W1 – Cartesian Vectors Unit 5

MCV4U

Iensen

SOLUTIONS

1) Express each vector in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

a) [–2, 0]

$= -2\hat{\imath}$

b) [0, 3]

$= 3\hat{\jmath}$

c) [3, 2]

$= 3\hat{\imath} + 2\hat{\jmath}$

d) [–1, 6]

$= -\hat{\imath} + 6\hat{\jmath}$

2) Express each vector as a position vector [*a*, *b*].

a) $3\hat{\imath} + 2\hat{\jmath}$

$[3, 2]$

b) $4\hat{\jmath}$

$[0, 4]$

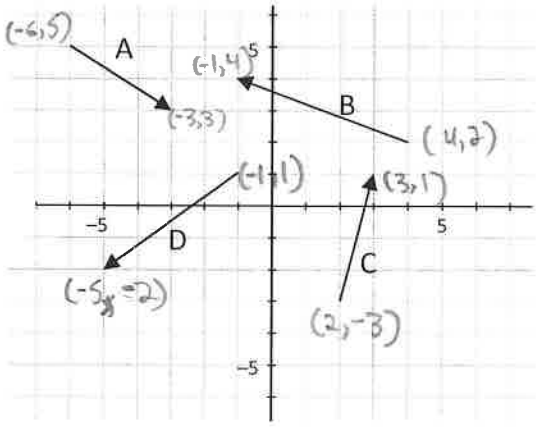
c) $-7\hat{\imath} + 3\hat{\jmath}$

$[-7, 3]$

d) $-9\hat{\imath}$

$[-9, 0]$

3) Write the coordinates of each Cartesian vector and determine the magnitude.



$\vec{A} = [-3-(-6), 3-5] = [3, -2]$

$|\vec{A}| = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}$

$\vec{B} = [-1-4, 4-2] = [-5, 2]$

$|\vec{B}| = \sqrt{(-5)^2 + (2)^2} = \sqrt{29}$

$\vec{C} = [3-2, 1-(-3)] = [1, 4]$

$|\vec{C}| = \sqrt{(1)^2 + (4)^2} = \sqrt{17}$

$\vec{D} = [-5-(-1), -2-1] = [-4, -3]$

$|\vec{D}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

4) Given the vector $\vec{v} = [2, -5]$.

a) State the vertical and horizontal vector components of $\vec{v}$.

horizontal: $v_x = 2$

vertical: $v_y = -5$

b) Find two vectors that are collinear with $\vec{v}$.

$2\vec{v} = [2(2), 2(-5)] = [4, -10]$

$3\vec{v} = [3(2), 3(-5)] = [6, -15]$

5) If $\vec{u}$ = [–3, 5] and $\vec{v}$ = [2, 9].

a) $\vec{u} + \vec{v}$

$= [-3+2, 5+9]$

$= [-1, 14]$

b) $\hat{u}$

$= \frac{1}{|\vec{u}|}\vec{u}$

$= \frac{1}{\sqrt{34}}[-3, 5]$

$= \left[\frac{-3}{\sqrt{34}}, \frac{5}{\sqrt{34}}\right]$

c) $-3\vec{u} + 4\vec{v}$

$= -3[-3,5] + 4[2,9]$

$= [9, -15] + [8, 36]$

$= [17, 21]$

d) $7\vec{u} + 6\hat{\imath} - 8\hat{\jmath} - 3\vec{v}$

$= 7(-3\hat{\imath}+5\hat{\jmath}) + 6\hat{\imath} - 8\hat{\jmath} - 3(2\hat{\imath}+9\hat{\jmath})$

$= -21\hat{\imath} + 35\hat{\jmath} + 6\hat{\imath} - 8\hat{\jmath} - 6\hat{\imath} - 27\hat{\jmath}$

$= -21\hat{\imath} + 0\hat{\jmath}$

$= [-21, 0]$

e) $|\vec{v}|$

$= \sqrt{(2)^2+(9)^2}$

$= \sqrt{85}$

f) $|-3\vec{u} - 2\vec{v}|$

$-3\vec{u} - 2\vec{v} = -3[-3,5] - 2[2,9]$

$= [9, -15] - [4, 18]$

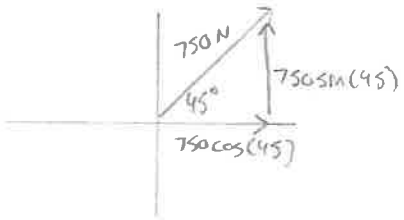
$= [5, -33]$

$|-3\vec{u} - 2\vec{v}| = \sqrt{(5)^2+(-33)^2}$

$= \sqrt{1114}$

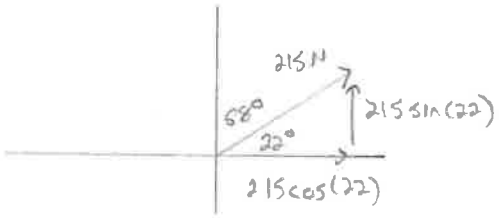
6. Write each force as a Cartesian vector.

a) 750 N applied 45° to the horizontal



$\vec{F} = [750\cos(45), 750\sin(45)]$

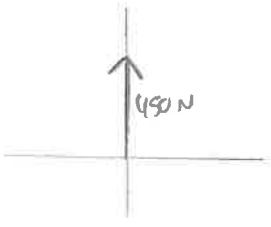
b) 215 N applied 68° to the vertical



$\vec{F} = [215\cos(22), 215\sin(22)]$

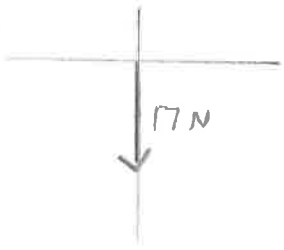
c) 450 N applied upwards

$[0, 450]$

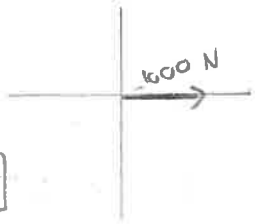


d) 17 N applied downwards

$[0, -17]$

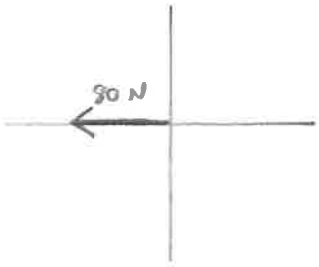


e) 1000 N east



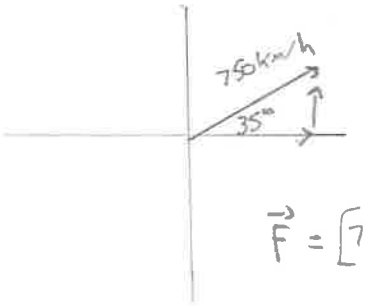
$[1000, 0]$

f) 80 N west



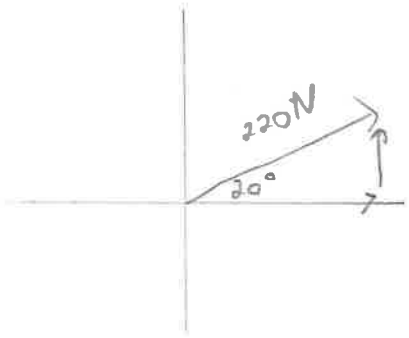
$[-80, 0]$

7) An aircraft is travelling at 750 km per hour at an angle of 35° to the level ground below. Find the force in component form as a Cartesian vector.



$\vec{F} = [750\cos(35), 750\sin(35)] \simeq [614.36, 430.18]$

A mom is pulling a sled exerting a force of 220 N along a rope that makes an angle of ~~20~~ 20° to the horizontal. Write this force in component form as a Cartesian vector.

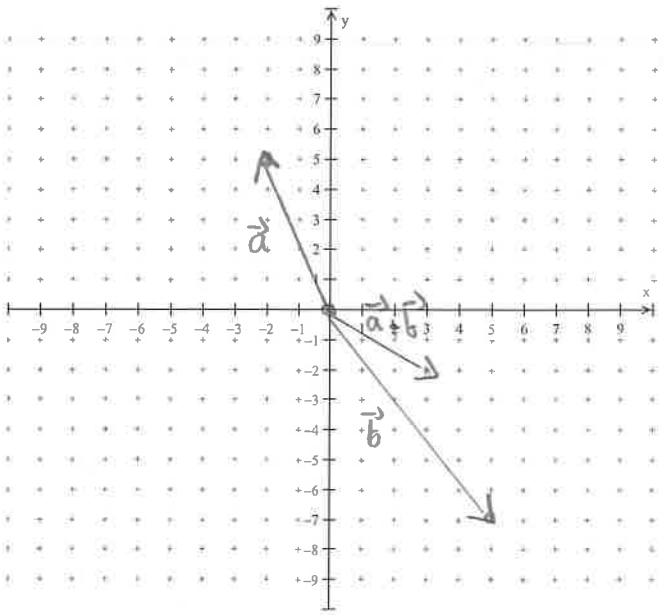


$\vec{F} = [220\cos 20, 220\sin 20] \simeq [206.73, 75.24]$

9) Let $\vec{a} = [-2, 5]$ and $\vec{b} = [5, -7]$.

a) Plot the two vectors.

b) Which is greater: $|\vec{a} + \vec{b}|$ or $|\vec{a}| + |\vec{b}|$?



$\vec{a} + \vec{b} = [-2+5, 5+(-7)]$

$= [3, -2]$

$|\vec{a}+\vec{b}| = \sqrt{13}$

$|\vec{a}| + |\vec{b}| = \sqrt{29} + \sqrt{74}$

$|\vec{a}| + |\vec{b}| > |\vec{a} + \vec{b}|$

10) Given the points $P(-6,1)$, $Q(-2,-1)$, and $R(-3,4)$, find...

a) $\overrightarrow{QP}$

$\overrightarrow{QP} = [-6-(-2), 1-(-1)]$

$= [-4, 2]$

b) $|\overrightarrow{RP}|$

$= \sqrt{[-6-(-3)]^2 + (1-4)^2}$

$= \sqrt{(-3)^2+(-3)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

c) perimeter of ΔPQR

$|\overrightarrow{RP}| = 3\sqrt{2}$

$|\overrightarrow{QP}| = \sqrt{(-4)^2+(2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$|\overrightarrow{QR}| = \sqrt{(-1)^2+(5)^2}$

$= \sqrt{26}$

Perimeter $= 3\sqrt{2} + 2\sqrt{5} + \sqrt{26}$

≈ 13.8 units

11) Which vector is NOT colinear with $\vec{a} = [6, -4]$?

$\vec{b} = [3, -2]$, $\vec{c} = [-6, -4]$, $\vec{d} = [-6,4]$, or $\vec{e} = [-9,6]$

$\frac{a_x}{b_x} = \frac{a_y}{b_y}$ $\quad \frac{6}{3} = \frac{-4}{-2}$ $\quad 2 = 2$ $\quad \vec{a} = 2\vec{b}$

$\frac{a_x}{c_x} = \frac{a_y}{c_y}$ $\quad \frac{6}{-6} = \frac{-4}{-4}$ $\quad -1 \neq 1$ & NOT collinear

$\frac{a_x}{d_x} = \frac{a_y}{d_y}$ $\quad \frac{6}{-6} = \frac{-4}{4}$ $\quad -1 = -1$ $\quad \vec{a} = -\vec{d}$

$\frac{a_x}{e_x} = \frac{a_y}{e_y}$ $\quad \frac{6}{-9} = \frac{-4}{6}$ $\quad -\frac{2}{3} = -\frac{2}{3}$ $\quad \vec{a} = -\frac{2}{3}\vec{e}$

ANSWER KEY:

1)a) $-2\hat{\imath}$ **b)** $3\hat{\jmath}$ **c)** $3\hat{\imath} + 2\hat{\jmath}$ **d)** $-\hat{\imath} + 6\hat{\jmath}$

2)a) [3, 2] **b)** [0, 4] **c)** [−7, 3] **d)** [−9, 0]

3)a) (3, −2); $\sqrt{13}$ **b)** (−5, 2); $\sqrt{29}$ **c)** (1, 4); $\sqrt{17}$ **d)** (−4,−3); 5

4) a) vertical: 2; horizontal: −5 **b)** e.g., [4, −10], [−6, 15]

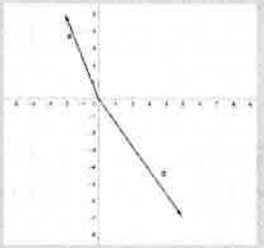
5. a) [−1,14] **b)** [illegible] **c)** [17, 21] **d)** [−21, 0] **e)** $\sqrt{85}$ **f)** $\sqrt{1114}$

6) a) [530.3, 530.33] **b)** [199, 80.5] **c)** [0, 450] **d)** [0, −17] **e)** [1000, 0] **f)** [−80, 0]

7) [614.36, 430.18]

8) [206.7, 75.2]

9) a)



b) $|\vec{a}| + |\vec{b}|$ is greater

10)a) [−4,2] **b)** $3\sqrt{2}$ units **c)** 13.8 units

11) $\vec{c}$