W1 – Derivative of a Polynomial Functions Unit 1

MCV4U

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SOLUTIONS

1) Circle the functions that have a derivative of zero:

A) $y = 8.7$ **B)** $y = -4 + x$ **C)** $y = \frac{5}{9}x$ **D)** $y = \sqrt{7}$ **E)** $y = -7.1\pi$

2) For each function, determine $\frac{dy}{dx}$

a) $y = x$

$\frac{dy}{dx} = 1$

b) $y = \frac{1}{4}x^2$

$\frac{dy}{dx} = 2\left(\frac{1}{4}\right)x$

$\frac{dy}{dx} = \frac{1}{2}x$

c) $y = -3x^4$

$\frac{dy}{dx} = 4(-3)x^3$

$\frac{dy}{dx} = -12x^3$

d) $y = \sqrt[5]{x^3}$

$y = x^{3/5}$

$\frac{dy}{dx} = \frac{3}{5}x^{3/5-1}$

$\frac{dy}{dx} = \frac{3}{5}x^{-2/5}$

$\frac{dy}{dx} = \frac{3}{5\sqrt[5]{x^2}}$

e) $y = \frac{5}{x}$

$y = 5x^{-1}$

$\frac{dy}{dx} = -5x^{-2}$

$\frac{dy}{dx} = \frac{-5}{x^2}$

f) $y = \frac{4}{\sqrt{x}}$

$y = 4x^{-1/2}$

$\frac{dy}{dx} = -\frac{1}{2}(4)x^{-1/2-1}$

$\frac{dy}{dx} = -2x^{-3/2}$

$\frac{dy}{dx} = \frac{-2}{\sqrt{x^3}}$

3) Determine the slope of the tangent to the graph of each function at the indicated value.

a) $y = 6$ at $x = 12$

$\frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{x=12} = 0$

b) $f(x) = 2x^5$ at $x = \sqrt{3}$

$f'(x) = 10x^4$

$f'(\sqrt{3}) = 10(\sqrt{3})^4$

$f'(\sqrt{3}) = 10(3)^2$

$f'(\sqrt{3}) = 90$

c) $y = \frac{1}{3x}$ at $x = -2$

$y = \frac{1}{3}x^{-1}$

$\frac{dy}{dx} = -\frac{1}{3}x^{-2}$

$\frac{dy}{dx} = \frac{-1}{3x^2}$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-1}{3(-2)^2}$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-1}{12}$

4) Find the derivative of each function

a) $f(x) = 2x^2 + x^3$

$f'(x) = 4x + 3x^2$

b) $y = \frac{4}{5}x^5 - 3x$

$y' = 5(\frac{4}{5})x^4 - 3$

$y' = 4x^4 - 3$

c) $h(t) = -1.1x^4 + 78$

$h'(t) = -4.4x^3$

d) $p(a) = \frac{a^5}{15} - 2\sqrt{a}$

$p'(a) = 5(\frac{1}{15})a^4 - \frac{1}{2}(2)a^{-1/2}$

$p'(a) = \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$

e) $k(s) = -\frac{1}{s^2} + 7s^4$

$k(s) = -1s^{-2} + 7s^4$

$k'(s) = 2s^{-3} + 28s^3$

$k'(s) = \frac{2}{s^3} + 28s^3$

5)a) Determine the point at which the slope of the tangent to each parabola is zero.

i) $y = 6x^2 - 3x + 4$

$y' = 12x - 3$

$0 = 12x - 3$

$x = \frac{1}{4}$

$y = 6(\frac{1}{4})^2 - 3(\frac{1}{4}) + 4$

$y = \frac{29}{8}$

$(\frac{1}{4}, \frac{29}{8})$

ii) $y = \frac{3}{4}x^2 + 2x + 3$

$y' = \frac{3}{2}x + 2$

$0 = \frac{3}{2}x + 2$

$x = \frac{-4}{3}$

$y = \frac{3}{4}(\frac{-4}{3})^2 + 2(\frac{-4}{3}) + 3$

$y = 5/3$

$(\frac{-4}{3}, \frac{5}{3})$

b) Use technology to look at the graph of each parabola. What does the point found in part a) correspond to on each of these graphs?

The vertex.

6) Simplify and then differentiate

a) $f(x) = \frac{10x^4 - 6x^3}{2x^2}$

$f(x) = \frac{10x^4}{2x^2} - \frac{6x^3}{2x^2}$

$f(x) = 5x^2 - 3x$

$f'(x) = 10x - 3$

b) $(5x + 2)^2$

$f(x) = 25x^2 + 20x + 4$

$f'(x) = 50x + 20$

7) A skydiver jumps out of a plane that is flying 2500 meters above the ground. The skydiver's height, h, in meters, above th4e ground after t seconds is $h(t) = 2500 - 4.9t^2$.

a) Determine the rate of change of the height of the skydiver at $t = 5$s

$h'(t) = -9.8t$

$h'(5) = -9.8(5)$

$h'(5) = -49$ m/s

b) The skydiver's parachute opens at 1000m above the ground. After how many seconds does this happen?

$1000 = 2500 - 4.9t^2$

$t^2 = \frac{1500}{4.9}$

$t = \sqrt{\frac{1500}{4.9}}$

$t \simeq 17.5$ seconds

c) What is the rate of change of the height of the skydiver at the time found in part b)?

$h'(17.5) \simeq -9.8(17.5)$

$h'(17.5) \simeq -171.5$ m/s

8) Determine the equation of the tangent line to the graph of $y = -6x^4 + 2x^3 + 5$ at the point $(-1, -3)$.

$y' = -24x^3 + 6x^2$

$y'(-1) = -24(-1)^3 + 6(-1)^2$

$y'(-1) = 30$

$m = 30$

$y = mx + b$

$-3 = 30(-1) + b$

$-3 = -30 + b$

$b = 27$

$\boxed{y = 30x + 27}$

9) Determine the equation of the tangent line to the graph of $y = -1.5x^3 + 3x - 2$ at the point $(2, -8)$.

$y' = -4.5x^2 + 3$

$y'(2) = -4.5(2)^2 + 3$

$y'(2) = -18 + 3$

$y'(2) = -15$

$m = -15$

$y = mx + b$

$-8 = -15(2) + b$

$b = 22$

$\boxed{y = -15x + 22}$

10) A flaming arrow is shot into the air to mark the beginning of a festival. Its height, h, in meters, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 24.5t + 2$.

a) Determine the height of the arrow at $t = 2$s.

$h(2) = -4.9(2)^2 + 24.5(2) + 2$

$h(2) = 31.4\,m$

b) How long does it take the arrow to land on the ground?

$0 = -4.9t^2 + 24.5t + 2$

$$t = \frac{-24.5 \pm \sqrt{(24.5)^2 - 4(-4.9)(2)}}{2(-4.9)}$$

$\cancel{t_1 \doteq -0.08}$ $\boxed{t_2 \doteq 5.08 \text{ seconds}}$

c) How fast is the arrow travelling when it hits the ground?

$h'(t) = -9.8t + 24.5$

$h'(5.08) = -9.8(5.08) + 24.5$

$h'(5.08) = -25.284\ m/s$

Answers:

1) A, D, E
2)a) $\frac{dy}{dx} = 1$ **b)** $\frac{dy}{dx} = \frac{1}{2}x$ **c)** $\frac{dy}{dx} = -12x^3$ **d)** $\frac{dy}{dx} = \frac{3}{5\sqrt[5]{x^2}}$ **e)** $\frac{dy}{dx} = -\frac{5}{x^2}$ **f)** $\frac{dy}{dx} = -\frac{2}{\sqrt{x^3}}$
3)a) 0 **b)** 90 **c)** $-\frac{1}{12}$
4)a) $f'(x) = 4x + 3x^2$ **b)** $\frac{dy}{dx} = 4x^4 - 3$ **c)** $h'(t) = -4.4t^3$ **d)** $p'(a) = \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$ **e)** $k'(s) = \frac{2}{s^3} + 28s^3$
5)a)i) (0.25, 3.625) **ii)** $(-0.53, -1.93)$ **b)** the vertex
) $f'(x) = 10x - 3$ **b)** $f'(x) = 50x + 20$
a) -49m/s **b)** 17.5 seconds **c)** -171.5m/s
8) $y = 30x + 27$
9) $y = -15x + 22$
10)a) 31.4m **b)** 5.08s **c)** velocity is -25.28m/s