W1 – 4.3 Co-function Identities
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SOLUTIONS

1) Simplify.

a) $\sin x\left(\frac{1}{\cos x}\right)$

$= \frac{\sin x}{\cos x}$

$= \tan x$

b) $(\cos x)(\sec x)$

$= \cos x\left(\frac{1}{\cos x}\right)$

$= 1$

c) $1 - \cos^2 x$

$= \sin^2 x$

d) $1 - \sin^2 x$

$= \cos^2 x$

e) $\frac{\tan x}{\sin x}$

$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x}$

$= \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$= \frac{1}{\cos x}$

$= \sec x$

f) $(1 - \sin x)(1 + \sin x)$

$= 1 - \sin^2 x$

$= \cos^2 x$

g) $\left(\frac{1}{\tan x}\right)\sin x$

$= \cot x(\sin x)$

$= \frac{\cos x}{\sin x}(\sin x)$

$= \cos x$

h) $\frac{1+\tan^2 x}{\tan^2 x}$

$= \frac{\sec^2 x}{\tan^2 x}$

$= \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

$= \frac{1}{\sin^2 x}$

$= \csc^2 x$

i) $\frac{\sin x \cos x}{1-\sin^2 x}$

$= \frac{\sin x \cos x}{\cos^2 x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

j) $\frac{1-\cos^2 x}{\sin x \cos x}$

$= \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

2) Prove the following identities.

a) $\sin^2 x\,(1 + \cot^2 x) = 1$

LS	RS
$= \sin^2 x + \sin^2 x \cot^2 x$	$= 1$
$= \sin^2 x + \sin^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right)$	
$= \sin^2 x + \cos^2 x$	
$= 1$	

LS = RS

b) $1 - \cos^2 x = \tan x \cos x \sin x$

LS	RS
$= 1 - \cos^2 x$	$= \tan x \cos x \sin x$
$= \sin^2 x$	$= \left(\frac{\sin x}{\cos x}\right)(\cos x)(\sin x)$
	$= \sin^2 x$

LS = RS

c) $\cos x \tan^3 x = \sin x \tan^2 x$

LS	RS
$= \cos x \tan^3 x$	$= \sin x \tan^2 x$
$= \cos x \left(\frac{\sin^3 x}{\cos^3 x}\right)$	$= \sin x \left(\frac{\sin^2 x}{\cos^2 x}\right)$
$= \frac{\sin^3 x}{\cos^2 x}$	$= \frac{\sin^3 x}{\cos^2 x}$

LS=RS

d) $1 - 2\cos^2\theta = \sin^4\theta - \cos^4\theta$

LS	RS
$= 1 - 2\cos^2\theta$	$= (\sin^2\theta)^2 - (\cos^2\theta)^2$
	$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$
	$= (\sin^2\theta - \cos^2\theta)(1)$
	$= 1 - \cos^2\theta - \cos^2\theta$
	$= 1 - 2\cos^2\theta$

LS=RS

e) $\cot x + \frac{\sin x}{1+\cos x} = \csc x$

LS	RS
$= \frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$	$= \csc x$
	$= \frac{1}{\sin x}$
$= \frac{(1+\cos x)(\cos x) + \sin x(\sin x)}{\sin x(1+\cos x)}$	
$= \frac{\cos x + \cos^2 x + \sin^2 x}{\sin x(1+\cos x)}$	
$= \frac{\cos x + 1}{\sin x(1+\cos x)}$	
$= \frac{1}{\sin x}$	

LS=RS

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

LS	RS
$= \frac{\left(\frac{1}{\cos x}\right)}{\sin x} + \frac{\left(\frac{1}{\sin x}\right)}{\cos x}$	$= \frac{2}{\sin x \cos x}$
$= \frac{1}{\cos x \sin x} + \frac{1}{\sin x \cos x}$	
$= \frac{2}{\sin x \cos x}$	

LS=RS

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

LS	RS
$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$	$= 1 - \frac{\sin x}{\cos x}$
	$= \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}$
$= \frac{\cos x - \sin x}{\cos x}$	$= \frac{\cos x - \sin x}{\cos x}$

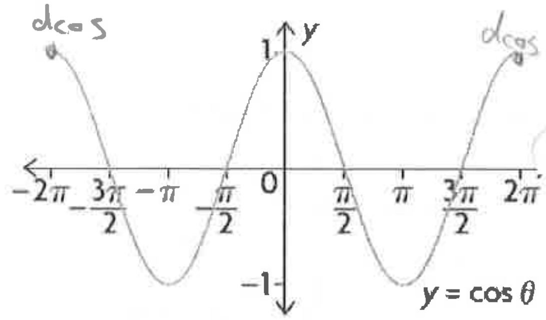
LS=RS

h) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

LS	RS
$= \frac{1}{1+\cos x} + \frac{1}{1-\cos x}$	$= 2\csc^2 x$
$= \frac{1-\cos x + 1+\cos x}{(1+\cos x)(1-\cos x)}$	$= 2\left(\frac{1}{\sin^2 x}\right)$
$= \frac{2}{1-\cos^2 x}$	$= \frac{2}{\sin^2 x}$
$= \frac{2}{\sin^2 x}$	

LS=RS

3)a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



① $y = \cos(\theta - 2\pi)$

② $y = \cos(\theta + 2\pi)$

③ $y = \cos(\theta - 4\pi)$

b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

$\cos x = \sin(x + \frac{\pi}{2})$

① $\cos(\theta - 2\pi) = \sin[(\theta - 2\pi) + \frac{\pi}{2}] = \sin(\theta - \frac{3\pi}{2})$

② $\cos(\theta + 2\pi) = \sin[(\theta + 2\pi) + \frac{\pi}{2}] = \sin(\theta + \frac{5\pi}{2})$

③ $\cos(\theta - 4\pi) = \sin[\theta - 4\pi) + \frac{\pi}{2}] = \sin(\theta - \frac{7\pi}{2})$

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin\frac{\pi}{6}$

$= \cos(\frac{\pi}{2} - \frac{\pi}{6})$

$= \cos(\frac{2\pi}{6})$

$= \cos(\frac{\pi}{3})$

b) $\cos\frac{5\pi}{12}$

$= \sin(\frac{\pi}{2} - \frac{5\pi}{12})$

$= \sin(\frac{\pi}{12})$

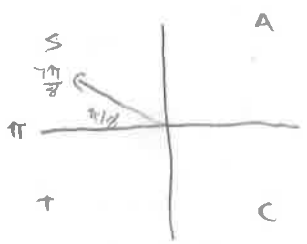
c) $\cos\frac{5\pi}{16}$

$= \sin(\frac{\pi}{2} - \frac{5\pi}{16})$

$= \sin(\frac{3\pi}{16})$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

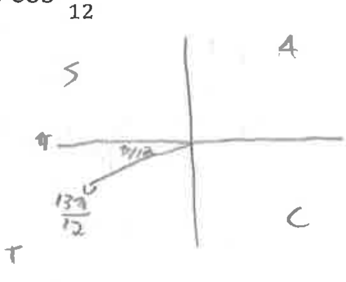
a) $\sin\frac{7\pi}{8}$



$\sin(\pi - x) = \sin x$

$\therefore \sin(\pi - \frac{\pi}{8}) = \sin(\frac{\pi}{8})$

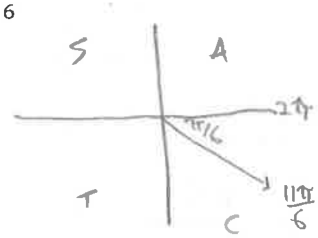
b) $\cos\frac{13\pi}{12}$



$\cos(\pi + x) = -\cos x$

$\therefore \cos(\pi + \frac{\pi}{12}) = -\cos(\frac{\pi}{12})$

c) $\cos\frac{11\pi}{6}$



$\cos(2\pi - x) = \cos x$

$\cos(2\pi - \frac{\pi}{6}) = \cos(\frac{\pi}{6})$

6) Given that $\sin\frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{2\pi}{6}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

$\therefore \sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

7) Given that $\sin\frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{2\pi}{3} = -\frac{1}{2}$

$\cos\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$

$= \sin\left(-\frac{\pi}{6}\right)$

$= -\sin\left(\frac{\pi}{6}\right)$

$\therefore \cos\left(\frac{2\pi}{3}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

8) Given that $\csc\frac{\pi}{4} = \sqrt{2}$, use an equivalent trigonometric expression to show that $\sec\frac{3\pi}{4} = -\sqrt{2}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$\cos\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$

$= \sin\left(-\frac{\pi}{4}\right)$

$= -\sin\left(\frac{\pi}{4}\right)$

If $\cos\frac{3\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$,

then $\sec\frac{3\pi}{4} = -\sqrt{2}$

9) Given that $\cos\frac{3\pi}{11} \sim 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\sin\frac{5\pi}{22}$

$= \cos\left(\frac{\pi}{2} - \frac{5\pi}{22}\right)$

$= \cos\left(\frac{6\pi}{22}\right)$

$= \cos\left(\frac{3\pi}{11}\right)$

$\simeq 0.6549$

b) $\sin\frac{17\pi}{22}$

$= \cos\left(\frac{\pi}{2} - \frac{17\pi}{22}\right)$

$= \cos\left(-\frac{6\pi}{22}\right)$

$= \cos\left(\frac{6\pi}{22}\right)$

$= \cos\left(\frac{3\pi}{11}\right)$

$\simeq 0.6549$

Answer Key

1)a) $\tan x$ **b)** 1 **c)** $\sin^2 x$ **d)** $\cos^2 x$ **e)** $\sec x$ **f)** $\cos^2 x$ **g)** $\cos x$ **h)** $\csc^2 x$ **i)** $\tan x$ **j)** $\tan x$

3) Answers will vary depending but possible solutions are:
a) $y = \cos(\theta + 2\pi)$, $y = \cos(\theta - 2\pi)$, $y = \cos(\theta - 4\pi)$
b) $y = \sin(\theta + \frac{5\pi}{2})$, $y = \sin(\theta - \frac{3\pi}{2})$, $y = \sin(\theta - \frac{7\pi}{2})$

4)a) $\cos\frac{\pi}{3}$ **b)** $\sin\frac{\pi}{12}$ **c)** $\sin\frac{3\pi}{16}$ **5)a)** $\sin\frac{\pi}{8}$ **b)** $-\cos\frac{\pi}{12}$ **c)** $\cos\frac{\pi}{6}$

9)a) $\sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} \sim 0.6549$ **b)** $\sin\frac{17\pi}{22} = \cos\left(-\frac{3\pi}{11}\right) = \cos\left(\frac{3\pi}{11}\right) \sim 0.6549$