W1 – Derivatives of Sine and Cosine Unit 3

MCV4U

Iensen

SOLUTIONS

1) Find the derivative with respect to x for each function.

a) $y = 4\sin x$

$y' = 4\cos x$

b) $f(x) = -3\cos x$

$F'(x) = -3(-\sin x)$

$f'(x) = 3\sin x$

c) $y = \cos x - \sin x$

$y' = -\sin x - \cos x$

d) $y = x^2 - 3\sin x$

$y' = 2x - 3\cos x$

e) $y = \cos x + 7\pi \sin x - 3x$

$y' = -\sin x + 7\pi\cos x - 3$

f) $f(x) = \frac{\pi}{4}\cos x - \frac{\pi}{3}\sin x$

$f'(x) = \frac{\pi}{4}(-\sin x) - \frac{\pi}{3}\cos x$

$f'(x) = -\frac{\pi}{4}\sin x - \frac{\pi}{3}\cos x$

Find the equation of the line that is tangent to the function $y = \cos x$ and passes through the point $\left(\frac{\pi}{3}, \frac{1}{2}\right)$.

Slope:

$y' = -\sin x$

$y'\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$

$y'\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$m = -\frac{\sqrt{3}}{2}$

Eqn:

$y = mx + b$

$\frac{1}{2} = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{3}\right) + b$

$\frac{1}{2} = -\frac{\sqrt{3}\pi}{6} + b$

$\frac{3}{6} + \frac{\sqrt{3}\pi}{6} = b$

$b = \frac{3+\sqrt{3}\pi}{6}$

$y = -\frac{\sqrt{3}}{2}x + \frac{3+\pi\sqrt{3}}{6}$

3) Find the equation of the line that is tangent to the function $y = -4\sin x$ at $x = \frac{\pi}{4}$.

Point:

$y\left(\frac{\pi}{4}\right) = -4\sin\left(\frac{\pi}{4}\right)$

$y\left(\frac{\pi}{4}\right) = -4\left(\frac{1}{\sqrt{2}}\right)$

$y\left(\frac{\pi}{4}\right) = -4\left(\frac{\sqrt{2}}{2}\right)$

$y\left(\frac{\pi}{4}\right) = -2\sqrt{2}$

$\left(\frac{\pi}{4}, -2\sqrt{2}\right)$

Slope:

$y' = -4\cos x$

$y'\left(\frac{\pi}{4}\right) = -4\cos\left(\frac{\pi}{4}\right)$

$y'\left(\frac{\pi}{4}\right) = -4\left(\frac{1}{\sqrt{2}}\right)$

$y'\left(\frac{\pi}{4}\right) = -2\sqrt{2}$

$m = -2\sqrt{2}$

Eqn: $y = mx + b$

$-2\sqrt{2} = -2\sqrt{2}\left(\frac{\pi}{4}\right) + b$

$-2\sqrt{2} = -\frac{\pi\sqrt{2}}{2} + b$

$-2\sqrt{2} + \frac{\pi\sqrt{2}}{2} = b$

$y = -2\sqrt{2}x + \frac{\pi\sqrt{2}}{2} - 2\sqrt{2}$

4) Determine an equation for the tangent to the function $f(x) = \tan x$ at $x = \frac{\pi}{4}$.

Point:

$f(\frac{\pi}{4}) = \tan(\frac{\pi}{4})$

$f(\frac{\pi}{4}) = 1$

$(\frac{\pi}{4}, 1)$

Slope:

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4})$

$f'(\frac{\pi}{4}) = \frac{1}{\cos^2(\frac{\pi}{4})}$

$f'(\frac{\pi}{4}) = \frac{1}{(\frac{1}{\sqrt{2}})^2}$

$m = 2$

Eq^n:

$y = mx + b$

$1 = 2(\frac{\pi}{4}) + b$

$1 - \frac{\pi}{2} = b$

$y = 2x + 1 - \frac{\pi}{2}$

5) Find an equation of a line that is tangent to $y = 2\sin x$ and whose slope is a max value.

$y = 2\sin x$ has a max slope when $y' = 2\cos x$ is a max.

$2 = 2\cos x$

$1 = \cos x$

$x = 0, 2\pi, 4\pi, 6\pi, \ldots$

max when $x = 2k\pi$, $k \in \mathbb{Z}$

(use any answer)

Point:

$y(0) = 2\sin(0)$

$y(0) = 0$

$(0, 0)$

Slope:

$y'(0) = 2\cos(0)$

$y'(0) = 2(1)$

$y'(0) = 2$

Eq^n:

$y = mx + b$

$0 = 2(0) + b$

$b = 0$

$y = 2x$

Answers:

1)a) $\frac{dy}{dx} = 4\cos x$ **b)** $f'(x) = 3\sin x$ **c)** $\frac{dy}{dx} = -\sin x - \cos x$ **d)** $\frac{dy}{dx} = 2x - 3\cos x$ **e)** $\frac{dy}{dx} = -\sin x + 7\pi\cos x - 3$ **f)** $\frac{dy}{dx} = -\frac{\pi}{4}\sin x - \frac{\pi}{3}\cos x$

2) $y = -\frac{\sqrt{3}}{2}x + \frac{\pi\sqrt{3}+3}{6}$

3) $y = -2\sqrt{2}x + \frac{\sqrt{2}\pi}{2} - 2\sqrt{2}$

4) $y = 2x + 1 - \frac{\pi}{2}$

5) $y = 2x$; note: there are an infinite number of solutions. The slope is at a max value at any $x = 2k\pi$ where $k \in \mathbb{Z}$. Depending on which x value you choose, you will get a different y-int.