5.1 Worksheet – Probability Distributions

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Jensen

1) Which of the following are valid probability distributions? Explain

a)

x	$P(x)$
0	0.5
1	0.25
2	0.25

b)

x	$P(x)$
0.5	0.2
0.2	0.3
0.3	0.25

c)

x	$P(x)$
0	0.3
1	0.25
2	0.25
3	0.2

a & c are valid. b is not valid because the probabilities do not add to 1.

2) Given the following probability distributions, determine the expected values.

a)

x	$P(x)$
5	0.3
10	0.25
15	0.45

b)

x	$P(x)$
1 000	0.25
100 000	0.25
1 000 000	0.25
10 000 000	0.25

c)

x	$P(x)$
1	$\frac{1}{6}$
2	$\frac{1}{5}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{20}$

a) $E(x) = \sum x \cdot P(x) = 5(0.3) + 10(0.25) + 15(0.45) = 10.75$

b) $E(x) = \sum x \cdot P(x) = 1000(0.25) + 100000(0.25) + 1000000(0.25) + 10000000(0.25) = 2\ 775\ 250$

c) $E(x) = \sum x \cdot P(x) = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{3}\right) + 5\left(\frac{1}{20}\right) = 2.9$

3) A spinner has eight equally-sized sectors, numbered 1 through 8.

a) Create a probability distribution for the outcome of a spin

Spin, x	$P(x)$
1	$\frac{1}{8}$
2	$\frac{1}{8}$
3	$\frac{1}{8}$
4	$\frac{1}{8}$
5	$\frac{1}{8}$
6	$\frac{1}{8}$
7	$\frac{1}{8}$
8	$\frac{1}{8}$

b) What is the probability that the arrow on the spinner will stop on a prime number?

$$P(prime) = P(2) + P(3) + P(5) + P(7) = \frac{4}{8} = \frac{1}{2}$$

c) What is the expected outcome?

$$E(x) = \sum x \cdot P(x) = 1\left(\frac{1}{8}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right) + 7\left(\frac{1}{8}\right) + 8\left(\frac{1}{8}\right) = 4.5$$

4) A lottery has a $1 000 000 first prize, a $25 000 second prize, and five $1 000 third prizes. A total of 2 000 000 tickets are sold.

a) Create a probability distribution for the amount of money you could win

Winnings, x	**Probability, $P(x)$**
1 000 000	$\frac{1}{2\,000\,000}$
25 000	$\frac{1}{2\,000\,000}$
1 000	$\frac{5}{2\,000\,000}$
0	$\frac{1\,999\,993}{2\,000\,000}$

b) Calculate the expected winnings

$$E(x) = \sum x \cdot P(x) = 1\,000\,000\left(\frac{1}{2\,000\,000}\right) + 25\,000\left(\frac{1}{2\,000\,000}\right) + 1\,000\left(\frac{5}{2\,000\,000}\right) + 0\left(\frac{1\,999\,993}{2\,000\,000}\right)$$

$$= \frac{1\ 030\ 000}{2\ 000\ 000}$$

$$= 0.515$$

The expected winnings is about $0.52

c) If you buy a ticket for $2.00, what is your expected profit?

$Expected\ profit = expected\ winnings - cost\ of\ ticket = 0.52 - 2 = -1.48$

You should expect to lose $1.48.

5) A game consists of rolling a die. If an even number shows, you receive double the value of the upper face in points. If an odd number shows, you lose points equivalent to triple the value of the upper face.

a) Create a probability distribution for the amount of points you receive on a single roll

Points, x	$P(x)$
-3	$\frac{1}{6}$
4	$\frac{1}{6}$
-9	$\frac{1}{6}$
8	$\frac{1}{6}$
-15	$\frac{1}{6}$
12	$\frac{1}{6}$

b) How many points would you expect to get on a single roll?

$$E(x) = \sum x \cdot P(x) = -3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) - 9\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) - 15\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) = -0.5$$

6) Make a probability distribution from the following frequency distribution to represent the number of fish caught in a 6-hour period. Then calculate the expected number of fish caught in 6 hours.

# of fish caught	0	1	2	3	4
frequency	88	72	30	8	2

# of fish caught, x	0	1	2	3	4
$P(x)$	$\frac{88}{200}$	$\frac{72}{200}$	$\frac{30}{200}$	$\frac{8}{200}$	$\frac{2}{200}$

$$E(x) = \sum x \cdot P(x) = 0\left(\frac{88}{200}\right) + 1\left(\frac{72}{200}\right) + 2\left(\frac{30}{200}\right) + 3\left(\frac{8}{200}\right) + 4\left(\frac{2}{200}\right) = \frac{164}{200} = 0.82$$

5.2 Worksheet – Hypergeometric Probability Distributions

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1) A customer randomly selects two RAM modules from a shipment of six known to contain two defective modules.

a) Create the probability distribution for x, the number of defective modules in the purchase.

# of defective modules purchased, *x*	$\boldsymbol{P(x)}$
0	$\frac{\binom{2}{0}\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15}$
1	$\frac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}} = \frac{8}{15}$
2	$\frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}} = \frac{1}{15}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{2}{x}\binom{4}{2-x}}{\binom{6}{2}}$$

b) Compute the expected number of defective RAM modules the customer would purchase.

$E(X) = \sum x \cdot P(x) = 0\left(\frac{6}{15}\right) + 1\left(\frac{8}{15}\right) + 2\left(\frac{1}{15}\right) = \frac{10}{15} = \frac{2}{3}$ or 0.67

2) A drawer contains four red socks and two blue socks. Three socks are drawn from the drawer without replacement.

a) Create a probability distribution in which the random variable represents the number of red socks.

# of Red Socks (X)	$\boldsymbol{P(X)}$
0	$\frac{\binom{4}{0}\binom{2}{3}}{\binom{6}{3}} = \frac{0}{20} = 0$
1	$\frac{\binom{4}{1}\binom{2}{2}}{\binom{6}{3}} = \frac{4}{20} = \frac{1}{5}$
2	$\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} = \frac{12}{20} = \frac{3}{5}$
3	$\frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = \frac{4}{20} = \frac{1}{5}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{4}{x}\binom{2}{3-x}}{\binom{6}{3}}$$

b) Determine the expected number of red socks if three are drawn from the drawer without replacement.

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{0}{20}\right) + 1\left(\frac{4}{20}\right) + 2\left(\frac{12}{20}\right) + 3\left(\frac{4}{20}\right) = \frac{40}{20} = 2$$

3) There are five cats and seven dogs in a pet shop. Four pets are chosen at random for a visit to a children's hospital.

a) Create a probability distribution for the number of dogs chosen for a random visit to the hospital.

# of Dogs (X)	$P(X)$
0	$\frac{\binom{7}{0}\binom{5}{4}}{\binom{12}{4}} = \frac{5}{495} = \frac{1}{99}$
1	$\frac{\binom{7}{1}\binom{5}{3}}{\binom{12}{4}} = \frac{70}{495} = \frac{14}{99}$
2	$\frac{\binom{7}{2}\binom{5}{2}}{\binom{12}{4}} = \frac{210}{495} = \frac{14}{33}$
3	$\frac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}} = \frac{175}{495} = \frac{35}{99}$
4	$\frac{\binom{7}{4}\binom{5}{0}}{\binom{12}{4}} = \frac{35}{495} = \frac{7}{99}$

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{7}{x}\binom{5}{4-x}}{\binom{12}{4}}$$

b) What is the probability that at least one dog is chosen to go?

$$P(\geq 1\ dog) = 1 - P(0\ dogs) = 1 - \frac{1}{99} = \frac{98}{99}$$

c) What is the expected number of dogs chosen?

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{5}{495}\right) + 1\left(\frac{70}{495}\right) + 2\left(\frac{210}{495}\right) + 3\left(\frac{175}{495}\right) + 4\left(\frac{35}{495}\right) = \frac{1155}{495} = 2.33$$

4) A 12-member jury for a criminal case will be selected from a pool of 14 men and 11 women.

a) What is the probability that the jury will have an equal number of men and women?

$$P(6\ men, 6\ women) = \frac{\binom{14}{6}\binom{11}{6}}{\binom{25}{12}} = \frac{1\ 387\ 386}{5\ 200\ 300} = 0.2668$$

b) What is the probability that at least 3 jurors will be women?

$$P(\geq 3\ women) = 1 - P(0\ women) - P(1\ woman) - P(2\ women)$$

$$P(\geq 3\ women) = 1 - \frac{\binom{14}{12}\binom{11}{0}}{\binom{25}{12}} - \frac{\binom{14}{11}\binom{11}{1}}{\binom{25}{12}} - \frac{\binom{14}{10}\binom{11}{2}}{\binom{25}{12}}$$

$$P(\geq 3\ women) = 1 - \frac{91}{5\ 200\ 300} - \frac{4\ 004}{5\ 200\ 300} - \frac{55\ 055}{5\ 200\ 300}$$

$$P(\geq 3\ women) = \frac{5\ 141\ 150}{5\ 200\ 300}$$

$$P(\geq 3\ women) = 0.9886$$

c) What is the expected number of women? *(Note: the formula $E(x) = r\left(\frac{a}{n}\right)$ can be used for hypergeometric distributions)*

$E(x) = 12\left(\frac{11}{25}\right) = 5.28$

5) The door prizes at a dance are four \$10 gift certificates, five \$20 gift certificates, and three \$50 gift certificates. The prize envelopes are mixed together in a bag, and five prizes are drawn at random.

a) Create a probability distribution for the number of \$10 gift certificates drawn.

$$P(x) = \frac{\binom{a}{x}\binom{b}{r-x}}{\binom{n}{r}}$$

$$P(x) = \frac{\binom{4}{x}\binom{8}{5-x}}{\binom{12}{5}}$$

# of \$10 Certificates Drawn (X)	$\boldsymbol{P(X)}$
0	$\frac{\binom{4}{0}\binom{8}{5}}{\binom{12}{5}} = \frac{56}{792} = \frac{7}{99}$
1	$\frac{\binom{4}{1}\binom{8}{4}}{\binom{12}{5}} = \frac{280}{792} = \frac{35}{99}$
2	$\frac{\binom{4}{2}\binom{8}{3}}{\binom{12}{5}} = \frac{336}{792} = \frac{14}{33}$
3	$\frac{\binom{4}{3}\binom{8}{2}}{\binom{12}{5}} = \frac{112}{792} = \frac{14}{99}$
4	$\frac{\binom{4}{4}\binom{8}{1}}{\binom{12}{5}} = \frac{8}{792} = \frac{1}{99}$

b) What is the expected number of \$10 gift certificates drawn?

$$E(X) = \sum x \cdot P(x) = 0\left(\frac{56}{792}\right) + 1\left(\frac{280}{792}\right) + 2\left(\frac{336}{792}\right) + 3\left(\frac{112}{792}\right) + 4\left(\frac{8}{792}\right) = \frac{1320}{792} = 1.67$$

OR

$$E(x) = r\left(\frac{a}{n}\right) = 5\left(\frac{4}{12}\right) = 1.67$$

5.3 Worksheet – Binomial Probability Distributions

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1) For each term, identify i) the number of trials ii) the probability p of a success iii) the number of successes

a) $\binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4$

i) $n = 10$ ii) $p = \frac{1}{2}$ iii) $k = 6$

b) $\binom{7}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4$

i) $n = 7$ ii) $p = \frac{1}{3}$ iii) $k = 3$

2) Mail-order marketing companies have a response rate of 15% to their advertising flyers.

a) Compute the probability that exactly 3 people out of a sample of 20 respond to the flyers they receive.

$n = 20$

$p = 0.15$

$k = 3$

$P(X = 3) = \binom{20}{3}(0.15)^3(0.85)^{17} = 0.243$

OR

$P(X = 3) = binompdf(n = 20, p = 0.15, k = 3) = 0.243$

b) Find the expected number of people in a sample of 20 who will respond to the flyers.

$E(x) = np = 20(0.15) = 3$

c) Compute the probability that at least 3 people out of a sample of 20 respond to the flyers they receive.

$P(X \geq 3) = 1 - P(X \leq 2)$
$= 1 - binomcdf(n = 20, p = 0.15, k = 2)$
$= 1 - 0.4049$
$= 0.5951$

3) A study published in a consumer magazine indicated that when a husband and a wife shop for a car, the husband exerts the primary influence in the decision 70% of the time. Five couples who will be purchasing a car are selected at random. Determine the probability of each of the following.

a) In exactly two of the couples, the husband will exert the primary influence on the decision.

$n = 5$

$p = 0.7$

$k = 2$

$P(X = 2) = \binom{5}{2}(0.7)^2(0.3)^3 = 0.1323$

OR

$P(X = 2) = binompdf(n = 5, p = 0.7, k = 2) = 0.1323$

b) In all five couples, the husband will exert the primary influence on the decision.

$P(X = 5) = \binom{5}{5}(0.7)^5(0.3)^0 = 0.16807$

OR

$P(X = 2) = binompdf(n = 5, p = 0.7, k = 5) = 0.16807$

c) Find the expected number of couples in which the husband will exert primary influence.

$E(x) = np = 5(0.7) = 3.5$

4) A baseball player has a batting average of 0.280. Find each of the following probabilities

a) Exactly 4 hits in her next 10 times at bat

$P(X = 4) = \binom{10}{4}(0.28)^4(0.72)^6 = 0.1798$

OR

$P(X = 4) = binompdf(n = 10, p = 0.28, k = 4) = 0.1798$

b) More than 4 hits in her next 10 times at bat

$P(X > 4) = 1 - P(X \leq 4)$
$= 1 - binomcdf(n = 10, p = 0.28, k = 4)$
$= 1 - 0.8819$
$= 0.1181$

c) Less than 6 hits in her next 10 times at bat

$P(X < 6) = P(X \leq 5)$
$= binomcdf(n = 10, p = 0.28, k = 5)$
$= 0.9658$

d) What is the player's expected number of hits in her next 10 times at bat?

$E(x) = np = 10(0.28) = 2.8$

5) Ten percent of the keyboards a computer company manufactures are defective. Determine the probability that one or more of the next three keyboards to come off the assembly line will be defective.

$P(X \geq 1) = 1 - P(0)$
$= 1 - binompdf(n = 3, p = 0.1, k = 0)$
$= 1 - 0.729$
$= 0.271$

6) Determine the probability, correct to four decimal places, that a die rolled six times in a row will produce the following:

a) one 3

$$P(X = 1) = \binom{6}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5 = 0.4019$$

OR

$$P(X = 1) = binompdf\left(n = 6, p = \frac{1}{6}, k = 1\right) = 0.4019$$

b) at least five 3's

$P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - binompdf\left(n = 6, p = \frac{1}{6}, k = 4\right)$
$= 1 - 0.9993$
$= 0.0007$

c) two 3's or less

$$P(X \leq 2) = binomcdf\left(n = 6, p = \frac{1}{6}, k = 2\right) = 0.9377$$

7) A multiple-choice quiz has 10 questions. Each question has four possible answers. Sam is certain that he knows the correct answer for Questions, 3, 5, and 8. If he guesses on the other questions, determine the probability that he gets at least 50% on the quiz.

To get at least 50%, Sam will need to guess at least 2 of the remaining seven questions correctly.

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - binomcdf(n = 7, p = 0.25, k = 1) = 1 - 0.4449 = 0.5551$$

8) A survey indicates that 41% of women in the United States consider reading as their favorite leisure-time activity. You randomly select four U.S. women and ask them if reading is their favorite leisure-time activity.

a) Create a binomial probability distribution for the number of women who respond yes.

# of women whose favorite leisure activity is reading (X)	**P(X)**
0	$\binom{4}{0}(0.41)^0(0.59)^4 = 0.121$ $binompdf(n = 4, p = 0.41, k = 0) = 0.121$
1	$\binom{4}{1}(0.41)^1(0.59)^3 = 0.337$ $binompdf(n = 4, p = 0.41, k = 1) = 0.337$
2	$\binom{4}{2}(0.41)^2(0.59)^2 = 0.351$ $binompdf(n = 4, p = 0.41, k = 2) = 0.351$
3	$\binom{4}{3}(0.41)^3(0.59)^1 = 0.163$ $binompdf(n = 4, p = 0.41, k = 3) = 0.163$
4	$\binom{4}{4}(0.41)^4(0.59)^0 = 0.028$ $binompdf(n = 4, p = 0.41, k = 4) = 0.028$

b) What is the probability that at least two of them respond yes?

$P(\geq 2) = P(2) + P(3) + P(4) = 0.351 + 0.163 + 0.028 = 0.542$

c) What is the expected number of women in the group of 4 that would choose reading as their favourite leisure time activity?

$E(x) = np = 4(0.41) = 1.64$

5.4 Worksheet – Geometric Probability Distributions

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1) To start her old lawn mower, Rita has to pull a cord and hope for some luck. On any particular pull, the mower has a 20% chance of starting.

a) Find the probability that it takes her exactly 3 pulls to start the mower.

$P(Y = k) = (1 - p)^{k-1}p$
$P(pulls = 3) = (1 - 0.2)^{3-1}(0.2)$
$P(pulls = 3) = (0.8)^2(0.2)$
$P(pulls = 3) = 0.128$

OR

$P(pulls = 3) = geometpdf(p = 0.2, k = 3) = 0.128$

b) Find the probability that it takes her 10 or fewer pulls to start the mower.

$P(pulls \leq 10) = geometcdf(p = 0.2, k = 10) = 0.8926$

2) Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot.

a) How many spins do you expect it to take until Marti wins? Justify your answer.

$$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{38}\right)} = 38$$

b) What is the probability that it takes 5 spins before Marti wins?

$P(Y = k) = (1 - p)^{k-1}p$
$P(spins = 5) = \left(1 - \frac{1}{38}\right)^{5-1}\left(\frac{1}{38}\right)$
$P(spins = 5) = \left(\frac{37}{38}\right)^4\left(\frac{1}{38}\right)$
$P(spins = 5) = 0.0237$

OR

$P(spins = 5) = geometpdf\left(p = \frac{1}{38}, k = 5\right) = 0.0237$

c) What is the probability that it will take Marti more than 50 spins to win?

$P(spins > 50) = 1 - P(\leq 50) = 1 - geometcdf\left(p = \frac{1}{38}, k = 50\right) = 1 - 0.7364 = 0.2636$

3) To finish a board game, Sarah needs to land on the last square by rolling a sum of 2 with two dice.

a) What is the probability that it takes her 8 tries before she wins?

$P(Y = k) = (1 - p)^{k-1}p$
$P(rolls = 8) = \left(1 - \frac{1}{36}\right)^{8-1}\left(\frac{1}{36}\right)$
$P(rolls = 8) = \left(\frac{35}{36}\right)^{7}\left(\frac{1}{36}\right)$
$P(rolls = 8) = 0.0228$

OR

$P(rolls = 8) = geometpdf\left(p = \frac{1}{36}, k = 8\right) = 0.0228$

b) What is the probability that she wins in under 5 tries?

$P(rolls < 5) = P(rolls \le 4) = geometcdf\left(p = \frac{1}{36}, k = 4\right) = 0.1066$

c) How many rolls would you expect it to take until she wins?

$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{36}\right)} = 36$

4) Suppose that 1 out of 50 cards in a scratch-and-win promotion gives a prize.

a) What is the probability of you not winning until your fourth try?

$P(Y = k) = (1 - p)^{k-1}p$
$P(cards = 4) = \left(1 - \frac{1}{50}\right)^{4-1}\left(\frac{1}{50}\right)$
$P(cards = 4) = \left(\frac{49}{50}\right)^{3}\left(\frac{1}{50}\right)$
$P(cards = 4) = 0.0188$

OR

$P(cards = 4) = geometpdf\left(p = \frac{1}{50}, k = 4\right) = 0.0188$

b) What is the probability that of winning in 10 tries or less?

$P(rolls \le 10) = geometcdf\left(p = \frac{1}{50}, k = 10\right) = 0.1829$

c) What is the expected number of scratch-and-win cards you need to play until winning?

$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{50}\right)} = 50$

5) A top NHL hockey player scores on 93% of his shots in a shooting competition.

a) What is the probability that the player will not miss the goal until his 20th try?

$P(Y = k) = (1 - p)^{k-1}p$
$P(shots = 20) = (1 - 0.07)^{20-1}(0.07)$
$P(shots = 20) = (0.93)^{19}(0.07)$
$P(shots = 20) = 0.0176$

OR

$P(shots = 20) = geometpdf(p = 0.07, k = 20) = 0.0176$

b) What is the probability that he takes more than 20 shots before missing?

$P(shots > 20) = 1 - P(shots \leq 20) = 1 - geometcdf(p = 0.07, k = 20) = 1 - 0.7658 = 0.2342$

c) What is the expected number of shots taken until he gets his first miss?

$E(Y) = \frac{1}{p} = \frac{1}{0.07} = 14.3$

5.5 Worksheet – Binomial Theorem

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1) Use Pascal's Identity to write an expression of the form $\binom{n}{r}$ that is equivalent to each of the following

a) $\binom{10}{2} + \binom{10}{3}$

$= \binom{11}{3}$

b) $\binom{20}{18} + \binom{20}{19}$

$= \binom{21}{19}$

c) $\binom{15}{14} + \binom{15}{13}$

$= \binom{16}{14}$

2) In the expansion of $(x + y)^{10}$, write the value of the exponent k in the term that contains

a) x^4y^k

$4 + k = 10$
$k = 6$

b) x^ky^8

$k + 8 = 10$
$k = 2$

c) x^ky^{4k}

$k + 4k = 10$
$5k = 10$
$k = 2$

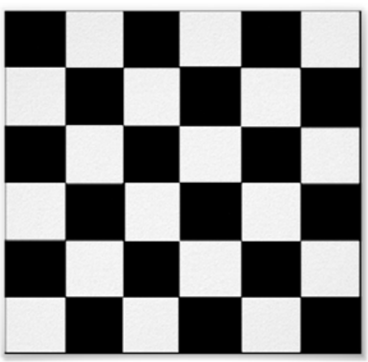
d) $x^{k-2}y^{3k}$

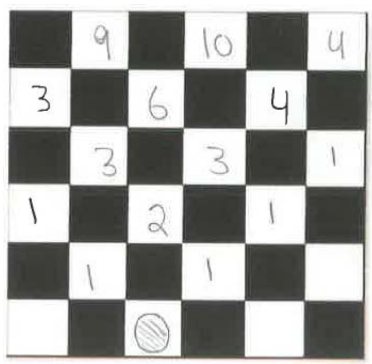
$(k - 2) + 3k = 10$
$4k = 12$
$k = 3$

3) a) Imagine that a checker is placed in the bottom left corner of a 6-by-6 checker board. The piece may be moved one square at a time diagonally left or right to the next row up. Calculate the number of different paths to the top row.

$n(paths) = 5 + 4 + 1 = 10$

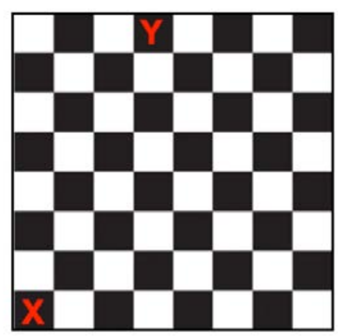
b) Suppose the checker began in the third square from the left in the bottom row. Calculate the number of possible paths to the top row from this position.





$n(paths) = 9 + 10 + 4 = 23$

4) How many paths are there from X to Y…



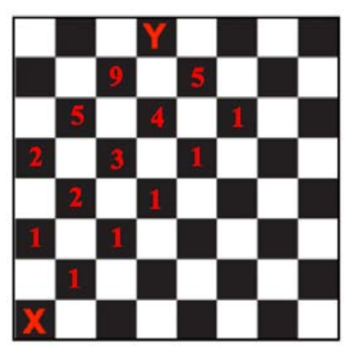
a) As a checkerboard where you can only move diagonally forward

$n(paths) = 9 + 5 = 14$

b) As a grid where you can only move north and east

Must move 7 North and 3 East

$n(routes) = \binom{10}{7} = 120$ *OR* $n(routes) = \frac{10!}{7!3!} = 120$



5) Faizel wants to travel from his house to the hardware store that is six blocks east and five blocks south of his home. If he walks east and south, how many different routes can he follow from his home to the store?

Faizel has to travel 11 blocks, 6 of which are east and 5 of which are south.

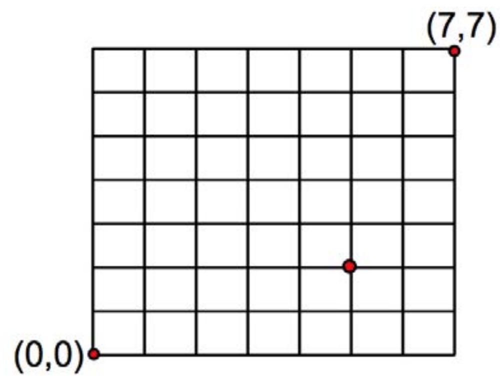
$n(routes) = \binom{11}{6} = 462$ OR $n(routes) = \binom{11}{5} = 462$ OR $n(routes) = \frac{11!}{6!5!} = 462$

6) a) Starting at (0, 0) and moving only North and East, how many routes pass through (5, 2) and end at (7, 7)?

$n(routes\ pass\ through\ 5{,}2) = n(routes\ from\ 0{,}0\ to\ 5{,}2) \times n(routes\ from\ 5{,}2\ to\ 7{,}7)$

$n(routes) = \binom{7}{2} \times \binom{7}{2}$

$n(routes) = 441$



b) Starting at (0, 0) and moving only North and East, how many routes avoid (5, 2) and end at (7, 7) ?

$n(routes\ avoid\ 5{,}2) = n(routes\ from\ 0{,}0\ to\ 7{,}7) - n(routes\ pass\ through\ 5{,}2)$

$n(routes\ avoid\ 5{,}2) = \binom{14}{7} - 441$

$n(routes\ avoid\ 5{,}2) = 3432 - 441$

$n(routes\ avoid\ 5{,}2) = 2991$

7) Expand and simplify each of the following using the Binomial Theorem

a) $(a + 2b)^4$

$= \binom{4}{0}(a)^{4-0}(2b)^0 + \binom{4}{1}(a)^{4-1}(2b)^1 + \binom{4}{2}(a)^{4-2}(2b)^2 + \binom{4}{3}(a)^{4-3}(2b)^3 + \binom{4}{4}(a)^{4-4}(2b)^4$

$= \binom{4}{0}(a)^4(2b)^0 + \binom{4}{1}(a)^3(2b)^1 + \binom{4}{2}(a)^2(2b)^2 + \binom{4}{3}(a)^1(2b)^3 + \binom{4}{4}(a)^0(2b)^4$

$= 1(a)^4(1) + 4(a)^3(2)(b) + 6(a)^2(4)(b)^2 + 4(a)^1(8)(b)^3 + 1(1)(16)(b)^4$

$= a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4$

b) $(x-y)^6$

$= \binom{6}{0}(x)^{6-0}(-1y)^0 + \binom{6}{1}(x)^{6-1}(-1y)^1 + \binom{6}{2}(x)^{6-2}(-1y)^2 + \binom{6}{3}(x)^{6-3}(-1y)^3 + \binom{6}{4}(x)^{6-4}(-1y)^4 + \binom{6}{5}(x)^{6-5}(-1y)^5 + \binom{6}{6}(x)^{6-6}(-1y)^6$

$= \binom{6}{0}(x)^{6}(-1y)^0 + \binom{6}{1}(x)^{5}(-1y)^1 + \binom{6}{2}(x)^{4}(-1y)^2 + \binom{6}{3}(x)^{3}(-1y)^3 + \binom{6}{4}(x)^{2}(-1y)^4 + \binom{6}{5}(x)^{1}(-1y)^5 + \binom{6}{6}(x)^{0}(-1y)^6$

$= 1(x)^6(1) + 6(x)^5(-1)(y)^1 + 15(x)^4(1)(y)^2 + 20(x)^3(-1)(y)^3 + 15(x)^2(1)(y)^4 + 6(x)^1(-1)(y)^5 + 1(1)(1)(y)^6$

$= x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$

c) $\left(c+\frac{1}{c}\right)^4$

$= \binom{4}{0}(c)^{4-0}\left(\frac{1}{c}\right)^0 + \binom{4}{1}(c)^{4-1}\left(\frac{1}{c}\right)^1 + \binom{4}{2}(c)^{4-2}\left(\frac{1}{c}\right)^2 + \binom{4}{3}(c)^{4-3}\left(\frac{1}{c}\right)^3 + \binom{4}{4}(c)^{4-4}\left(\frac{1}{c}\right)^4$

$= \binom{4}{0}(c)^{4}\left(\frac{1}{c}\right)^0 + \binom{4}{1}(c)^{3}\left(\frac{1}{c}\right)^1 + \binom{4}{2}(c)^{2}\left(\frac{1}{c}\right)^2 + \binom{4}{3}(c)^{1}\left(\frac{1}{c}\right)^3 + \binom{4}{4}(c)^{0}\left(\frac{1}{c}\right)^4$

$= 1(c)^4(1) + 4(c)^3(c)^{-1} + 6(c)^2(c)^{-2} + 4(c)^1(c)^{-3} + 1(1)(c)^{-4}$

$= c^4 + 4c^2 + 6c^0 + 4c^{-2} + c^{-4}$

$= c^4 + 4c^2 + 6 + \frac{4}{c^2} + \frac{1}{c^4}$

d) $\left(d-\frac{1}{d}\right)^5$

$= \binom{5}{0}(d)^{5-0}\left(\frac{-1}{d}\right)^0 + \binom{5}{1}(d)^{5-1}\left(\frac{-1}{d}\right)^1 + \binom{5}{2}(d)^{5-2}\left(\frac{-1}{d}\right)^2 + \binom{5}{3}(d)^{5-3}\left(\frac{-1}{d}\right)^3 + \binom{5}{4}(d)^{5-4}\left(\frac{-1}{d}\right)^4 + \binom{5}{5}(d)^{5-5}\left(\frac{-1}{d}\right)^5$

$= 1(d)^5(1) + 5(d)^4(-1)(d)^{-1} + 10(d)^3(1)(d)^{-2} + 10(d)^2(-1)(d)^{-3} + 5(d)^1(1)(d)^{-4} + 1(d)^0(-1)(d)^{-5}$

$= d^5 - 5d^3 + 10d - 10d^{-1} + 5d^{-3} - 1d^{-5}$

$= d^5 - 5d^3 + 10d - \frac{10}{d} + \frac{5}{d^3} - \frac{1}{d^5}$

8) Find an expression for the general term, in simplified form, for each of the following…

a) $(x+y)^{10}$

$t_{r+1} = \binom{10}{r}(x)^{10-r}(y)^r$

b) $(x-y)^{10}$

$t_{r+1} = \binom{10}{r}(x)^{10-r}(-1y)^r$

$t_{r+1} = \binom{10}{r}(-1)^r(x)^{10-r}(y)^r$

c) $\left(z + \frac{1}{z}\right)^8$

$t_{r+1} = \binom{8}{r}(z)^{8-r}\left(\frac{1}{z}\right)^r$

$t_{r+1} = \binom{8}{r}(1)^r(z)^{8-r}(z)^{-r}$

$t_{r+1} = \binom{8}{r}(z)^{8-2r}$

d) $\left(w^2 + \frac{1}{w}\right)^9$

$t_{r+1} = \binom{9}{r}(w^2)^{9-r}\left(\frac{1}{w}\right)^r$

$t_{r+1} = \binom{9}{r}(1)^r(w)^{18-2r}(w)^{-r}$

$t_{r+1} = \binom{9}{r}(w)^{18-3r}$

9) Find an expression for the indicated term in the expansion of each of the following...

a) the third term of $(x^2 - 2)^7$

$t_{r+1} = \binom{n}{r}a^{n-r}b^r$

$t_{2+1} = \binom{7}{2}(x^2)^{7-2}(-2)^2$

$t_3 = 21(x^2)^5(4)$

$t_3 = 84x^{10}$

b) the middle term of $(c - d)^8$

There are 9 terms in the expansion, therefore the 5th term is the middle term.

$t_{r+1} = \binom{n}{r}a^{n-r}b^r$

$t_{4+1} = \binom{8}{4}(c)^{8-4}(-1d)^4$

$t_5 = 70(c)^4(1)(d)^4$

$t_5 = 70c^4d^4$

c) the tenth term of $\left(\frac{x}{3}-\frac{3}{x}\right)^{12}$

$$t_{r+1} = \binom{n}{r}a^{n-r}b^{r}$$

$$t_{9+1} = \binom{12}{9}\left(\frac{x}{3}\right)^{12-9}\left(\frac{-3}{x}\right)^{9}$$

$$t_{10} = 220\left(\frac{x^3}{27}\right)\left(\frac{-19\ 683}{x^9}\right)$$

$$t_{10} = \frac{-160\ 380}{x^6}$$

10) In the expansion of $\left(4x-\frac{2}{x}\right)^{8}$, find the following...

a) the term containing x^6

General term:

$$t_{r+1} = \binom{n}{r}a^{n-r}b^{r}$$

$$t_{r+1} = \binom{8}{r}(4x)^{8-r}\left(\frac{-2}{x}\right)^{r}$$

$$t_{r+1} = \binom{8}{r}(4)^{8-r}(-2)^{r}(x)^{8-r}(x)^{-r}$$

$$t_{r+1} = \binom{8}{r}(4)^{8-r}(-2)^{r}(x)^{8-2r}$$

Term with x^6:

$$t_{1+1} = \binom{8}{1}(4)^{8-1}(-2)^{1}(x)^{8-2(1)}$$

$$t_2 = 8(16\ 384)(-2)(x)^6$$

$$t_2 = -262\ 144x^6$$

b) the constant term

General Term:

$$t_{r+1} = \binom{8}{r}(4)^{8-r}(-2)^r(x)^{8-2r}$$

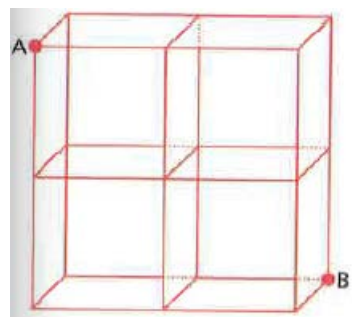
Term with x^0:

$$t_{4+1} = \binom{8}{4}(4)^{8-4}(-2)^4(x)^{8-2(4)}$$

$$t_5 = 70(256)(16)x^0$$

$$t_5 = 286\ 720$$

11) Determine the number of possible paths from point A to point B in the diagram to the left if travel may occur only along the edges of the cubes and if the path must always move closer to B.



2 right, 2 down, 1 away (5 total moves)

$$n(routes) = \binom{5}{2}\times\binom{3}{2}\times\binom{1}{1} = 10\times3\times1 = 30$$