Chapter 7

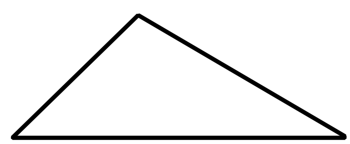
Geometric Relationships

Intro

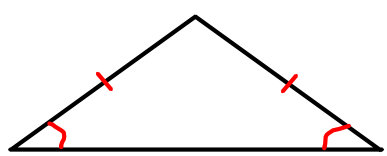
Part 1: Classifying Triangles

Classifying Using Side Lengths

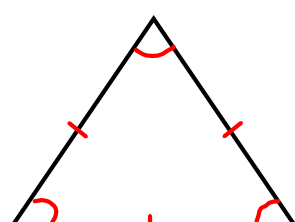
Scalene Triangle
- no equal sides or angles



Isosceles Triangle
- 2 equal sides
- 2 equal angles

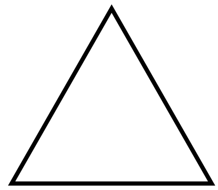


Equilateral Triangle
- 3 equal sides
- 3 equal angles

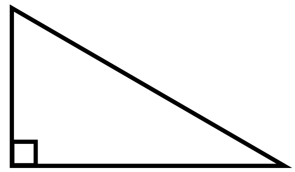


Classifying Using Angle Measures

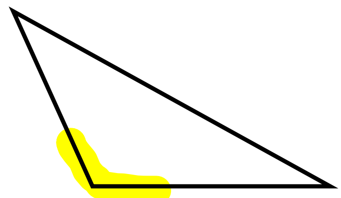
Acute Triangle
- 3 acute angles
(less than 90 degrees)



Right Triangle
- one right angle
(90 degrees)



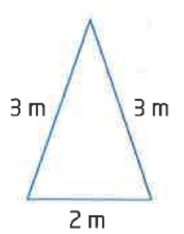
Obtuse Triangle
- one obtuse angle
(between 90 and 180 degrees)



Example 1

Classify Each Triangle Using its Side Lengths

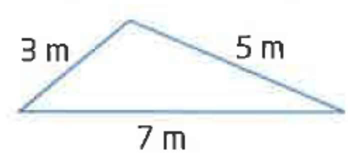
a)



Isosceles

2 equal sides

b)



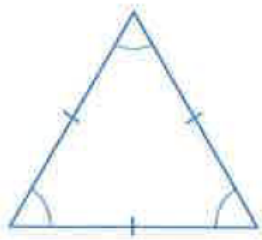
Scalene

No equal sides

Example 2

Classify Each Triangle in Two ways Using its Angle Measures

a)



b)

Equilateral (3 equal angles)

Acute (all angles < 90)

Isosceles (2 equal angles)

Obtuse (1 angle > 90)

Part 2: Classifying Polygons

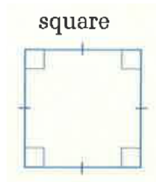
A ***polygon*** is a closed figure formed by three or more line segments.

A ***regular polygon*** has all sides equal and all angles equal.

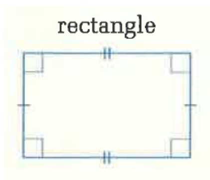
Number of Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon

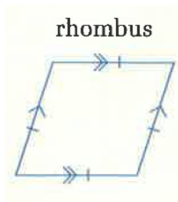
Some **quadrilaterals** have special names.

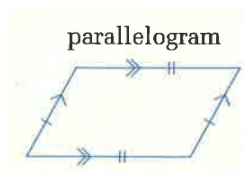
A regular quadrilateral is a square.

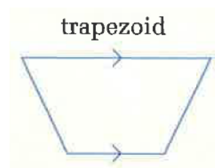


An irregular quadrilateral may be a ***rectangle***, ***rhombus***, ***parallelogram***, or ***trapezoid***





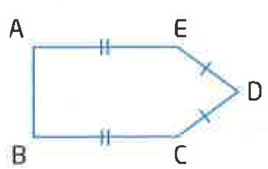




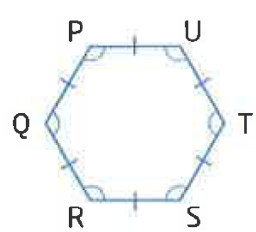
Example 3

Classify each polygon according to its number of sides and whether it is regular or irregular.

a)



b)



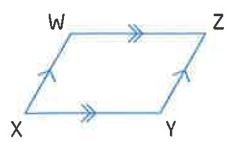
Irregular Pentagon

Regular Hexagon

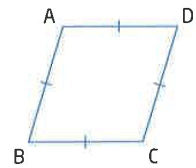
Example 4

Classify each quadrilateral.

a)



b)



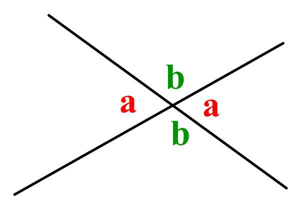
Parallelogram

Rhombus

Part 3: Angle Properties

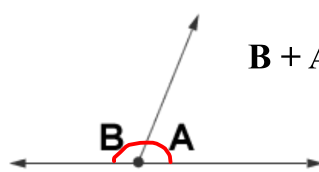
Opposite Angles:

- When 2 angles intersect, the opposite angles are equal.



Supplementary Angles:

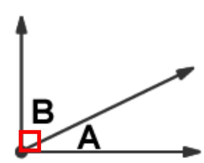
- angles that add to 180 degrees
- angles on a straight line are supplementary



B + A = 180

Complementary Angles:

- angles that add to 90 degrees

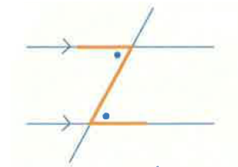


B + A = 90

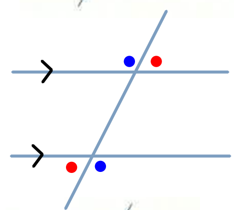
Part 4: Parallel Line Theorems

When a transversal crosses parallel lines, many pairs of angles are related…

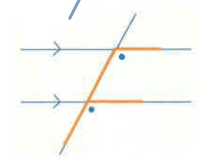
Alternate Interior Angles are equal
- Z pattern



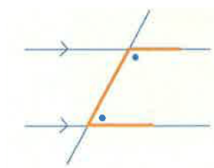
Alternate Exterior Angles are equal



Corresponding Angles are equal
- F pattern

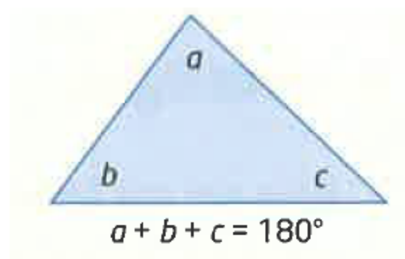


Co-Interior Angles add to 180
- C pattern

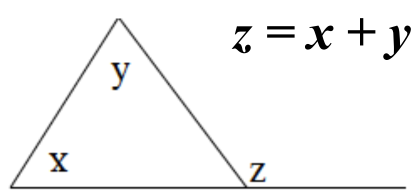


Part 6: Triangle Theorems

The sum of the **interior angles** of a triangle is **180** degrees.



The **exterior angle** is equal to the sum of the 2 opposite interior angles.

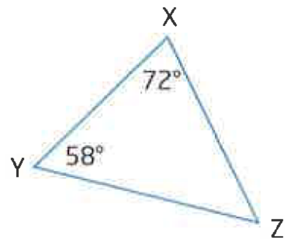


$$z = x + y$$

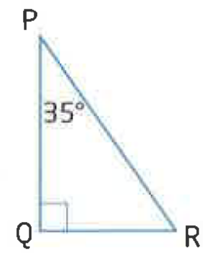
Example 5

Find the measure of the third angle in each triangle...

a)



b)



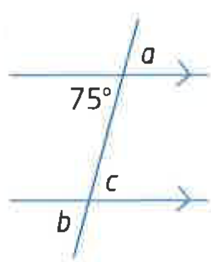
$\angle Z = 180 - 58 - 72$
$= 50°$

$\angle R = 180 - 90 - 35$
$= 55$

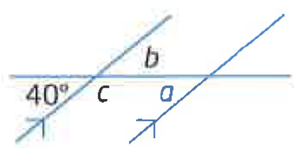
Example 6

Find the measure of the angles *a*, *b*, and *c*. Give reasons for your answers…

a)



b)



$\angle a = 75°$ (opposite angle)

$\angle c = 75°$ (alternate interior)

$\angle b = 75°$ (corresponding angle)

$\angle c = 180 - 40 = 140°$ (supplementary)
$\angle b = 40°$ (opposite angle)

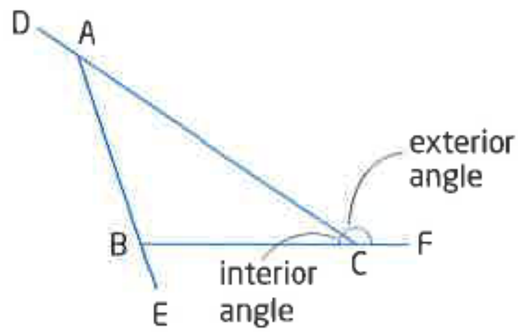
$\angle a = 180 - \angle c = 40°$ (co-interior)

7.1 - Angle Relationships in Triangles

Interior and Exterior Angles

Interior Angle - angle formed on the inside of a polygon by two sides meeting at a vertex.

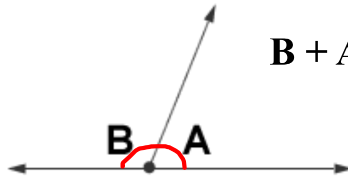
Exterior Angle - angle formed on the outside of a geometric shape by extending one of the sides past a vertex.



You must remember….

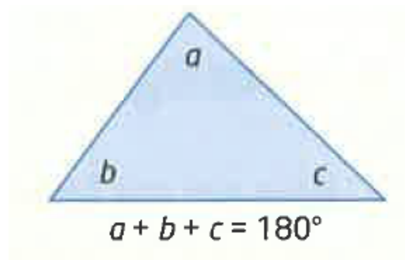
Supplementary Angles:

- angles that add to 180 degrees
- angles on a straight line are supplementary



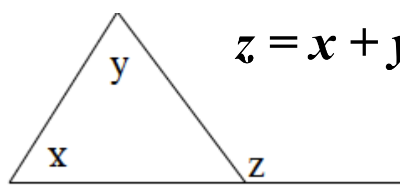
B + A = 180

The sum of the **interior angles** of a triangle is **180** degrees.



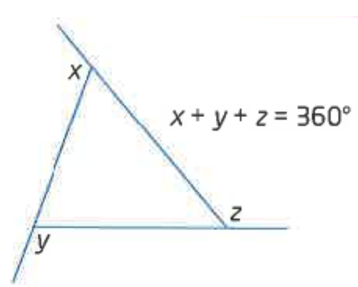
New Exterior Angle Rules...

The **exterior angle** is equal to the sum of the 2 opposite interior angles.



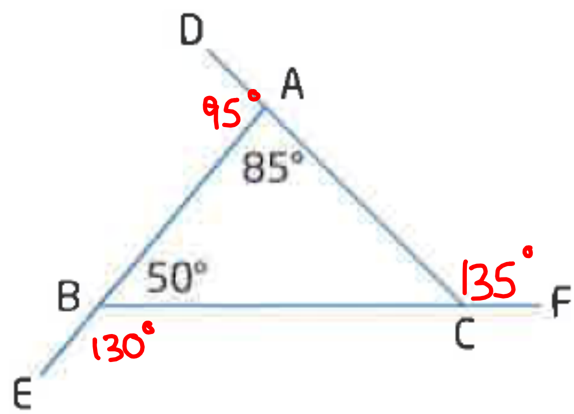
$$z = x + y$$

The sum of the **exterior angles** of a triangle is 360 degrees.



Example 1

Find the measures of the exterior angles in ΔABC



Note: at vertex A and B, the interior and exterior angles are supplementary angles (form an angle of 180 degrees)

∠DAB $= 180 - 85 = 95^\circ$ (supplementary)

∠EBC $= 180 - 50 = 130^\circ$ (supplementary)

$\angle ACF$

Method 1:

Since the exterior angle at a vertex of a triangle is equal to the sum of the interior angles at the other two vertices...

$\angle ACF = 85 + 50 = 135^\circ$

Method 2:

Since the sum of the exterior angles of a triangle is 360 degrees...

$\angle ACF = 360 - 130 - 95 = 135^\circ$

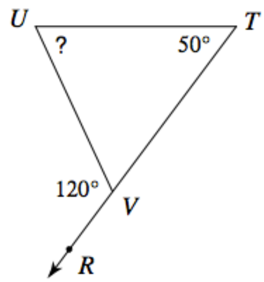
The measures of the three exterior angles are:

$\angle DAB$ = 95°

$\angle EBC$ = 130°

$\angle ACF$ = 135°

Example 2 Find the measure of the indicated angle

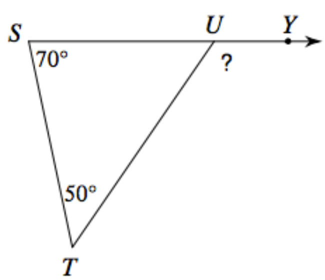


$\angle VUT + 50 = 120$

$\angle VUT = 120 - 50$

$\angle VUT = 70°$

Example 3 Find the measure of the indicated angle

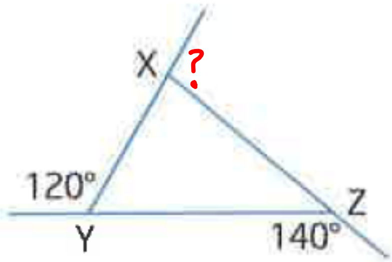


$\angle TUY = 50 + 70$

$\angle TUY = 120°$

Example 4

Find the measure of the exterior angle at vertex X.

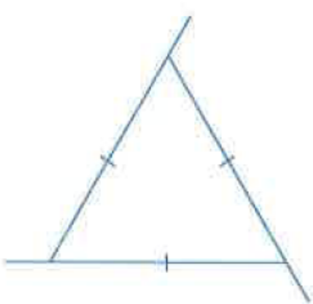


$? = 360 - 120 - 140$

$? = 100°$

Example 5

What is the measure of each exterior angle of an equilateral triangle?



All angles in an equilateral triangle are EQUAL.

Therefore all three interior angles are...

$= \frac{180}{3} = 60°$

At each vertex, the interior angle and exterior angle are supplementary, meaning they sum to 180°.

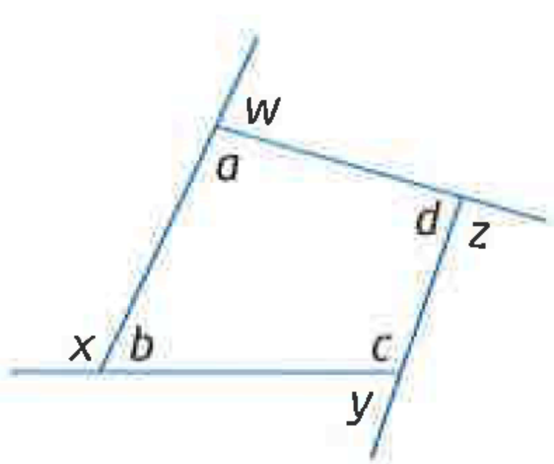
Therefore all three exterior angles are... $= 180 - 60 = 120°$

7.2 Angle Relationships in Quadrilaterals

Angle Relationships in Quadrilaterals

The sum of the **interior** angles of a quadrilateral is 360 degrees.

The sum of the **exterior** angles of a quadrilateral is also 360 degrees.



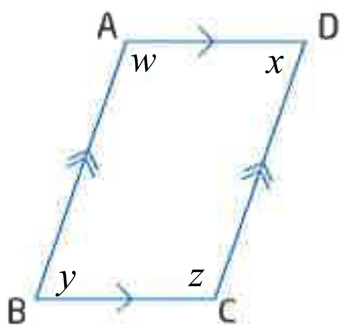
Interior angles:
$a + b + c + d = 360°$

Exterior angles:
$w + x + y + z = 360°$

Angle Relationships in Parallelograms

Adjacent angles in a parallelogram are supplementary (add to 180).

Opposite angles in a parallelogram are equal.



Adjacent angles:

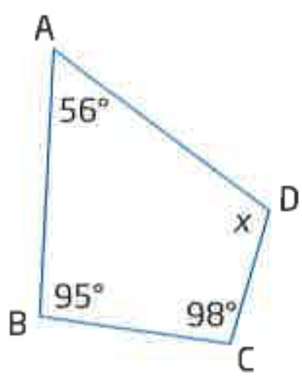
$w + x = 180$
$w + y = 180$
$y + z = 180$
$z + x = 180$

Opposite angles:

$w = z$
$x = y$

Example 1

Find the measure of the unknown angle

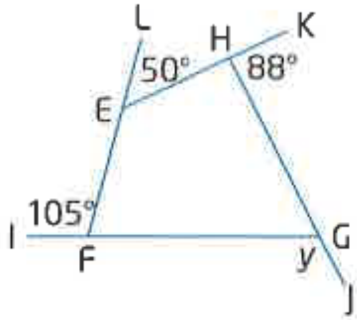


$x + 56 + 95 + 98 = 360$

$x = 360 - 56 - 95 - 98$

$x = 111°$

Example 2 Find the measure of the unknown angle

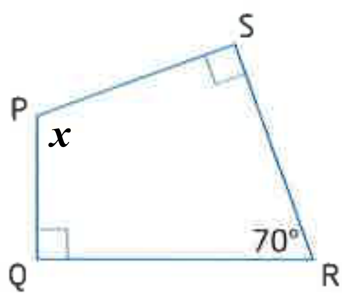


$y + 105 + 50 + 88 = 360$

$y = 360 - 105 - 50 - 88$

$y = 117°$

Example 3 Find the measure of the unknown angle

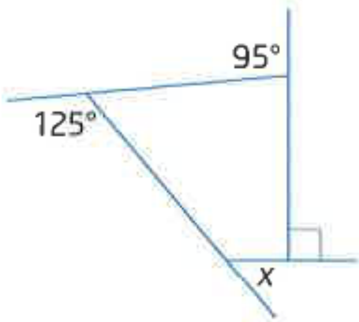


$x + 90 + 90 + 70 = 360$

$x = 360 - 90 - 90 - 70$

$x = 110°$

Example 4 Find the measure of the unknown angle

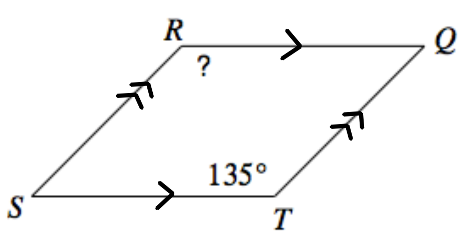


$x + 125 + 95 + 90 = 360$

$x = 360 - 125 - 95 - 90$

$x = 50^\circ$

Example 5 Find the measure of the unknown angle

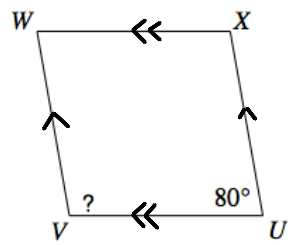


$\angle SRQ = \angle STQ$

$\angle SRQ = 135^\circ$

Opposite angles are equal in parallelograms

Example 6 Find the measure of the unknown angle



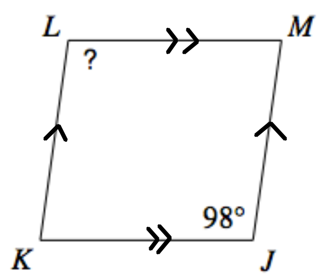
$\angle WVU + 80 = 180$

$\angle WVU = 180 - 80$

$\angle WVU = 100°$

Adjacent angles are supplementary in a parallelogram

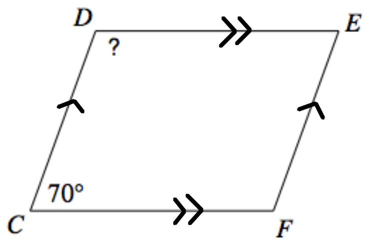
Example 7 Find the measure of the unknown angle



$? = 98°$ (opposite)

Example 8

Find the measure of the unknown angle



$? + 70 = 180$ (adjacent)

$? = 180 - 70$

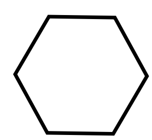
$? = 110°$

7.3 Angle Relationships in Polygons

Types of Polygons

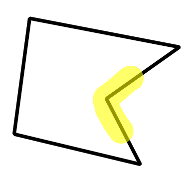
Convex Polygon: All interior angles measure less than 180 degrees.

- no part of any line segment joining two points on the polygon goes outside the polygon.

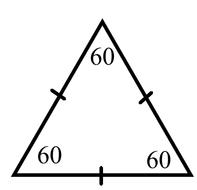


Concave Polygon: Can have interior angles greater than 180 degrees.

- parts of some line segments joining two points on the polygon go outside the polygon.



Regular Polygon: All sides are equal and all interior angles are equal.



Angle Properties in Polygons

The sum of the exterior angles of a convex polygon is 360 degrees.

For a polygon with n sides, the sum of the interior angles, in degrees, is $180(n - 2)$

For a regular polygon with n sides, the measure of each interior angle is equal to: $\frac{180(n-2)}{n}$

For a regular polygon with n sides, the measure of each exterior angle is equal to: $\frac{360}{n}$

Example 1

$\boxed{180(n-2)}$

Calculate the sum of the interior angles of an octagon

8 sides

sum of interior angles $= 180(n-2)$

$= 180(8-2)$

$= 180(6)$

$= 1080°$

Example 2

$\boxed{\dfrac{180(n-2)}{n}}$

Calculate the measure of each of the interior angles of a *regular* octagon.

$$\text{interior angle} = \frac{180(n-2)}{n}$$

$$= \frac{180(8-2)}{8}$$

$$= \frac{1080}{8}$$

$$= 135^\circ$$

Example 3

$\boxed{\dfrac{360}{n}}$

Calculate the measure of each of the exterior angles of a *regular* octagon.

$$\text{exterior angle} = \frac{360}{n}$$

$$= \frac{360}{8}$$

$$= 45^\circ$$

Example 4

How many sides does a polygon have if each of its interior angles measure 140 degrees?

$$\text{interior angle} = \frac{180(n-2)}{n}$$

$$140 = \frac{180(n-2)}{n}$$

$$140n = 180(n-2)$$

$$140n = 180n - 360$$

$$360 = 180n - 140n$$

$$\frac{360}{40} = \frac{\cancel{40}n}{\cancel{40}}$$

$$9 = n$$

The regular polygon has 9 sides.

Example 5

The measure of one of the exterior angles of a regular polygon is 30 degrees. How many sides does it have?

$$\text{exterior angle} = \frac{360}{n}$$

$$30 = \frac{360}{n}$$

$$30n = 360$$

$$n = \frac{360}{30}$$

$$n = 12$$

The regular polygon has 12 sides.

Example 6

6 sides

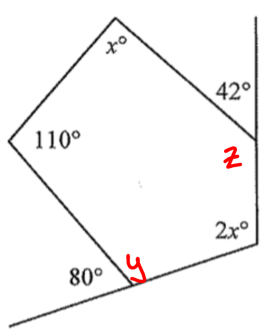
Five angles of a hexagon have measures 100°, 110°, 120°, 130°, and 140°. What is the measure of the sixth angle?

$$\text{sum of interior angles} = 180(n-2)$$
$$= 180(6-2)$$
$$= 720°$$

$$6^{th}\text{ angle} = 720-100-110-120-130-140$$
$$= 120°$$

Example 7

Solve for x



$y = 180 - 80$ (supplementary)
$= 100°$

$z = 180-42$ (supplementary)
$= 138°$

$$\text{sum of interior angles} = 180(n-2)$$
$$= 180(5-2)$$
$$= 540°$$

$$x + 110 + 100 + 2x + 138 = 540$$
$$3x = 540 - 110 - 100 - 138$$
$$3x = 192$$
$$x = \frac{192}{3}$$
$$x = 64°$$

Complete the following chart and then complete the worksheet

Polygon	Number of Sides	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	$180(3-2) = 180^\circ$	360°
Quadrilateral	4	$180(4-2) = 360^\circ$	360°
Pentagon	5	$180(5-2) = 540^\circ$	360°
Hexagon	6	$180(6-2) = 720^\circ$	360°
Heptagon	7	$180(7-2) = 900^\circ$	360°
Octagon	8	$180(8-2) = 1080^\circ$	360°
Enneagon	9	$180(9-2) = 1260^\circ$	360°
Decagon	10	$180(10-2) = 1440^\circ$	360°