W2 – 4.4 Compound Angle Formulas

MHF4U

Jensen

1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}+\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

e) $\cos\frac{2\pi}{9}\cos\frac{5\pi}{18}-\sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18}+\sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

c) $\cos\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

d) $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

e) $\tan\left(\frac{\pi}{4}+\pi\right)$

f) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12}$

b) $\sin\frac{5\pi}{12}$

c) $\cos\frac{11\pi}{12}$

d) $\cos\frac{5\pi}{12}$

e) $\sin\frac{13\pi}{12}$

f) $\cos\frac{17\pi}{12}$

g) $\sin\frac{19\pi}{12}$

h) $\cos\frac{23\pi}{12}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x+y)$

b) $\sin(x-y)$

c) $\cos(x+y)$

d) $\cos(x-y)$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

Answer Key

1)a) $\sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **b)** $\sin\left(\frac{\pi}{4}-\frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$
c) $\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$ **d)** $\cos\left(\frac{\pi}{4}-\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
e) $\cos\left(\frac{2\pi}{9}+\frac{5\pi}{18}\right) = \cos\frac{\pi}{2} = 0$ **f)** $\cos\left(\frac{10\pi}{9}-\frac{5\pi}{18}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

3)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **c)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **e)** 1 **f)** $\frac{\sqrt{3}}{3}$

4)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **c)** $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **e)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **f)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **g)** $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ **h)** $\frac{1+\sqrt{3}}{2\sqrt{2}}$

5) $\cos x = \frac{4}{5}$ and $\sin y = \frac{12}{13}$

6)a) $\frac{63}{65}$ **b)** $-\frac{33}{65}$ **c)** $-\frac{16}{65}$ **d)** $\frac{56}{65}$