

W1 – 6.1/6.2 – Intro to Logarithms and Review of Exponentials

MHF4U

Jensen

SOLUTIONS

- 1) Sketch a graph of each function. Then, sketch a graph of the inverse of each function. Label each graph with its equation.

a)  $y = 2^x$

$$f(x) = 2^x$$

x	y
-2	0.25
-1	0.5
0	1
1	2
2	4

b)  $y = 4^x$

$$g(x) = 4^x$$

x	y
-2	0.0625
-1	0.25
0	1
1	4
2	16

Find eqn of  $f^{-1}(x)$

$$x = 2^y$$

$$\log x = \log 2^y$$

$$\log x = y \log 2$$

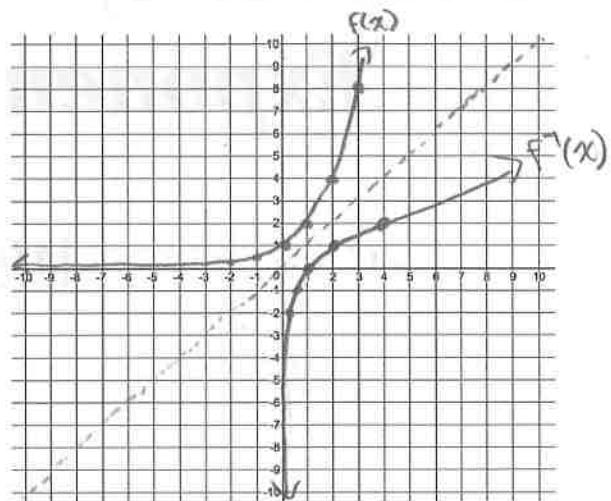
$$\frac{\log x}{\log 2} = y$$

$$y = \log_2 x$$

$$f^{-1}(x) = \log_2 x$$

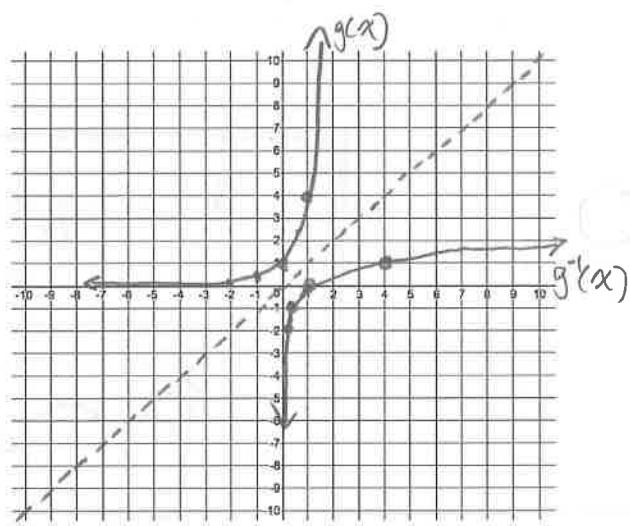
$$f^{-1}(x) = \log_2 x$$

x	y
0.25	-2
0.5	-1
1	0
2	1
4	2



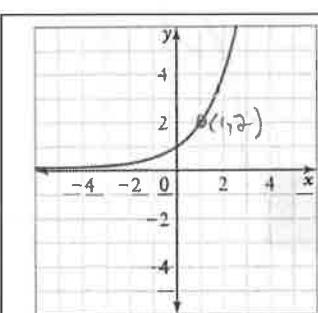
$$g^{-1}(x) = \log_4 x$$

x	y
0.0625	-2
0.25	-1
1	0
4	1
16	2



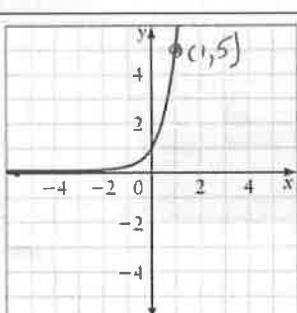
- 2) Match each equation to its corresponding graph.

A)  $y = 5^x$



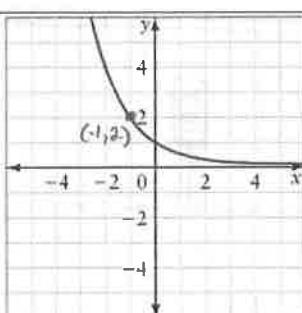
C

B)  $y = \left(\frac{1}{2}\right)^x$



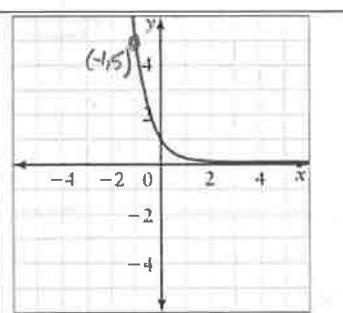
A

C)  $y = 2^x$



B

D)  $y = \left(\frac{1}{5}\right)^x$



D

3) An influenza virus is spreading according to the function  $N = 10(2)^t$ , where  $N$  is the number of people infected and  $t$  is the time, in days.

How many people have the virus at each time?

i) initially, when  $t = 0$

$$N = 10$$

ii) after 1 day

$$\begin{aligned} N &= 10(2)^1 \\ &= 20 \end{aligned}$$

iii) after 2 days

$$\begin{aligned} N &= 10(2)^2 \\ &= 40 \end{aligned}$$

iv) after 3 days

$$\begin{aligned} N &= 10(2)^3 \\ &= 80 \end{aligned}$$

b) After how many days will 40960 people be infected?

$$\begin{aligned} 40960 &= 10(2)^t \\ 4096 &= 2^t \\ \log 4096 &= \log 2^t \end{aligned}$$

$$\begin{aligned} \log 4096 &= t \log(2) \\ t &= \frac{\log 4096}{\log 2} \\ t &= 12 \text{ days} \end{aligned}$$

4) Rewrite each equation in logarithmic form

$$4^3 = 64$$

$$\text{b)} 128 = 2^7$$

$$\text{c)} 5^{-2} = \frac{1}{25}$$

$$\log_4 64 = 3$$

$$\log_2 128 = 7$$

$$\log_5 \left(\frac{1}{25}\right) = -2$$

$$\text{d)} \left(\frac{1}{2}\right)^2 = 0.25$$

$$\text{e)} 6^x = y$$

$$\text{f)} 10^5 = 100\,000$$

$$\log_{\frac{1}{2}} 0.25 = 2$$

$$\log_6 y = x$$

$$\log_{10} 100\,000 = 5$$

$$\text{g)} \frac{1}{27} = 3^{-3}$$

$$\log_3 \left(\frac{1}{27}\right) = -3$$

5) Evaluate each logarithm

a)  $\log_2 64$

$$= \log_2(2^6)$$

$$= 6$$

b)  $\log_3 27$

$$= \log_3(3^3)$$

$$= 3$$

c)  $\log_2\left(\frac{1}{4}\right)$

$$= \log_2(2^{-2})$$

$$= -2$$

d)  $\log_4\left(\frac{1}{64}\right)$

$$= \log_4(4^{-3})$$

$$= -3$$

e)  $\log_5 125$

$$= \log_5(5^3)$$

$$= 3$$

f)  $\log_2 1024$

$$= \log_2(2^{10})$$

$$= 10$$

6) Evaluate each common logarithm

a)  $\log 1000$

$$= \log(10^3)$$

$$= 3$$

b)  $\log\left(\frac{1}{10}\right)$

$$= \log(10^{-1})$$

$$= -1$$

c)  $\log 1$

$$= \log(10^0)$$

$$= 0$$

d)  $\log 0.001$

$$= \log(10^{-3})$$

$$= -3$$

e)  $\log 10^{-4}$

$$= -4$$

f)  $\log 1\ 000\ 000$

$$= \log(10^6)$$

$$= 6$$

7) Rewrite in exponential form

a)  $\log_7 49 = 2$

$$7^2 = 49$$

b)  $5 = \log_2 32$

$$2^5 = 32$$

c)  $\log 10\ 000 = 4$

$$10^4 = 10\ 000$$

d)  $w = \log_b z$

$$b^w = z$$

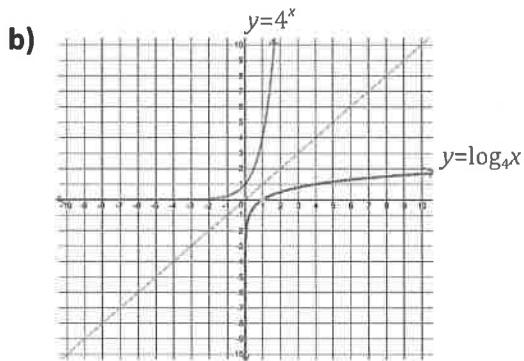
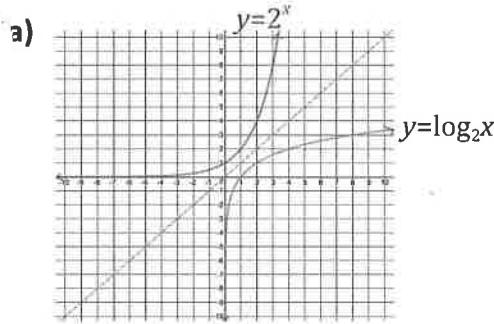
e)  $\log_2 8 = 3$

$$2^3 = 8$$

f)  $-2 = \log\left(\frac{1}{100}\right)$

$$10^{-2} = \frac{1}{100}$$

## ANSWER KEY



2) C A B D

3)a)i) 10 ii) 20 iii) 40 iv) 80 b) 12 days

4)a)  $\log_4 64 = 3$  b)  $\log_2 128 = 7$  c)  $\log_5 \left(\frac{1}{25}\right) = -2$  d)  $\log_{\frac{1}{2}} 0.25 = 2$  e)  $\log_6 y = x$

f)  $\log_{10} 100\,000 = 5$  g)  $\log_3 \left(\frac{1}{27}\right) = -3$

5)a) 6 b) 3 c) -2 d) -3 e) 3 f) 10

6)a) 3 b) -1 c) 0 d) -3 e) -4 f) 6

a)  $7^2 = 49$  b)  $2^5 = 32$  c)  $10^4 = 10\,000$  d)  $b^w = z$  e)  $2^3 = 8$  f)  $10^{-2} = \frac{1}{100}$