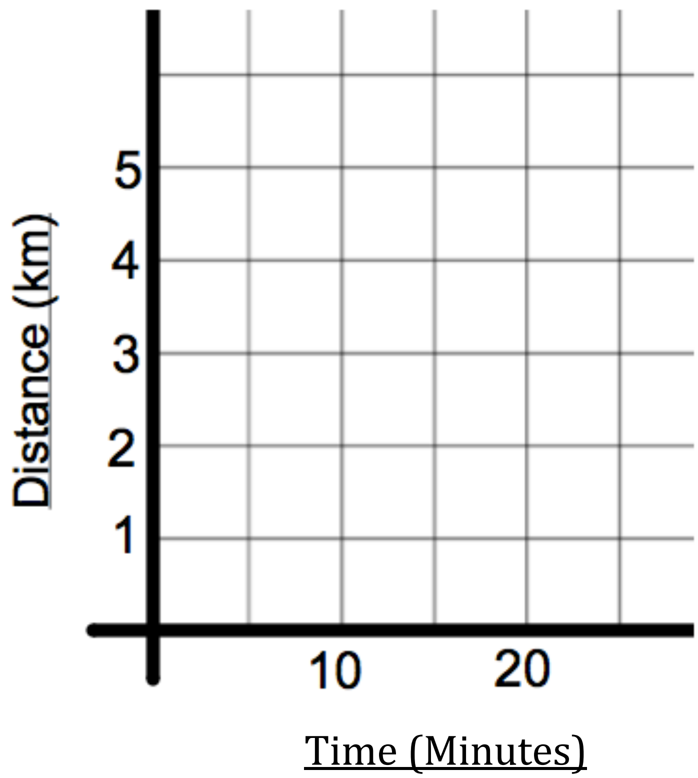
Dependent variable: ___________________________

Independent variable: _________________________

$$\text{Rate of change} = \frac{\Delta dependent\ variable}{\Delta independent\ variable}$$

Rate of change (speed) =

b) Graph distance as it relates to time



c) Calculate the slope of the line from the graph

$$\textbf{Slope} = \frac{rise}{run}$$

d) Explain the meaning of the rate of change and how it relates to the slope of the graph

Section 5.5 – First Differences

MPM1D
Jensen

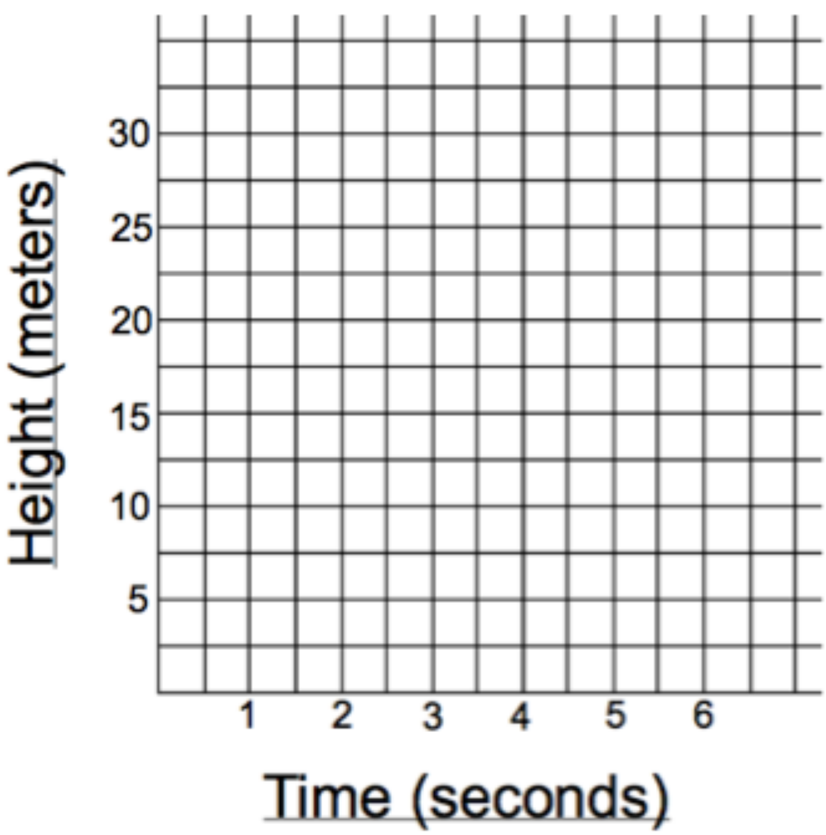
DO IT NOW

If a tennis ball falls out of the third story window of a building will its motion be linear?

The height of the ball over time is recorded in the following table.

Time (seconds)	*Height (meters)*
0	*30*
1	*29*
2	*26*
3	*21*
4	*14*
5	*5*
6	*0*

Graph the relation and determine if it represents linear motion.



Part 1:Recall

We know from graphing lines that **if the slope (rise and the run) is constant** then the relation will **form a straight line**.

$$slope = m = \frac{rise}{run} = \frac{\Delta x}{\Delta y}$$

Therefore, we need to determine if the changes in x and y are constant in a table to determine if a relation is linear.

Part 2: What are First differences

First differences are the differences between consecutive y-values in tables of values with evenly spaced x-values.

If the first differences of a relation are constant, the relation is ____________________________

If the first differences of a relation are not constant, the relation is ________________________

Notice that the differences between consecutive y-values are constant! This means it is a linear relation

Notice that the x-values change by a constant amount. This is a requirement to work with first differences!

x	y
0	0
1	3
2	6
3	9
4	12

First Differences

3-0 = 3
6-3 = 3
9-6 = 3
12-9 = 3

Part 3: Calculating First Differences

Complete a table of values for each equation given. Then determine if the first differences are constant and state whether the relation is linear or non linear.

Example 1:

$y = -2x + 7$

x	y
0	
1	
2	
3	
4	

First Differences

Conclusion:
the first differences are

____

therefore the relationship is

____

Example 2:

$y = x^2$

x	y
0	
1	
2	
3	
4	

First Differences

Conclusion:
the first differences are

____

therefore the relationship is

____

Example 3:

$y = 2^x$

x	y
0	
1	
2	
3	
4	

First Differences

Conclusion:
the first differences are

____

therefore the relationship is

____

Part 4: Check Your Understanding

Use first differences to determine which of these relations are linear and which are non linear.

Example 4:

x	y	First Differences
0	7	
1	3	
2	-1	
3	-5	
4	-9	

Type of relation:____________________

Example 5:

x	y	First Differences
2	-5	
3	10	
4	25	
5	40	
6	55	

Type of relation:____________________

Example 6:

x	y	First Differences
-2	-10	
-1	-2	
0	0	
1	2	
2	10	

Type of relation:____________________

Section 5.6 – Connecting Everything

MPM1D
Jensen

Part 1: Do It Now

a) Calculate the first differences

Type of relation:

x	y
0	-3
3	1
6	5
9	9
12	13

First Differences

b) Using the table of values, what is the constant of variation (slope)?

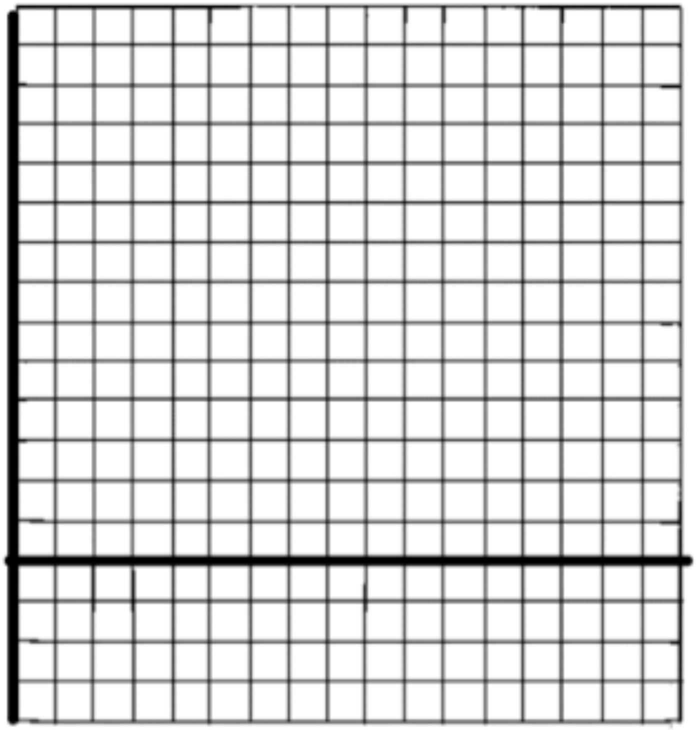
Remember: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

c) What is the initial value (y-intercept)?

d) Is this a direct variation or partial variation?

e) Write an equation for the relation in the form $y = mx + b$ using the constant of variation (m) and the initial value (b)

f) Graph the relation



g) Find the slope of the line from the graph. How does this relate to the constant of variation?

h) What is the y-intercept? How does this relate to the initial value?

i) Write the equation of the line in the form $y = mx + b$ using the slope and y-intercept

Part 2: The Rule of Four

A relation can be represented in a variety of ways so that it can be looked at from different points of view. A mathematical relation can be described in four ways:

1. Using words
2. Using a graph
3. Using a table of values
4. Using an equations

Part 3: Write an equation when the relation is represented in words

Remember that the equation of a line is $y = mx + b$

Considering that a line is really just a set of ordered pairs, (x , y), it makes sense that the equation of a line needs to contain the variables x and y. These variables will define the coordinates that make up the line.

This means that the only 2 values that need to be determined in order to write the equation of a linear relation are m and b.

When a linear relation is represented in words m is the rate of change and b is the initial value.

Linear relation represented in words:	***m*** **= rate of change (slope)**
	b **= initial value (y-intercept)**

Example 1: Write an equation for the following relationship by first identifying the value of m and b.

The Copy Centre charges $75 to design a poster plus 25 cents **for each copy**.

m = b =

And the equation of this linear relation is:

If The Copy Centre changed their cost per flyer to 35 cents for each copy the equation would become:

If The Copy Centre changed their design cost to $125 the equation will become:

Example 2: y varies directly with x. When $x=2$, $y=8$.

a) What is the initial value?

b) What is the slope of the line?

c) Write an Equation for this relationship

Example 3: y varies partially with x. When $x=0$, $y=3$, and when $x=2$, $y=8$.

a) What is the initial value?

b) What is the slope of the line?

c) Write an Equation for this relationship

Part 4: Write an equation when the relation is represented in a table of values

Remember:

$$slope = rate\ of\ change = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = initial\ value = y - intercept = value\ of\ y\ when\ x\ is\ 0$$

Example 4: Determine the equation of the following linear relations using the tables provided:

a)

x	y
0	6
1	9
2	12
3	15
4	18

m =

b =

Equation:

b)

x	y
-2	3
0	5
2	7
4	9
6	11

m =

b =

Equation:

What should we do if the initial value isn't in the table?

c)

x	y
2	-11
3	-8
4	-5

$m =$

$b =$

Equation:

d)

x	y
-8	-5
-6	-10
-4	-15

$m =$

$b =$

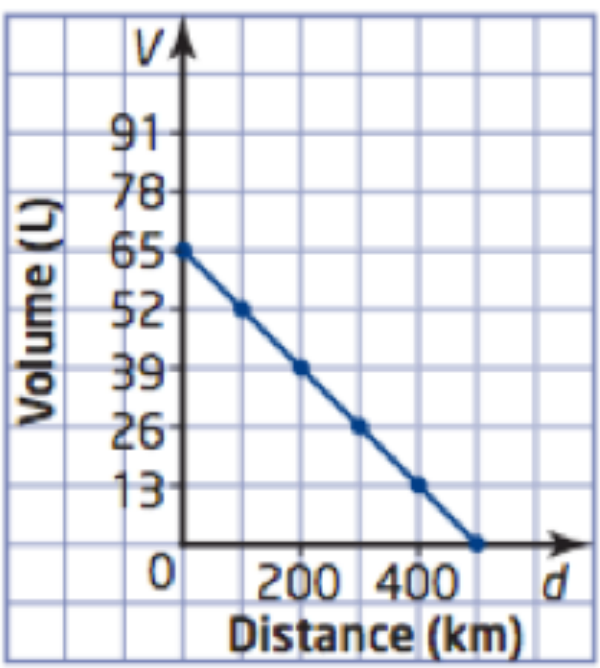
Equation:

Part 5: Write an equation when the relation is represented as a graph

Example 5: The graph shows the relationship between the volume of gasoline remaining in a car's fuel tank and the distance driven.

Remember: $m = slope = \frac{rise}{run}$

$b = initial\ value = y - intercept$



Slope:

***y*-intercept:**

Equation: