

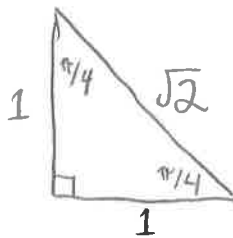
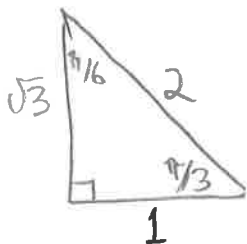
W2 - 4.2 Trig Ratios and Special Angles

MHF4U

Jensen

ANSWERS

1) Draw both special triangles using radian measures.



2) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) $\cos 3.43$

≈ -0.9587

b) $\sin 2.92$

≈ 0.2198

c) $\tan 5.61$

≈ -0.7975

d) $\csc 1.27$

$= \frac{1}{\sin 1.27} \approx 1.0470$

e) $\cot 4.53$

$= \frac{1}{\tan 4.53} \approx 0.1844$

f) $\sec 0.98$

$= \frac{1}{\cos 0.98} \approx 1.7953$

3) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) $\cot \frac{3\pi}{7} = \frac{1}{\tan(\frac{3\pi}{7})}$

≈ 0.2282

b) $\sec \frac{16\pi}{3} = \frac{1}{\cos(\frac{16\pi}{3})}$

$= -2$

c) $\csc \frac{5\pi}{11} = \frac{1}{\sin(\frac{5\pi}{11})}$

≈ 1.0103

4) Use the unit circle and the cast rule to find exact expressions for each ratio

a) $\sin \frac{2\pi}{3}$

$\sin(\frac{2\pi}{3}) = \sin(\frac{\pi}{3})$
 $= \frac{\sqrt{3}}{2}$

b) $\tan \frac{\pi}{6}$

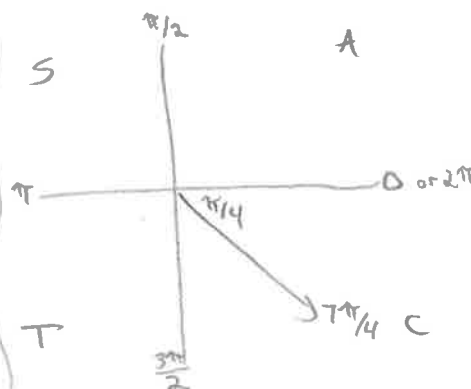
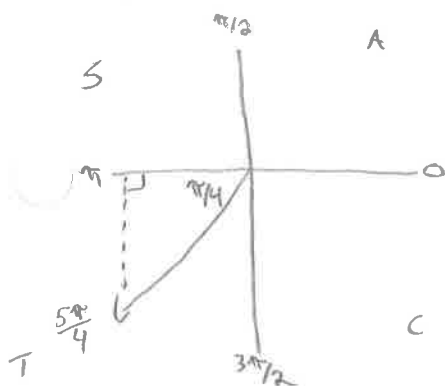
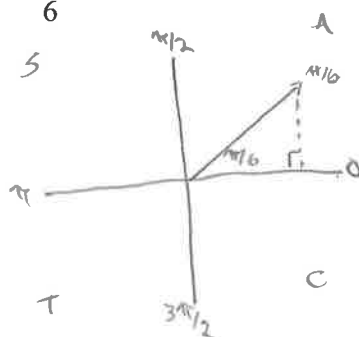
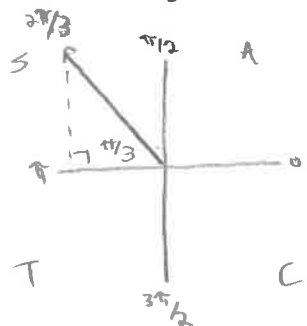
$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

c) $\cos \frac{5\pi}{4}$

$\cos(\frac{5\pi}{4}) = -\cos(\frac{\pi}{4})$
 $= -\frac{1}{\sqrt{2}}$

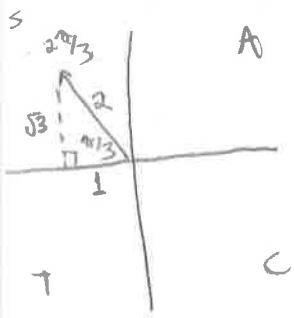
d) $\tan \frac{7\pi}{4}$

$\tan(\frac{7\pi}{4}) = -\tan(\frac{\pi}{4})$
 $= -1$



5) Use the unit circle and cast rule to determine exact values of the primary trig ratios for each angle.

a) $\frac{2\pi}{3}$

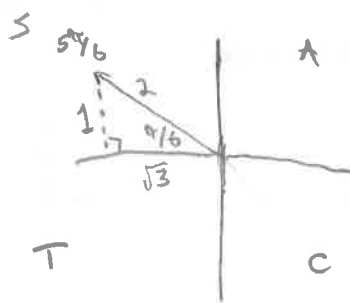


$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

b) $\frac{5\pi}{6}$

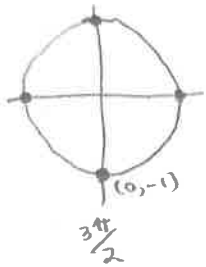


$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

c) $\frac{3\pi}{2}$

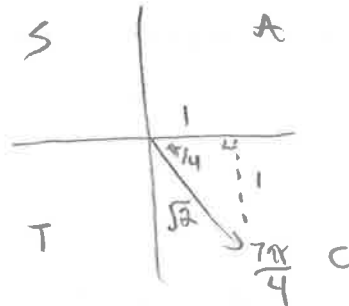


$$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$$

d) $\frac{7\pi}{4}$

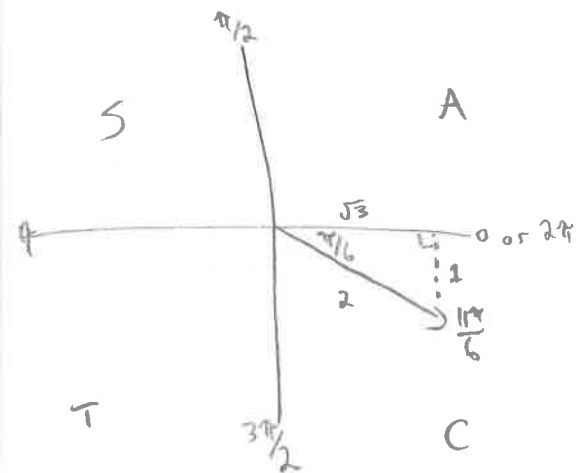


$$\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\tan\left(\frac{7\pi}{4}\right) = -1$$

6) Use the special triangles determine exact values for the six trigonometric ratios for $\frac{11\pi}{6}$.



$$\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\csc\left(\frac{11\pi}{6}\right) = -2$$

$$\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

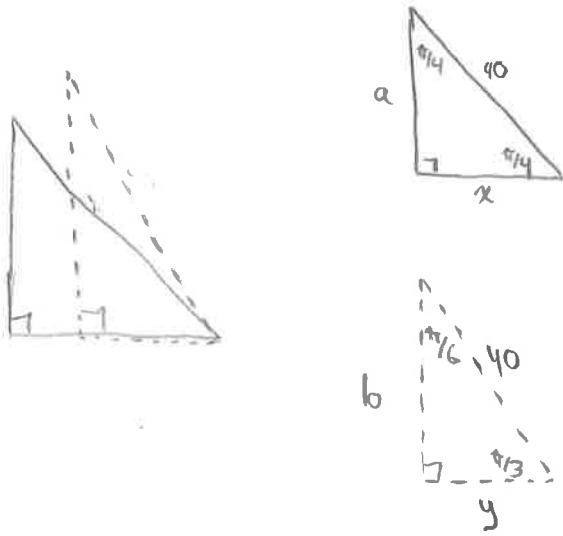
$$\sec\left(\frac{11\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

$$\tan\left(\frac{11\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

$$\cot\left(\frac{11\pi}{6}\right) = -\sqrt{3}$$

7) Lynda is flying her kite at the end of a 40-m string. The string makes an angle of $\frac{\pi}{4}$ with the ground. The wind speed increases, and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground.

Determine an exact expression for the horizontal distance that the kite moves between the two positions.



$$\cos \frac{\pi}{4} = \frac{x}{40}$$

$$\frac{1}{\sqrt{2}} = \frac{x}{40}$$

$$x = \frac{40}{\sqrt{2}} = 20\sqrt{2}$$

$$\cos \frac{\pi}{3} = \frac{y}{40}$$

$$\frac{1}{2} = \frac{y}{40}$$

$$y = 20$$

$$\begin{aligned} \text{horizontal distance} &= x - y \\ &= 20\sqrt{2} - 20 \\ &= 20(\sqrt{2} - 1) \text{ m} \end{aligned}$$

b) Determine an exact expression for the vertical distance that the kite moves between the two positions.

$$\sin \frac{\pi}{4} = \frac{a}{40}$$

$$\frac{1}{\sqrt{2}} = \frac{a}{40}$$

$$a = \frac{40}{\sqrt{2}}$$

$$a = 20\sqrt{2}$$

$$\sin \frac{\pi}{3} = \frac{b}{40}$$

$$\frac{\sqrt{3}}{2} = \frac{b}{40}$$

$$b = 20\sqrt{3}$$

$$\begin{aligned} \text{vertical distance} &= b - a \\ &= 20\sqrt{3} - 20\sqrt{2} \\ &= 20(\sqrt{3} - \sqrt{2}) \text{ m.} \end{aligned}$$

8) Determine an exact value for each expression

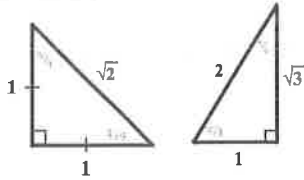
$$\begin{aligned} \text{a) } \frac{\sin \frac{\pi}{3} \tan \frac{\pi}{6}}{\cos \frac{\pi}{4}} &= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} \\ &= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$



$$\begin{aligned} \text{b) } \cot \frac{5\pi}{4} + \tan \frac{11\pi}{6} \tan \frac{5\pi}{3} &= \cot \frac{\pi}{4} - \tan \frac{\pi}{6} (-\tan \frac{\pi}{3}) \\ &= 1 - \left(\frac{1}{\sqrt{3}}\right)\left(-\frac{\sqrt{3}}{1}\right) \\ &= 1 - (-1) \\ &= 2 \end{aligned}$$

Answer Key

1)



2) a) -0.9587 b) 0.2198 c) -0.7975 d) 1.0470 e) 0.1844 f) 1.7953

3) a) 0.2282 b) -2.0000 c) 1.0103

4) a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $-\frac{1}{\sqrt{2}}$ d) -1

5) a) $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; $\cos \frac{2\pi}{3} = -\frac{1}{2}$; $\tan \frac{2\pi}{3} = -\sqrt{3}$

b) $\sin \frac{5\pi}{6} = \frac{1}{2}$; $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; $\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

c) $\sin \frac{3\pi}{2} = -1$; $\cos \frac{3\pi}{2} = 0$; $\tan \frac{3\pi}{2} = \text{undefined}$

d) $\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$; $\cos \frac{7\pi}{4} = \frac{1}{\sqrt{2}}$; $\tan \frac{7\pi}{4} = -1$

6) $\sin \frac{11\pi}{6} = -\frac{1}{2}$; $\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$; $\tan \frac{11\pi}{6} = \frac{-1}{\sqrt{3}}$; $\csc \frac{11\pi}{6} = -2$; $\sec \frac{11\pi}{6} = \frac{2}{\sqrt{3}}$; $\cot \frac{11\pi}{6} = -\sqrt{3}$

7) a) $20(\sqrt{2}-1)$ meters b) $20(\sqrt{3}-\sqrt{2})$ meters

8) a) $\frac{\sqrt{2}}{2}$ b) 2