Unit 2 – Linear Relations

Chapter 6 – Linear Relations
Lessons

MPM1D

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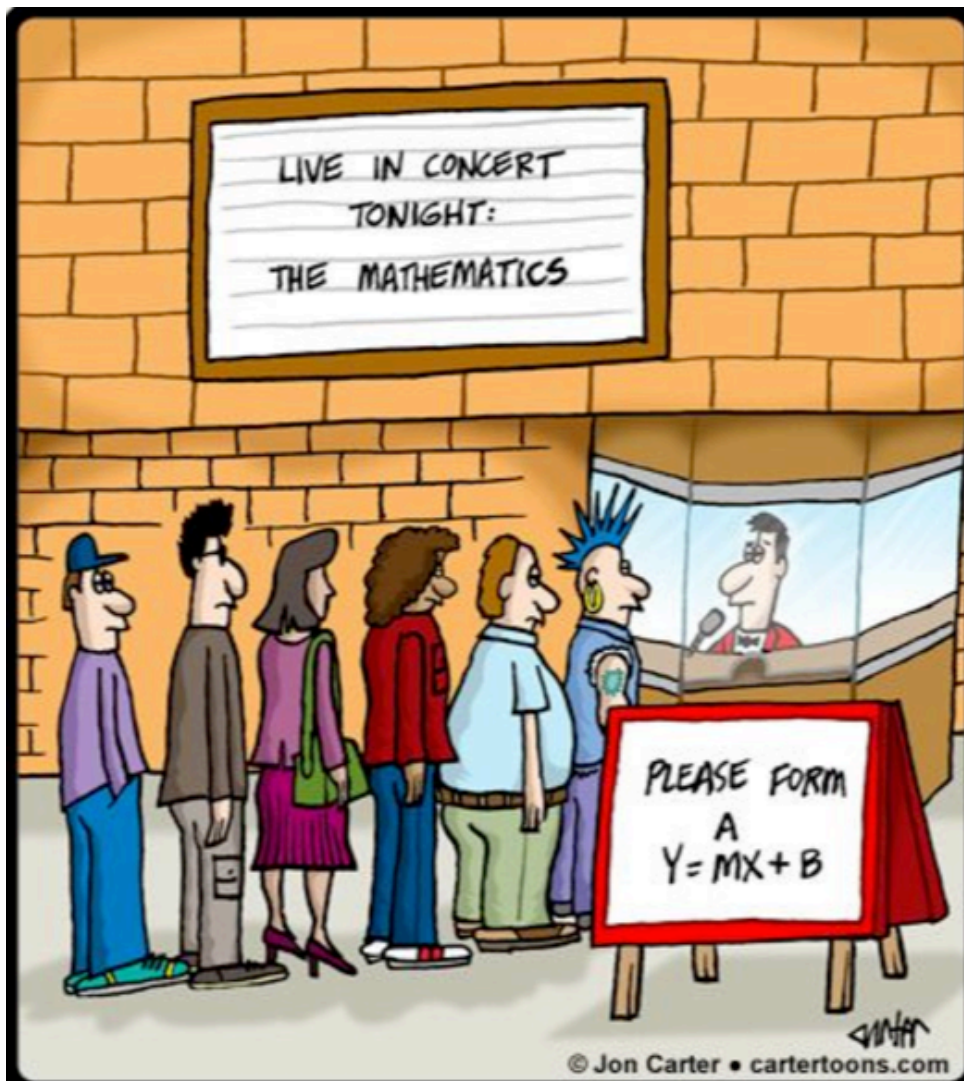
Chapter 6 Outline

Section	Subject	Learning Goals	Curriculum Expectations
6.1a	Equation of a Line in Slope y-Intercept Form	- understand the connection between the equation of a line in form $y = mx + b$ and its graph	B3.3, C1.2,C2.2, C3.1
6.1b	Equation of a Line in Slope y-Intercept Form	- understand the connection between the equation of a line in form $y = mx + b$ and its graph	B3.3, C1.2,C2.2, C3.1
6.2	Equation of a Line in Standard Form	- know how to write the equation of a line in slope y-intercept form given the equation in standard form - understand the connection between standard form and slope y-intercept form equations	C1.3
6.3	Graphing Using X and Y Intercepts	- be able to graph a linear relation using the x and y-intercepts given the equation in standard form	C3.1
6.4	Parallel and Perpendicular Lines	- know how to determine if two relations are parallel, perpendicular, or neither given equation in slope y-intercept for or standard form	C2.4
6.5	Equation of a Line Given Slope and a Point	- given slope and a point on a line, be able to determine the equation of the line	C3.2
6.6	Equation of a Line Given Two Points	- given two points on a line, determine the equation of the line	C3.2
6.7	Linear Systems	- understand what the solution to a linear system represents - be able to determine the point of intersection of two lines by graphing them	C3.5

Assessments	F/A/O	Ministry Code	P/O/C	KTAC
Note Completion	A		P	
Practice Worksheet Completion	F/A		P	
Quiz – Determine Equation of a Line	F		P	
Skill Builder #1 – Slope intercept form	F	B3.3, C1.2,C2.2, C3.1	P/O	K(50%) T(10%) A(10%) C(30%)
Skill Builder #2 – Point and Slope	F	C3.2	P/O	K(50%) T(10%) A(10%) C(30%)
Assignment #1– Slope Picture	O	B3.3, C1.2, C2.2, C2.4, C3.2, C3.5	P	K(33%), T(13%), A(33%), C(20%)
Assignment #2 – Linear Battlefield	O	B3.3, C3.1, C3.2	P/O	K(33%), T(20%), A(33%), C(13%)
Group Problem Solving - Linear Relations CSD	O	B3.3, C1.2, C1.3, C2.1, C2.2, C2.4, C3.1, C3.2, C3.3, C3.5	P/O/C	K(30%), T(20%), A(30%), C(20%)
PreTest Review	F/A			
Test – Linear Relations	O	B3.3, C1.2, C1.3, C2.1, C2.2, C2.4, C3.1, C3.2, C3.3, C3.5	P	K(45%), T(9%), A(28%), C(18%)

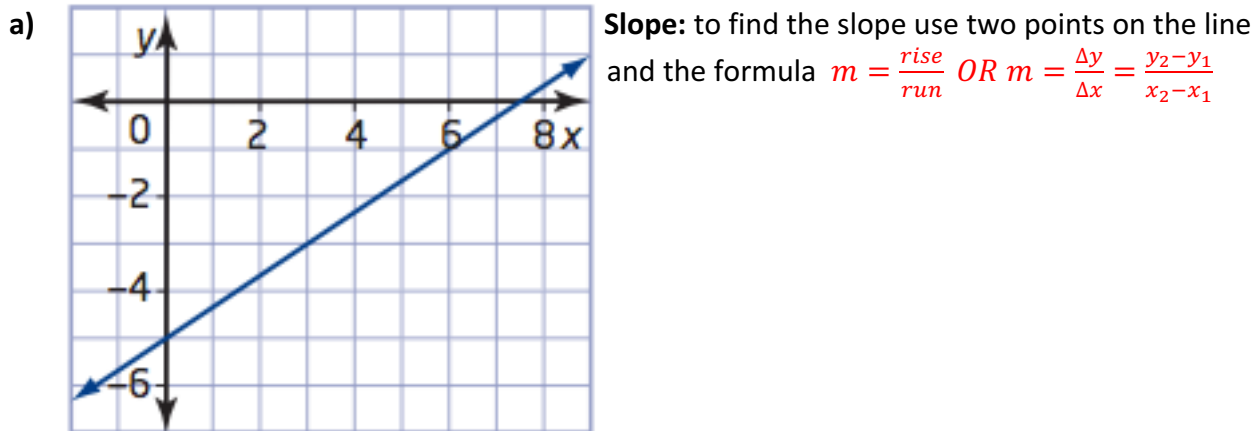**Learning Goals:** In this lesson students will be able to determine the equation of a line in the form $y = mx + b$ from its graph. The will be able to use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line.

Example 1: Complete the following chart

Equation	Slope	y-intercept
$y = -2x - 5$		
$y = x + 2$		
$y = \frac{2}{5}x + 8$		
$y = -\frac{1}{2}x$		
$y = 4$		

Example 2: Identify the slope and y-intercept of each line

a)

Slope: to find the slope use two points on the line and the formula $m = \frac{rise}{run}$ OR $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
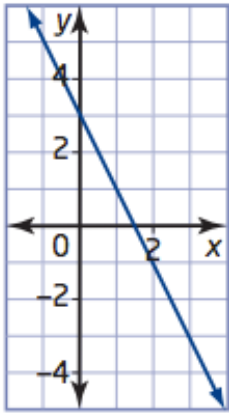
y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

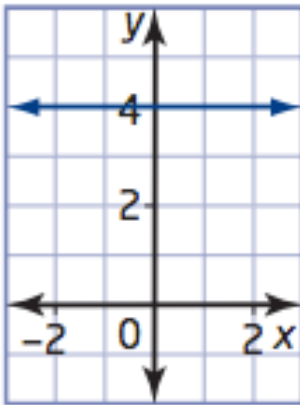Slope: y-intercept:

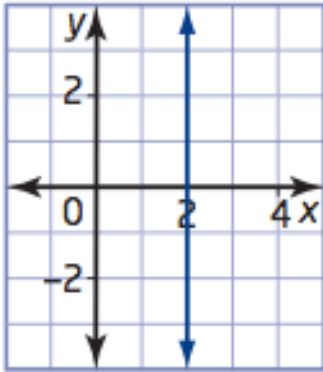Equation of the line:

b)

Slope: to find the slope use two points on the line
and the formula $m = \dfrac{rise}{run}$ OR $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

Slope:	**y-intercept:**
Equation of the line:	

c)

Slope: to find the slope use two points on the line
and the formula $m = \dfrac{rise}{run}$ OR $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis. (When $x = 0, y = ?$)

Slope:	**y-intercept:**
Equation of the line:	

Note: all horizontal lines have a slope of _____ and an equation of the form _____, where

b is the _____.

d)

Slope: to find the slope use two points on the line and the formula $m = \dfrac{rise}{run}$ OR $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

y-intercept: you can find the y-intercept by looking at the graph and checking where the line crosses the y-axis.
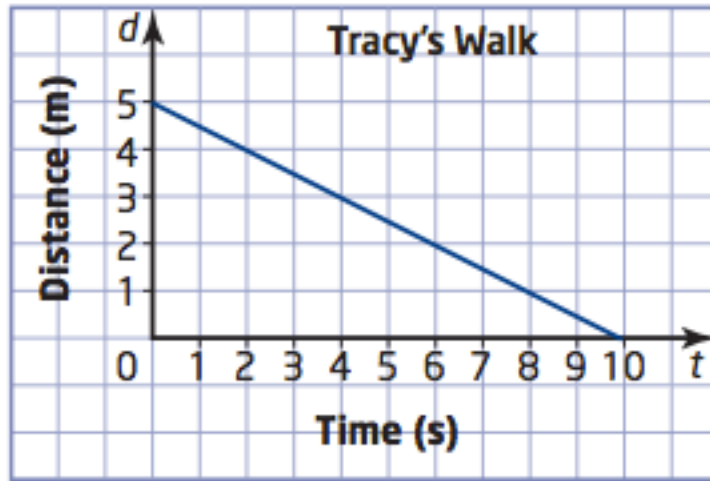
> **Slope:**   **y-intercept:**
>
>
> **Equation of the line:**

Note: All vertical lines have an _____ slope and an equation of the form

_____, where a is the _____.

Example 3: Interpreting a Linear Relation

Identify the slope and the vertical intercept of the linear relation and explain what they mean.

Slope: **y-intercept:**

The slope represents Tracy's _____. The negative value means that her distance from the sensor is

_____. Tracy's speed toward the sensor was _____.

y-intercept:

The y-intercept of _____ means that Tracy started walking at a distance of _____ from the

sensor.

Equation of the relation:

Learning Goals: In this lesson students will continue to develop their knowledge of the equation of a line while using slope and y-intercept to graph a line. Students will also develop an understanding of the equation of both vertical and horizontal lines.

Part 1: Graphing a Line Using the Slope and the y-Intercept:

Example 1: How can we graph $y = \frac{2}{3}x + 1$ without using a table of values?

a) The line $y = \frac{2}{3}x + 1$, has a slope: _____ and y-intercept: _____
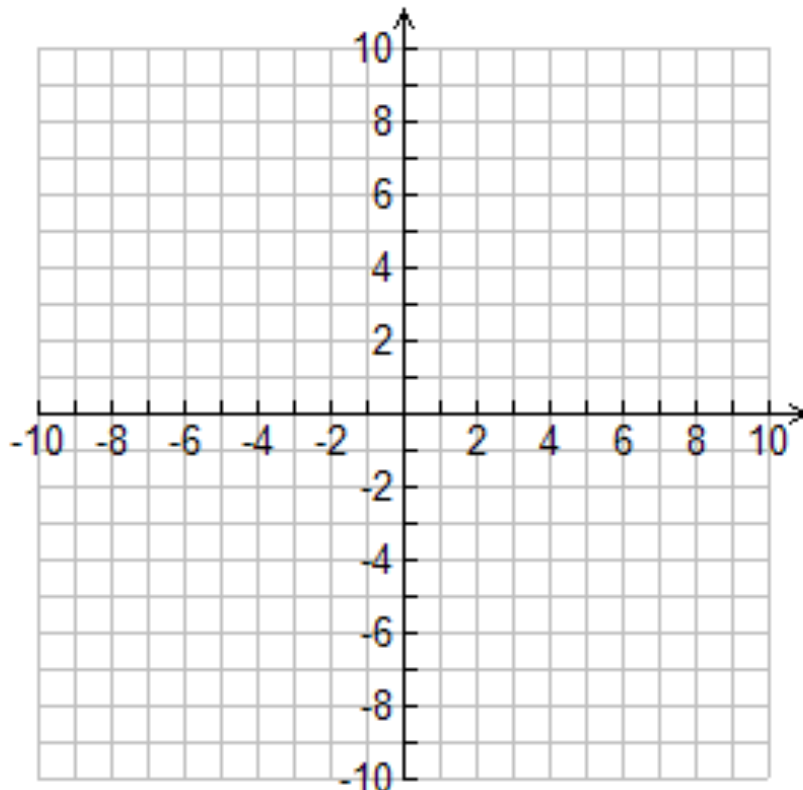
b) Plot the y-intercept on the given grid

c) How can the slope be used to determine other points on this line?

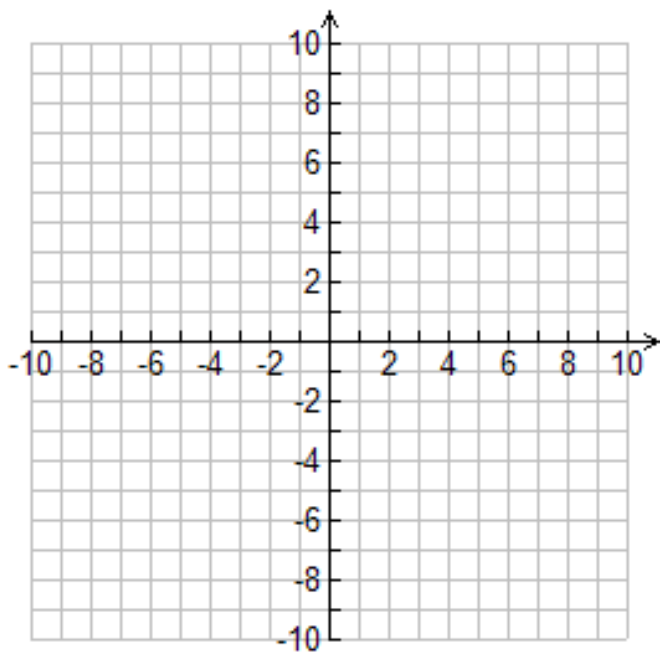 Use the slope of $\frac{2}{3}$ which has a rise of _____ and a run of _____ to plot another point on the line.

  You could also use the opposite slope to plot points on the other side of the y-intercept.

  The opposite slope, $\frac{-2}{-3}$, has a rise of _____ and a run of _____.

d) Use the slope to determine 2 other points on the line and draw in the line.

Example 2: Graph the line $y = -3x - 5$ using the slope and the y-intercept.
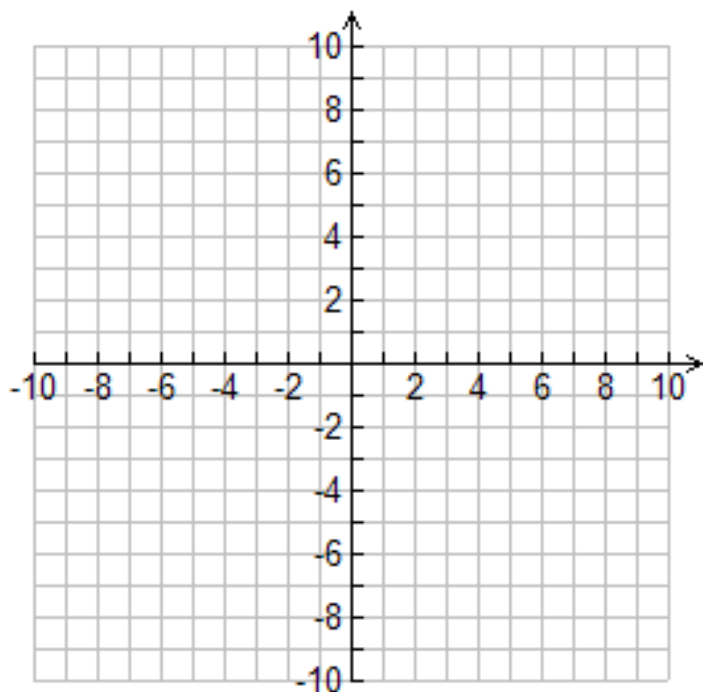

Slope:

y-intercept:

Part 2: Find the Equation of a Line Graphically Given Two Points

Example 3:

a) Plot the points A(-5, 3) and B(8, 3)on the given grid.
b) What is the y-intercept for the line that passes through A and B?

c) What is the slope for the line that passes through A and B?

d) What is the equation for the line that passes through A and B?
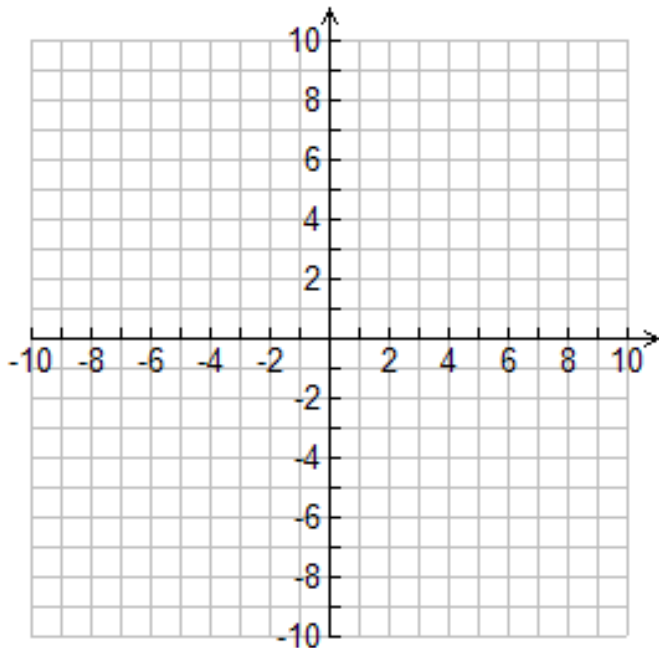

Note: the equation of a horizontal line is always in the form y=b. Every point on the line has a y-coordinate of 3.

Example 4: **a)** Plot the points A(5, 8) and B(5, -3) on the given grid.

b) What is the y-intercept for the line that passes through A and B?

c) What is the slope for the line that passes through A and B?

d) What is the equation for the line that passes through A and B?

> **Note:** the equation of a vertical line is always in the form of x = the x-intercept. Notice that every point on this line has an x-coordinate of 5.

Part 3: Consolidation

a) In general, a horizontal line has a slope that is _____ and an equation of the form _____ where 'b' is the _____.

b) In general, a vertical line has a slope that is _____ and an equation of the form _____ where 'a' is the _____.

c) State the steps required to graph a line using the slope and the y-intercept:

1.

2.

3.

The equation of a line can be written in two different forms:

1. Slope y-intercept form: $y = mx + b$

where m is the slope, and b is the y-intercept

2. Standard form: $Ax + By + c = 0$

where A, B, and c are _____ and A and B are both not _____.

You can change an equation from one form to the other by rearranging the equation.

Example 1:
Write the equation of the line 2x – 3y – 6 = 0 in slope y-intercept form by isolating the y.

Example 2: Write each equation in slope y-intercept form and state the slope and the y-intercept.

a) $3x + 5y - 15 = 0$

b) $7x - 3y + 21 = 0$

Example 3: Barney's Banquet Facility charges according to the equation $2x - y + 200 = 0$ where x is the number of people attending and y is the total cost.

a) Write the equation in slope y-intercept form.

b) What is the fixed cost?

c) What is the rate of change of the cost?

d) What is the total cost if 125 people attend a banquet at Barney's?

e) If the total cost is $920, how many people attend the banquet?

<u>Consolidate:</u>

1. There are two forms in which the equation of a line can be written. What are they?

2. It is possible to convert an equation from one form to the other by _____ the equation.

3. Write the slope-intercept form of the equation of each line:

a) $3x - 2y = -16$ **b)** $13x - 11y = -12$

c) $9x - 7y = -7$ **d)** $x - 3y = 6$

e) $6x + 5y = -15$ **f)** $4x - y = 1$

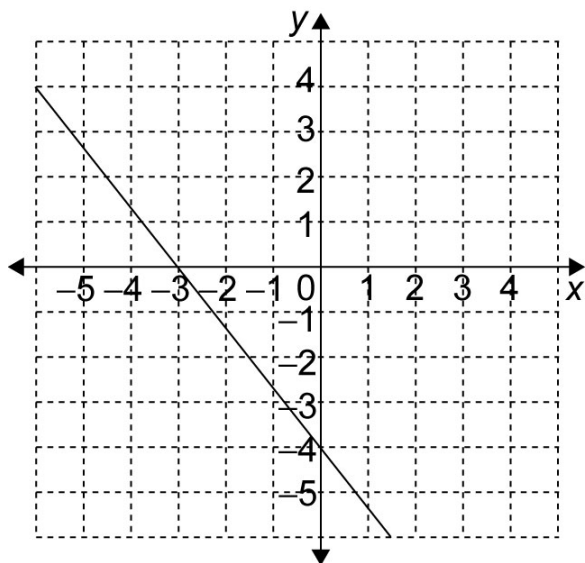**g)** $11x - 4y = 32$ **h)** $11x - 8y = -48$

Learning Goals: In this lesson students will learn how to calculate the x and y intercepts of a linear equation in Standard Form.

Part 1: Do It Now!

What are the x and y intercepts of the following line:
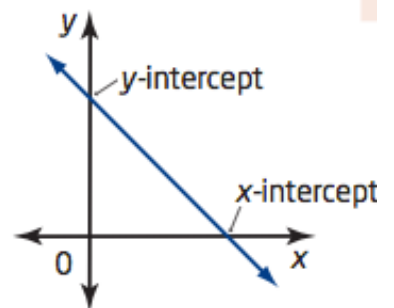

x-intercept:_____

y-intercept:_____

When a line is written in standard form, Ax + By + C = 0, or the form Ax + By = -C,

it is easy to graph the line using _____.

The **x-intercept** is the x-coordinate of the point where the line crosses the x-axis.

At the x-intercept, _____.

The **y-intercept** is the y-coordinate of the point where the line crosses the y-axis.
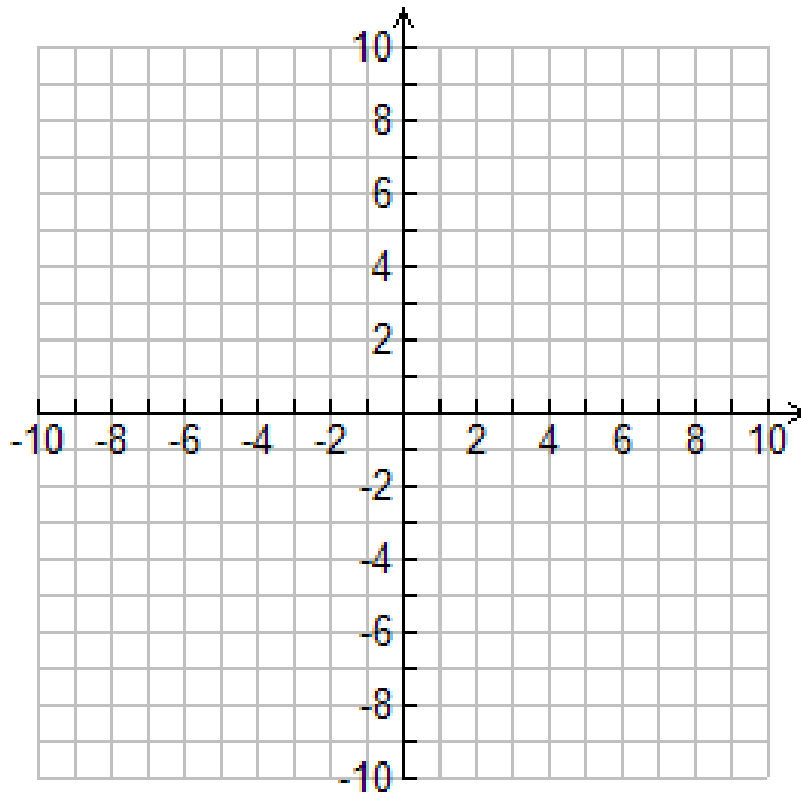
At the y-intercept, _____.

Example 1:

Determine the intercepts for the line $2x - 3y - 6 = 0$ and use these points to graph the line.

To find the x-intercept, set $y = 0$ and solve:

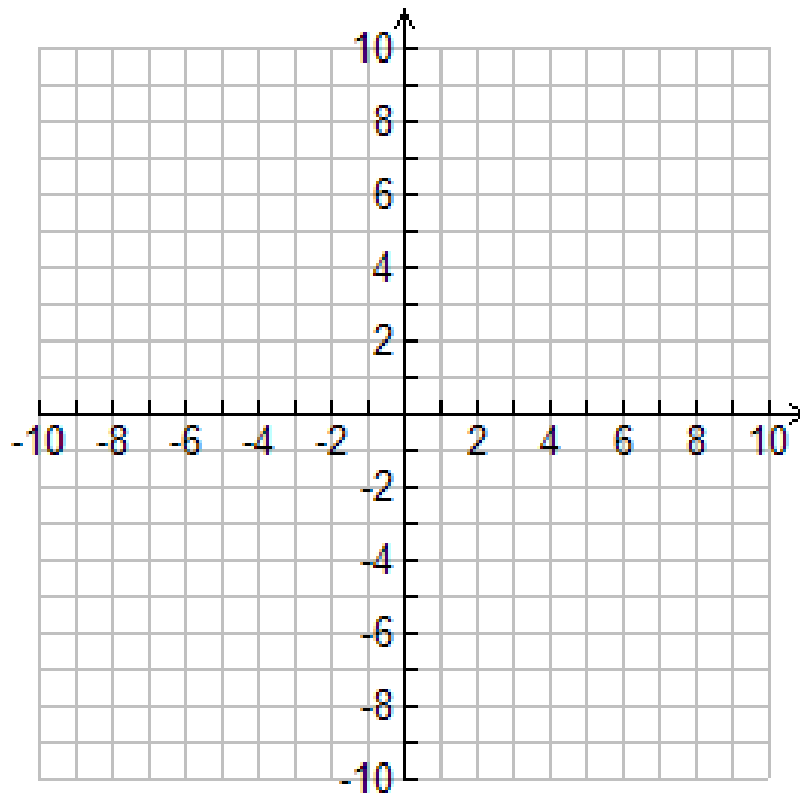**To find the y-intercept, set $x = 0$ and solve:**

Example 2:

Determine the intercepts for the line $2x - y = 7$ and use these points to graph the line.

To find the x-intercept,
set $y=0$ and solve:

To find the y-intercept,
set $x = 0$ and solve:

Example 3:   a) Determine the intercepts for the line $5x - 6y + 30 = 0$.
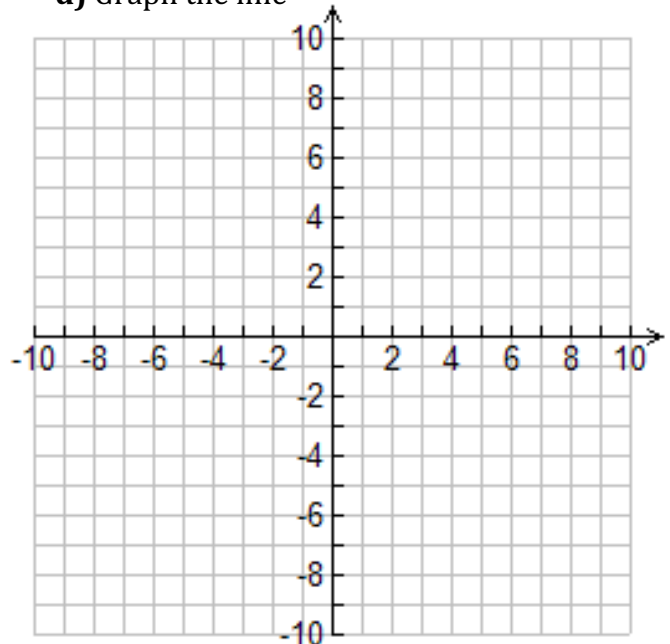
Remember:

Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

b) Use the intercepts to determine the slope of the line.

c) Write the equation of the line

d) Graph the line

Example 4: Determine the slope of the line whose x-intercept is -4 and y-intercept is -6.

Consolidate:
State the steps needed to graph a line using the intercepts.

Parallel Lines – **lines in the same plane that never meet.**
Perpendicular Lines – **Two lines that cross at 90 degrees.**

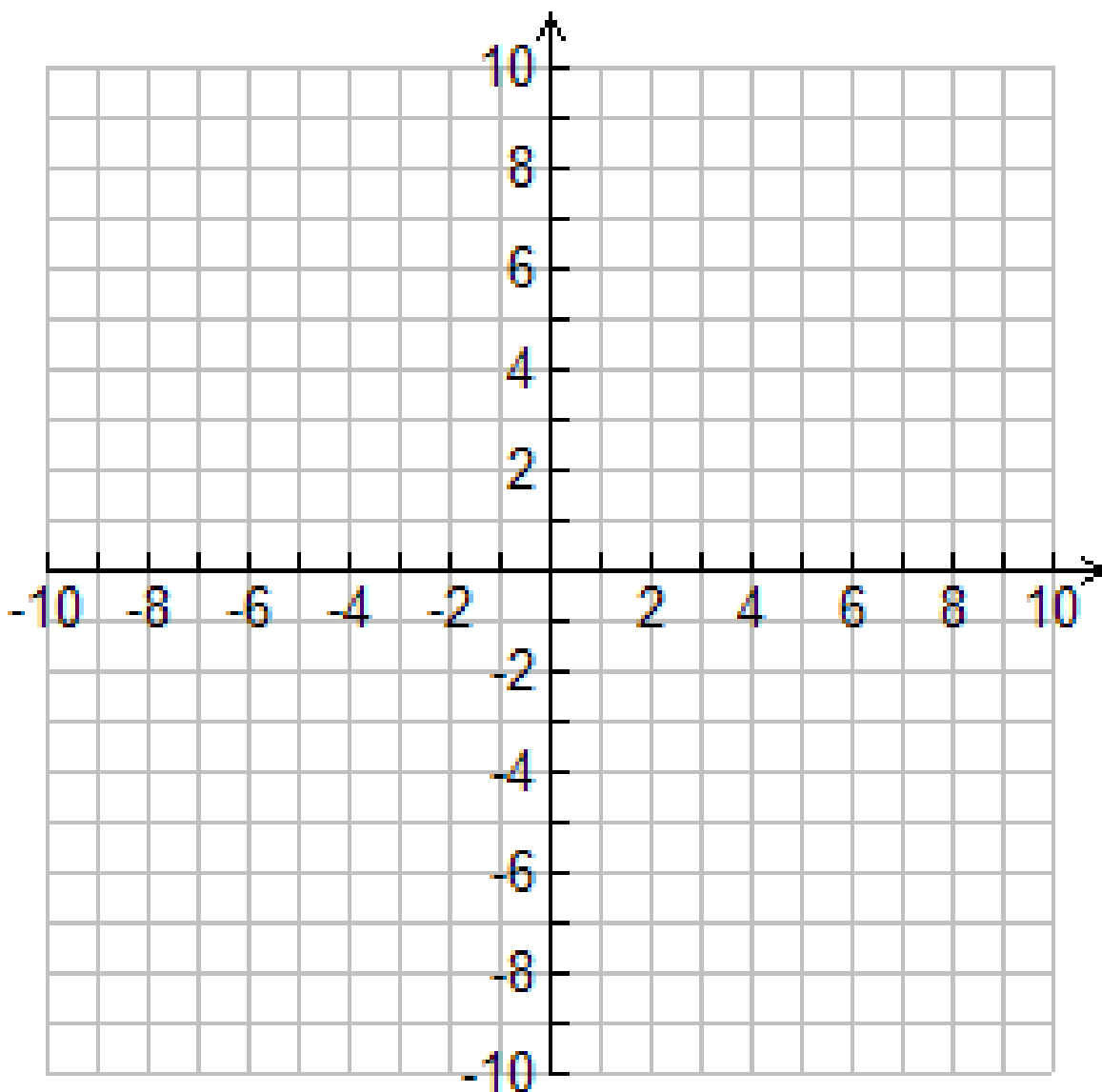<div style="border:1px solid black">

DO IT NOW!

Instructions: Draw and label each of the following lines on the grid below:

A) $y = 2x + 4$ **B)** $y = 2x + 8$ **C)** $y = \frac{1}{2}x + 4$ **D)** $y = -\frac{1}{2}x - 6$

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1) Which lines are parallel?

2) What do you notice about the slopes of lines that are parallel?

3) Which lines are perpendicular?

4) What do you notice about the slopes of lines that are perpendicular?

5) What is the product of the perpendicular slopes?

6) Does the y-intercept matter when deciding if two lines are parallel or perpendicular?

Consolidation:

Parallel lines will have _____ slopes.

Perpendicular lines will have slopes that are _____ _____.
Their product is -1.

Example 1:

a) The equation of a line is $y = 3x - 4$. What is the slope of a line that is parallel to this line?

b) The equation of a line is $y = -x + 15$. What is the slope of a line that is parallel to this line?

c) The equation of a line is $y = 2x + 1$. What is the slope of a line that is perpendicular to this line?

d) The equation of a line is $y = \frac{3}{5}x + 2$. What is the slope of a line that is perpendicular to this line?

d) The equation of a line is $y = -\frac{1}{7}x - \frac{3}{7}$. What is the slope of a line that is perpendicular to this line?

Example 2: The slopes of two lines are given. Determine whether the lines are parallel, perpendicular or neither.

a) $m = 3, m = -\frac{1}{3}$

b) $m = 5, m = -5$

c) $m = 6, m = \frac{1}{6}$
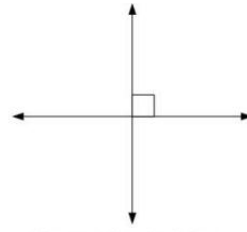
d) $m = -2, m = -2$

Example 3:
a) Determine the slope of a line that is parallel to the line $2x - 3y - 6 = 0$.

b) Determine the slope of a line that is perpendicular to the line $2x - 3y - 6 = 0$.

Consolidate:

Parallel Lines   Perpendicular Lines

a) Explain how you can determine if the two lines $3x - 4y - 12 = 0$ and $4x - 3y - 24 = 0$ are parallel, perpendicular or neither.

b) Determine if the two lines $3x - 4y - 12 = 0$ and $4x - 3y - 24 = 0$ are parallel, perpendicular or neither.

DO IT NOW!

Instructions: Determine the equation of the line, in slope y-intercept form, that has a slope of 3 and goes through the point (2, -5)

Note: You can write the equation of a line once you know the slope and y-intercept.

$$y = mx + b$$

Slope

y-intercept

Step 1: State what you know about the line

Step 2: Determine the y-intercept of the line

To do this we can use the equation $y = mx + b$, substitute in values for m, x and y and then solve for the b value. Use the point on the line that is given for the x and y values.

STEP 3: Write the equation of the line in slope y-intercept form.

Note: When writing the final equation of the line, plug in values for m and b, not for x and y.

Example 1: Find the equation of the line with a slope of $\frac{1}{2}$ that passes through (1, 5).

Step 1: State what you know about the line   **Step 2: Determine the y-intercept of the line**

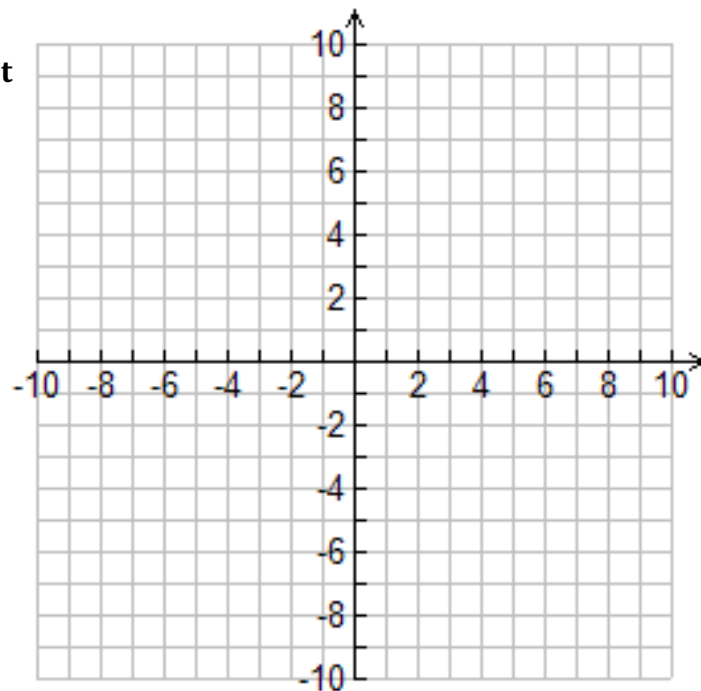**STEP 3: Write the equation of the line in slope y-intercept form.**

Example 2: Find the equation of the line with a slope of 3 and that passes through (0, 2). Then graph the line.

Step 1: State what you know about the line   **Step 2: Determine the y-intercept of the line**

STEP 3: Write the equation of the line in slope y-intercept form.

Step 4: Graph the line using the slope and y-intercept

Example 3: Determine the equation of a line that is parallel to the line y = -2x – 7 and passes through the point (1, -3).

Step 1: State what you know about the line

Remember: lines that are parallel have the same slope. They do not have the same y-intercept. You will still have to solve for that.

Step 2: Determine the y-intercept of the line

STEP 3: Write the equation of the line in slope y-intercept form.

Example 4: Determine the equation of a line that is perpendicular to the line $2x - y + 4 = 0$ and passes through the point (-2, 5).

Hint: to determine the slope you will need to put the equation into y=mx+b form so that you can see the slope and then take the negative reciprocal.

STEP 1: state what you know about the line

Slope of given line:

Slope of perpendicular line (find negative reciprocal):

Point on the perpendicular line:

Step 2: Determine the y-intercept of the line (make sure to use the slope of the perpendicular line)

STEP 3: STEP 3: Write the equation of the line in slope y-intercept form (make sure to use the slope of the perpendicular line).

Consolidation:

To write the equation of a line you need to know the _____ and _____.

You can use the slope of a line and a point on the line to calculate the _____.

To find the slope of a perpendicular line, find the _____.

Remember: You can write the equation of a line once you know the slope and y-intercept.

$$y = mx + b$$

Slope

y-intercept

DO IT NOW!

Instructions: Write the equation of the following lines:

a) Line with a slope of $\frac{3}{5}$ that passes through the point B(-5, 4).

b) Line that is parallel to the line $y = 2x - 7$ and passes through the point (1, -3).

c) Line that is perpendicular to the line 2x − 2y + 4 = 0 and passes through the point (-2, 5).

Today's Lesson: Find the equation of a line given two points on the line.

What do you need to write the equation of a line?

If you are not given the slope of a line, how can you find it?

How can you find the y-intercept?

Example 1: Determine the equation of a line that passes through the points M(4, -3) and N(2, 5).

Step 1: Calculate the slope

Step 2: Find the y-intercept

Step 3: Write the equation of the line

<u>**Example 2:**</u> Determine the equation of a line that passes through the points P(0, 4) and Q(7, 0).

Step 1: Calculate the slope   **Step 2: Find the y-intercept**

Step 3: Write the equation of the line

Example 3: Determine the equation of a line that passes through the points A(-4, 2) and B(8, 11).

Step 1: Calculate the slope   **Step 2: Find the y-intercept**

Step 3: Write the equation of the line

Example 4: On your own determine the equation of the line that passes through the points A(2,-4) and B(5,5)

Example 5:

a) An appliance repair company charges $205 for a repair that takes 3 hours. The same company charges $505 for a repair that takes 8 hours. Determine an equation that represents the cost of a repair based on the number of hours that the repair takes.

Hint: you can write two coordinate points with the information given. (ind. variable, dep. variable)

b) What is the cost of a repair that takes 7 hours?

c) If a repair costs $385, how many hours does it take?

Consolidate:

To write the equation of a line you need the _____ and _____.

If you are not given the slope you can find it if you have _____ on the line by using the

Formula:

Linear System: A set of two or more linear equations that are considered simultaneously

Point of Intersection: the point where two or more lines intersect

DO IT NOW!

Mike is considering joining a ski club for the winter season. He is considering the following two options:

Standard Rate: $50 per day and no registration fee
Frequent Skier Pass: $40 per day and $100 registration fee

a) Write an equation that relates the total cost, C, in dollars, and the number of days, n, that Mike goes skiing if he chooses the Standard Rate:

b) Write an equation if he chooses the Frequent Skier Pass option:

c) Graph both of the lines on the same graph
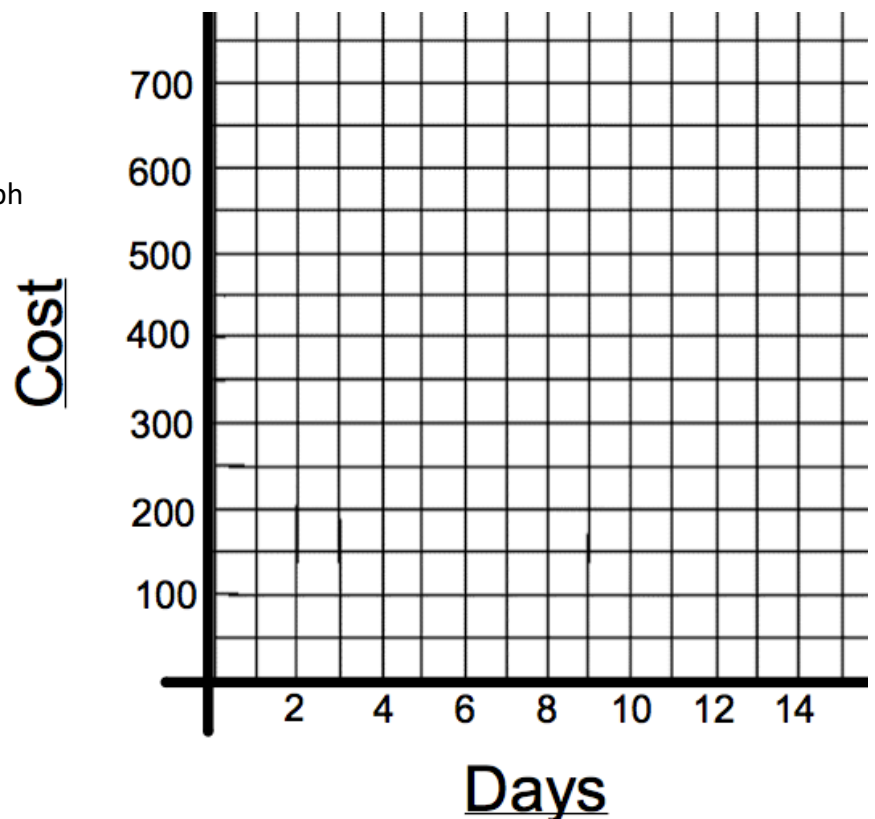
d) What is the point of intersection?

e) What is the cost of both plans at the point of intersection? What does this mean?

f) Look to the right of the point of intersection, which plan is cheaper?

g) Look to the left of the point of intersection, which plan is cheaper?

h) If Mike is going to go skiing 11 times this winter, which plan would you recommend to him?

Example 1: Graph $y = \frac{1}{2}x - 3$ and $x + y = -6$ on the same grid and identify the coordinates of the point of intersection.

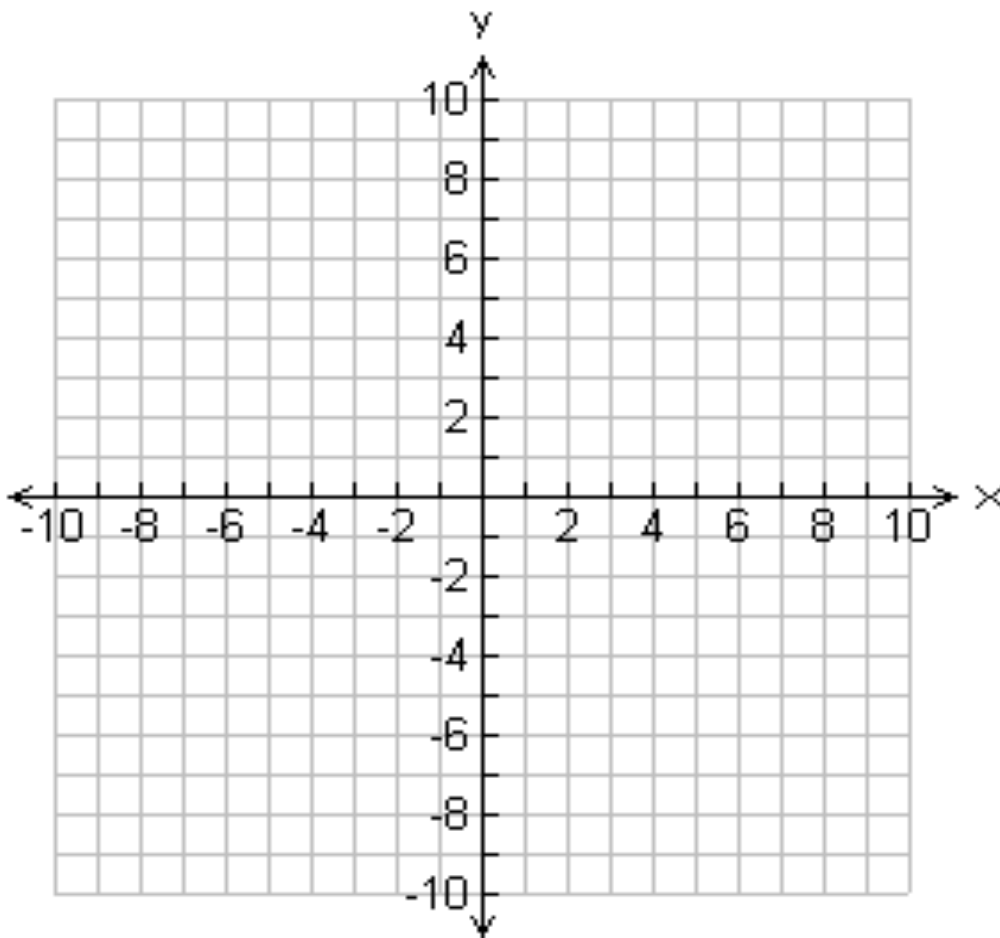**Line 1:** $y = \frac{1}{2}x - 3$ **Line 2:** $x + y = -6$

Slope: **Slope:**

y-intercept: **y-intercept:**

Point of Intersection: _____

Check your answer: To verify the solution, (-2, -4), substitute the coordinates into both equations and check that they hold true. Use the left side/right side method.

Check: $y = \frac{1}{2}x - 3$

> **L.S.**   **R.S.**

Check: $x + y = -6$

> **L.S.**   **R.S.**

Example 2: Graph $y = 2x - 2$ and $y = x + 1$ on the same grid and identify the coordinates of the point of intersection.
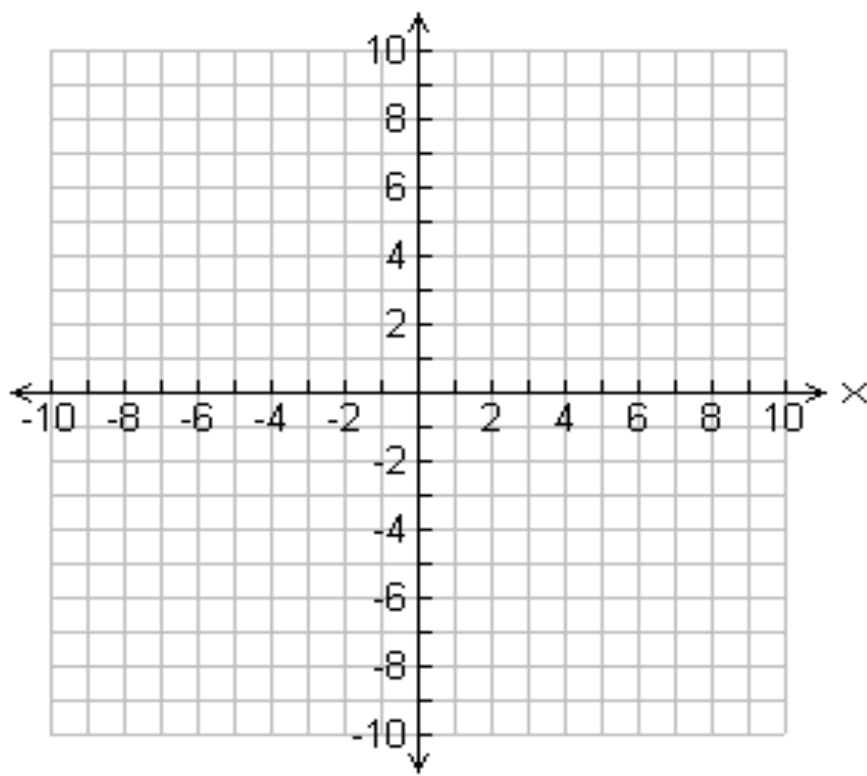
> **Line 1:** $y = 2x - 2$ **Line 2:** $y = x + 1$

> **Slope:**   **Slope:**

> **y-intercept:**   **y-intercept:**

Point of Intersection: _____

Check your answer: To verify the solution, (3,4), substitute the coordinates into both equations and check that they hold true. Use the left side/right side method.

Check: $y = 2x - 2$

          L.S.   R.S.

Check: $y = x + 1$

          L.S.   R.S.