Solutions

1) Consider the graph shown.

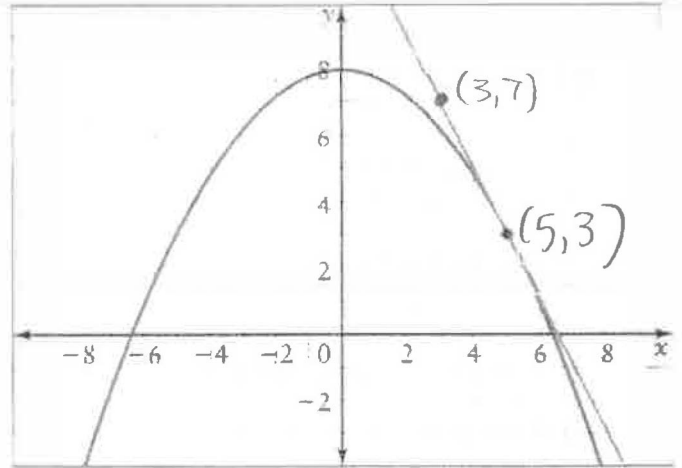**a)** State the coordinates of the tangent point

$(5,3)$

b) State the coordinates of another point on the tangent line

$(3,7)$

c) Use the points you found to find the slope of the tangent line

$$m = \frac{3-7}{5-3} = \frac{-4}{2} = -2$$

d) What does the slope of the tangent line represent?

instantaneous rate of change at $x=5$

2)a) At each of the indicated points on the graph, is the instantaneous rate of change positive, negative, or zero?

A: positive
B: zero
C: Negative

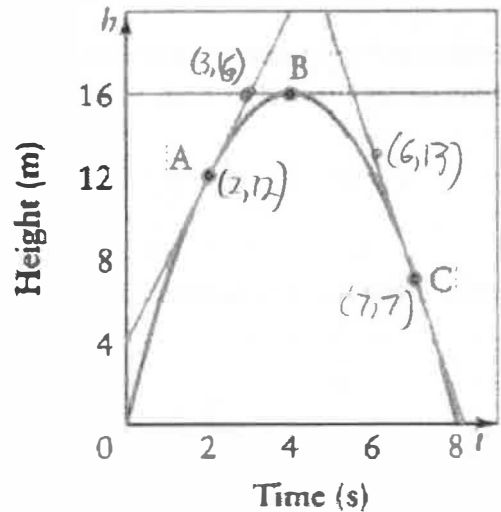**b)** Estimate the instantaneous rate of change at points A and C.

$\underline{A}$

$$\frac{dH}{dt}\Big|_{t=2} \approx \frac{16-12}{3-2} = 4 \, m/s$$

$\underline{C}$

$$\frac{dH}{dt}\Big|_{t=7} \approx \frac{13-7}{6-7} = -6 \, m/s$$

Height of a Tennis Ball

c) Interpret the values in part b) for the situation represented by the graph.
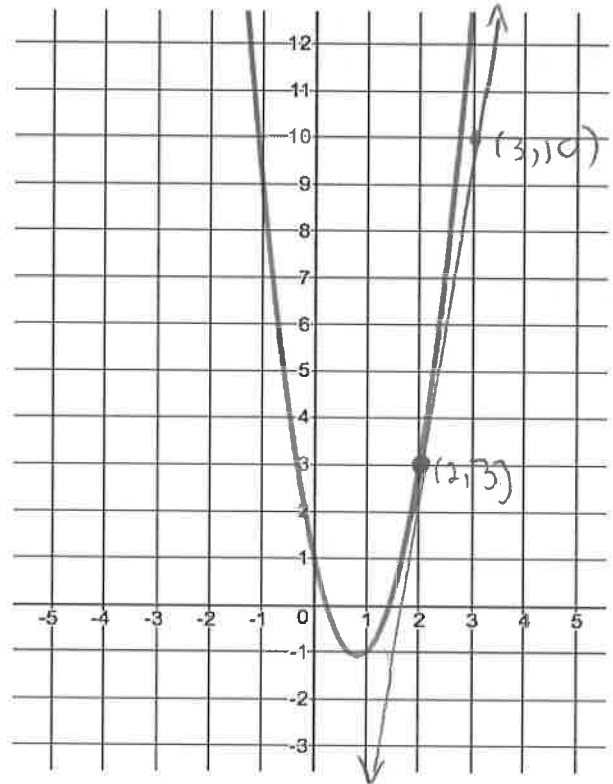
change in distance with respect to time gives a velocity.

3) Use the graph of each function to estimate the instantaneous rate of change at $x = 2$ by drawing a tangent line and calculating it's slope.

$3x^2 - 5x + 1$

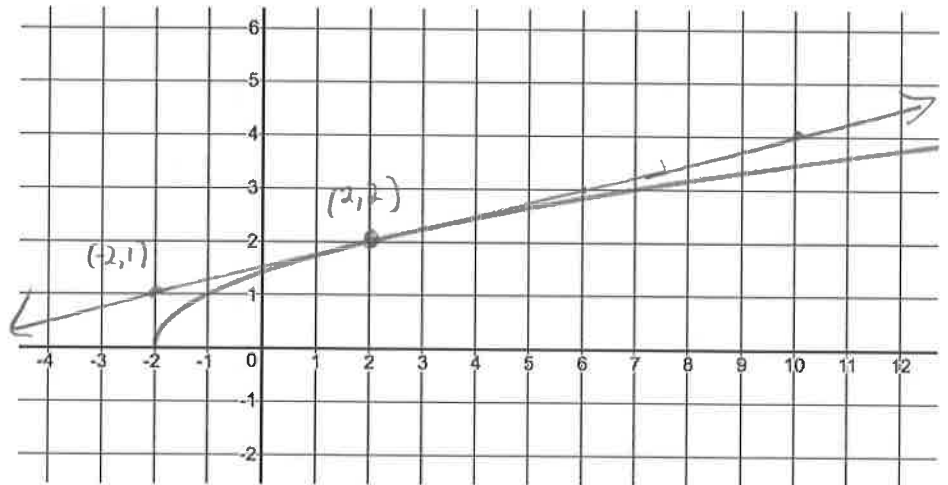$M. = \dfrac{\Delta y}{\Delta x} = \dfrac{10-3}{3-2} = 7$

$\dfrac{dy}{dx}\bigg|_{x=2} \approx 7$

b) $\sqrt{x+2}$

$M = \dfrac{2-1}{2-(-2)} = \dfrac{1}{4}$

$\dfrac{dy}{dx}\bigg|_{x=2} \approx \dfrac{1}{4}$

4) Verify your answers from question #3 by calculating the LIMIT of the secant slopes as you approach $x = 2$.

a) $3x^2 - 5x + 1$

Interval	Δy	Δx	Slope of secant $= \frac{\Delta y}{\Delta x}$
$2 \le x \le 2.5$	$= f(2.5) - f(2)$ $= 7.25 - 3$ $= 4.25$	$= 2.5 - 2$ $= 0.5$	$= \frac{4.25}{0.5}$ $= 8.5$
$2 \le x \le 2.1$	$= f(2.1) - f(2)$ $= 3.73 - 3$ $= 0.73$	$= 2.1 - 2$ $= 0.1$	$= \frac{0.73}{0.1}$ $= 7.3$
$2 \le x \le 2.01$	$= f(2.01) - f(2)$ $= 3.0703 - 3$ $= 0.0703$	$= 2.01 - 2$ $= 0.01$	$= \frac{0.0703}{0.01}$ $= 7.03$
$2 \le x \le 2.001$	$= f(2.001) - f(2)$ $= 3.007003 - 3$ $= 0.007003$	$= 2.001 - 2$ $= 0.001$	$= \frac{0.007003}{0.001}$ $= 7.003$

$$\frac{dy}{dx}\Big|_{x=2} \approx 7$$

b) $\sqrt{x+2}$

Interval	Δy	Δx	Slope of secant $= \frac{\Delta y}{\Delta x}$
$2 \le x \le 2.5$	$= f(2.5) - f(2)$ $= 2.121320344 - 2$ $= 0.121320344$	$= 2.5 - 2$ $= 0.5$	$= \frac{0.121320344}{0.5}$ $= 0.242640687$
$2 \le x \le 2.1$	$= f(2.1) - f(2)$ $= 2.024845673 - 2$ $= 0.024845673$	$= 2.1 - 2$ $= 0.1$	$= \frac{0.024845673}{0.1}$ $= 0.24845673$
$2 \le x \le 2.01$	$= f(2.01) - f(2)$ $= 2.002498439 - 2$ $= 0.002498439$	$= 2.01 - 2$ $= 0.01$	$= \frac{0.002498439}{0.01}$ $= 0.2498439$
$2 \le x \le 2.001$	$= f(2.001) - f(2)$ $= 2.000249984 - 2$ $= 0.000249984$	$= 2.001 - 2$ $= 0.001$	$= \frac{0.000249984}{0.001}$ $= 0.249984$

$$\frac{dy}{dx}\Big|_{x=2} \approx 0.25$$

5) Use the chart below to estimate the slope of the tangent to the curve $y = \sqrt{2-x}$ at $x = 1$. Have 4 (four) decimal place accuracy in the "slope of secant" column. (4 mks)

Interval	Change in y = Δy	Δx	$\dfrac{\Delta y}{\Delta x}$ = slope of secant
$0 \le x \le 1$	$= f(1) - f(0)$ $= 1 - 1.414213562$ $= -0.414213562$	$= 1-0$ $= 1$	$= \dfrac{-0.414213562}{1} \approx -0.4142$
$0.5 \le x \le 1$	$= f(1) - f(0.5)$ $= 1 - 1.224744871$ $= -0.224744871$	$= 1 - 0.5$ $= 0.5$	$= \dfrac{-0.224744871}{0.5} \approx -0.4495$
$0.9 \le x \le 1$	$= f(1) - f(0.9)$ $= 1 - 1.048808848$ $= -0.048808848$	$= 1 - 0.9$ $= 0.1$	$= \dfrac{-0.048808848}{0.1} \approx -0.4881$
$0.99 \le x \le 1$	$= f(1) - f(0.99)$ $= 1 - 1.004987562$ $= -0.004987562$	$= 1 - 0.99$ $= 0.01$	$= \dfrac{-0.004987562}{0.01} \approx 0.4988$
$0.999 \le x \le 1$	$= f(1) - f(0.999)$ $= 1 - 1.000499877$ $= -0.000499877$	$= 1 - 0.999$ $= 0.001$	$= \dfrac{-0.000499877}{0.001} \approx -0.4999$

Predicted Slope of the Tangent when $x = 1 \dots \dfrac{dy}{dx}\Big|_{x=1} = -0.5$ (follow the trend in the 4th column)

6) The data shows the percent of households that play games over the internet.

Year	1999	2000	2001	2002	2003
% of Households	12.3	18.2	24.4	25.7	27.9

a) Determine the average rate of change, in percent, of households that played games over the internet from 1999 to 2003.

$$M = \frac{\Delta \text{\% households}}{\Delta\ year} = \frac{27.9 - 12.3}{2003 - 1999} = \frac{15.6}{4} = 3.9\ \%/year$$

b) Estimate the instantaneous rate of change in percent of households that played games over the internet in the year 2000. Use the method of averaging a preceding and following interval AND the method of choosing a surrounding interval.

Method 1: averaging

for interval [1999, 2000]
$$M = \frac{\Delta y}{\Delta x} = \frac{18.2 - 12.3}{2000 - 1999}$$
$$= 5.9\ \%/year$$

for interval [2000, 2001]
$$M = \frac{\Delta y}{\Delta x} = \frac{24.4 - 18.2}{2001 - 2000}$$
$$= 6.2\ \%/year$$

$$\frac{dy}{dx}\Big|_{x=2000} \approx \frac{5.9 + 6.2}{2} = 6.05\ \%/year$$

Method 2: Surrounding

for interval [1999, 2001]
$$M = \frac{\Delta y}{\Delta x} = \frac{24.4 - 12.3}{2001 - 1999}$$
$$= 6.05\ \%/year$$

$$\therefore \frac{dy}{dx}\Big|_{x=2000} \approx 6.05\ \%/year.$$

7) Consider the data below describing the height of the world's tallest modern human, Robert Wadlow (1918-1940). At his death at 22 years of age, his height was 8 feet, 11.1 inches.

Age in years	4	8	10	13	16	18	19	21	22
Height in cm	160	190	200	220	240	250	260	268	272

a) Find average rate of change in Wadlow's height between the ages of 4 and 22. Show proper units and notation.

$$M = \frac{\Delta y}{\Delta x} = \frac{272 - 160}{22 - 4} = \frac{112}{18} \doteq 6.2 \text{ cm/year}$$

b) Estimate the instantaneous rate of change for Robert Wadlow's height when he was 16 years of age using 2 methods.

__Method 1: averaging__

for interval $[16, 18]$

$$M = \frac{\Delta y}{\Delta x} = \frac{250 - 240}{18 - 16}$$
$$= \frac{10}{2}$$
$$= 5 \text{ cm/year}$$

for interval $[13, 16]$

$$M = \frac{\Delta y}{\Delta x} = \frac{240 - 220}{16 - 13}$$
$$= \frac{20}{3}$$
$$= 6.67 \text{ cm/year}$$

$$\frac{dy}{dx}\bigg|_{x=16} \approx \frac{5 + 6.67}{2} = 5.835 \text{ cm/year}$$

__Method 2: surrounding__

for interval $[13, 18]$

$$M = \frac{\Delta y}{\Delta x} = \frac{250 - 220}{18 - 13}$$
$$= \frac{30}{5}$$
$$= 6 \text{ cm/year}$$

$$\therefore \frac{dy}{dx}\bigg|_{x=16} \approx 6 \text{ cm/year}.$$

__Answer Key__

1)a) (5, 3) **b)** (3, 7) **c)** $m = -2$ **d)** instantaneous rate of change at $x = 5$
2)a) at A the instantaneous rate of change is positive, at B the instantaneous rate of change is 0, and at C it is negative **b)** A: $m = 4$ m/s C: $m = -6$ m/s **c)** velocity at 2 seconds and 7 seconds
3&4)a) $\frac{dy}{dx} = 7$ **b)** $\frac{dy}{dx} = 0.25$
5) $\frac{dy}{dx} = -0.5$
6)a) $\frac{\Delta y}{\Delta x} = 3.9$ %/year **b)** averaging: $\frac{dy}{dx} = 6.05$ %/year surrounding: $\frac{dy}{dx} = 6.05$ %/year
7)a) $\frac{\Delta y}{\Delta x} = 6.2$ cm/year **b)** averaging: $\frac{dy}{dx} = 5.83$ cm/year surrounding: $\frac{dy}{dx} = 6$ cm/year