

W1 - 4.1 Radian Measure

MHF4U

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ANSWERS

1) Determine mentally the exact radian measure for each angle, given that 30° is exactly $\frac{\pi}{6}$ radians.

$$\begin{array}{llll} \text{a) } 60^\circ = 2(30^\circ) & \text{b) } 90^\circ = \frac{3\pi}{6} & \text{c) } 120^\circ = \frac{4\pi}{6} & \text{d) } 150^\circ = \frac{5\pi}{6} \\ & = 2\left(\frac{\pi}{6}\right) & = \frac{2\pi}{3} & \\ & = \frac{\pi}{3} & & \end{array}$$

2) Determine mentally the exact radian measure for each angle, given that 30° is exactly $\frac{\pi}{6}$ radians.

$$\begin{array}{llll} \text{a) } 15^\circ = \frac{\left(\frac{\pi}{6}\right)}{2} & \text{b) } 10^\circ = \frac{\left(\frac{\pi}{6}\right)}{3} & \text{c) } 7.5^\circ = \frac{\left(\frac{\pi}{6}\right)}{4} & \text{d) } 5^\circ = \frac{\left(\frac{\pi}{6}\right)}{6} \\ & = \frac{\pi}{12} & = \frac{\pi}{24} & = \frac{\pi}{36} \\ & & & \end{array}$$

3) Determine mentally the exact radian measure for each angle, given that 45° is exactly $\frac{\pi}{4}$ radians.

$$\begin{array}{llll} \text{a) } 90^\circ = \frac{2\pi}{4} & \text{b) } 135^\circ = \frac{3\pi}{4} & \text{c) } 180^\circ = \frac{4\pi}{4} & \text{d) } 225^\circ = \frac{5\pi}{4} \\ & = \frac{\pi}{2} & = \pi & \end{array}$$

4) Determine mentally the exact radian measure for each angle, given that 45° is exactly $\frac{\pi}{4}$ radians.

$$\begin{array}{llll} \text{a) } 22.5^\circ = \frac{\left(\frac{\pi}{4}\right)}{2} & \text{b) } 15^\circ = \frac{\left(\frac{\pi}{4}\right)}{3} & \text{c) } 9^\circ = \frac{\left(\frac{\pi}{4}\right)}{5} & \text{d) } 3^\circ = \frac{\left(\frac{\pi}{4}\right)}{15} \\ & = \frac{\pi}{8} & = \frac{\pi}{20} & = \frac{\pi}{60} \\ & & & \end{array}$$

5) Determine the EXACT radian measure for each angle

$$\begin{array}{lll} \text{a) } 40^\circ \times \frac{\pi}{180} & \text{b) } 10^\circ \times \frac{\pi}{180} & \text{c) } 315^\circ \times \frac{\pi}{180} \\ & = \frac{\pi}{18} & = \frac{7\pi}{4} \\ & & \\ \text{d) } 210^\circ \times \frac{\pi}{180} & \text{e) } 300^\circ \times \frac{\pi}{180} & \text{f) } 75^\circ \times \frac{\pi}{180} \\ & = \frac{5\pi}{3} & = \frac{5\pi}{12} \end{array}$$

6) Determine the APPROXIMATE radian measure, the nearest hundredth, for each angle.

$$\begin{aligned} \text{a) } 23^\circ &\times \frac{\pi}{180} \\ &\approx 0.40 \end{aligned}$$

$$\begin{aligned} \text{b) } 51^\circ &\times \frac{\pi}{180} \\ &\approx 0.89 \end{aligned}$$

$$\begin{aligned} \text{c) } 82^\circ &\times \frac{\pi}{180} \\ &= 1.43 \end{aligned}$$

$$\begin{aligned} \text{d) } 128^\circ &\times \frac{\pi}{180} \\ &\approx 2.23 \end{aligned}$$

$$\begin{aligned} \text{e) } 240^\circ &\times \frac{\pi}{180} \\ &\approx 4.19 \end{aligned}$$

$$\begin{aligned} \text{f) } 330^\circ &\times \frac{\pi}{180} \\ &\approx 5.76 \end{aligned}$$

7) Determine the EXACT degree measure for each angle.

$$\begin{aligned} \text{a) } \frac{\pi}{5} &\times \frac{180}{\pi} \\ &= 36^\circ \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{\pi}{9} &\times \frac{180}{\pi} \\ &= 20^\circ \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{5\pi}{12} &\times \frac{180}{\pi} \\ &= 75^\circ \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{5\pi}{18} &\times \frac{180}{\pi} \\ &= 50^\circ \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{3\pi}{4} &\times \frac{180}{\pi} \\ &= 135^\circ \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{3\pi}{2} &\times \frac{180}{\pi} \\ &= 270^\circ \end{aligned}$$

8) Determine the APPROXIMATE degree measure, to the nearest tenth, for each angle.

$$\begin{aligned} \text{a) } 2.34 &\times \frac{180}{\pi} \\ &\approx 134.1^\circ \end{aligned}$$

$$\begin{aligned} \text{b) } 3.14 &\times \frac{180}{\pi} \\ &\approx 179.9^\circ \end{aligned}$$

$$\begin{aligned} \text{c) } 5.27 &\times \frac{180}{\pi} \\ &\approx 301.9^\circ \end{aligned}$$

$$\begin{aligned} \text{d) } 7.53 &\times \frac{180}{\pi} \\ &\approx 431.4^\circ \end{aligned}$$

$$\begin{aligned} \text{e) } 0.68 &\times \frac{180}{\pi} \\ &\approx 39.0 \end{aligned}$$

$$\begin{aligned} \text{f) } 1.72 &\times \frac{180}{\pi} \\ &\approx 98.5^\circ \end{aligned}$$

9) A circle of radius 25 cm has a central angle of 4.75 radians. Determine the length of the arc that subtends this angle.

$$\theta = \frac{a}{r}$$

$$4.75 = \frac{a}{25}$$

$$a = 118.75 \text{ cm}$$

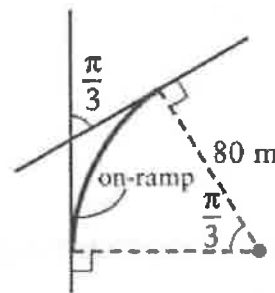
10) Two highways meet at an angle measuring $\frac{\pi}{3}$ radians, as shown. An on-ramp in the shape of a circular arc is to be built such that the arc has a radius of 80 m.

a) Determine an EXACT expression for the length of the on-ramp.

$$\theta = \frac{a}{r}$$

$$\frac{\pi}{3} = \frac{a}{80}$$

$$a = \frac{80\pi}{3} \text{ m}$$



b) Determine the length of the on-ramp, to the nearest tenth of a meter.

$$\approx 83.8 \text{ m}$$

11) David made a swing for his niece Sarah using ropes 2.4 m long, so that Sarah swings through an arc length of 1.2 meters. Determine the angle through which Sarah swings, in both radians and degrees.

$$\theta = \frac{a}{r}$$

$$\theta = \frac{1.2}{2.4}$$

$$\theta = 0.5 \text{ radians}$$

$$0.5 \times \frac{180}{\pi}$$

$$\approx 28.6^\circ$$

Answer Key

1)a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) $\frac{5\pi}{6}$

2) a) $\frac{\pi}{12}$ b) $\frac{\pi}{18}$ c) $\frac{\pi}{24}$ d) $\frac{\pi}{36}$

3)a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) π d) $\frac{5\pi}{4}$

4)a) $\frac{\pi}{8}$ b) $\frac{\pi}{12}$ c) $\frac{\pi}{20}$ d) $\frac{\pi}{60}$

5)a) $\frac{2\pi}{9}$ b) $\frac{\pi}{18}$ c) $\frac{7\pi}{4}$ d) $\frac{7\pi}{6}$ e) $\frac{5\pi}{3}$ f) $\frac{5\pi}{12}$

6)a) 0.4 b) 0.89 c) 1.43 d) 2.23 e) 4.19 f) 5.76

7)a) 36° b) 20° c) 75° d) 50° e) 135° f) 270°

8)a) 134.1° b) 179.9° c) 301.9° d) 431.4° e) 39° f) 98.5°

9) 118.75 cm

10)a) $\frac{80\pi}{3}$ m b) 83.8 m

11) 0.5 radians; 28.6°