W1 – 4.3 Co-function Identities

MHF4U

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1) Simplify.

a) $\sin x \left(\frac{1}{\cos x}\right)$ **b)** $(\cos x)(\sec x)$ **c)** $1 - \cos^2 x$ **d)** $1 - \sin^2 x$

e) $\frac{\tan x}{\sin x}$ **f)** $(1 - \sin x)(1 + \sin x)$ **g)** $\left(\frac{1}{\tan x}\right)\sin x$ **h)** $\frac{1+\tan^2 x}{\tan^2 x}$

i) $\frac{\sin x \cos x}{1-\sin^2 x}$ **j)** $\frac{1-\cos^2 x}{\sin x \cos x}$

2) Prove the following identities.

a) $\sin^2 x\,(1 + \cot^2 x) = 1$ **b)** $1 - \cos^2 x = \tan x \cos x \sin x$

c) $\cos x \tan^3 x = \sin x \tan^2 x$

d) $1 - 2\cos^2 \theta = \sin^4 \theta - \cos^4 \theta$

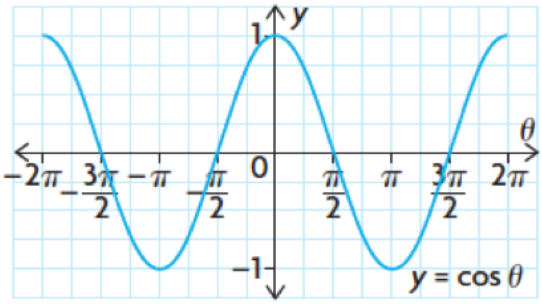
e) $\cot x + \frac{\sin x}{1+\cos x} = \csc x$

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

h) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

3)a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin\frac{\pi}{6}$ **b)** $\cos\frac{5\pi}{12}$ **c)** $\cos\frac{5\pi}{16}$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin\frac{7\pi}{8}$ **b)** $\cos\frac{13\pi}{12}$ **c)** $\cos\frac{11\pi}{6}$

6) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{\pi}{3}=\frac{1}{2}$

7) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{2\pi}{3}=-\frac{1}{2}$

8) Given that $\csc\frac{\pi}{4}=\sqrt{2}$, use an equivalent trigonometric expression to show that $\sec\frac{3\pi}{4}=-\sqrt{2}$

9) Given that $\cos\frac{3\pi}{11}\sim 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\sin\frac{5\pi}{22}$

b) $\sin\frac{17\pi}{22}$

Answer Key

1)a) $\tan x$ **b)** 1 **c)** $\sin^2 x$ **d)** $\cos^2 x$ **e)** $\sec x$ **f)** $\cos^2 x$ **g)** $\cos x$ **h)** $\csc^2 x$ **i)** $\tan x$ **j)** $\tan x$

3) Answers will vary depending but possible solutions are:
a) $y = \cos(\theta + 2\pi), y = \cos(\theta - 2\pi), y = \cos(\theta - 4\pi)$
b) $y = \sin(\theta + \frac{5\pi}{2}), y = \sin(\theta - \frac{3\pi}{2}), y = \sin(\theta - \frac{7\pi}{2})$

4)a) $\cos\frac{\pi}{3}$ **b)** $\sin\frac{\pi}{12}$ **c)** $\sin\frac{3\pi}{16}$ **5)a)** $\sin\frac{\pi}{8}$ **b)** $-\cos\frac{\pi}{12}$ **c)** $\cos\frac{\pi}{6}$

9)a) $\sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} \sim 0.6549$ **b)** $\sin\frac{17\pi}{22} = \cos\left(-\frac{3\pi}{11}\right) = \cos\left(\frac{3\pi}{11}\right) \sim 0.6549$