1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12} + \cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$

$= \sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}\right)$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12} - \cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$

$= \sin\left(\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{6}\right)$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12} - \sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12} + \sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$\cos\frac{2\pi}{9}\cos\frac{5\pi}{18} - \sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{2\pi}{9} + \frac{5\pi}{18}\right)$

$= \cos\left(\frac{9\pi}{18}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18} + \sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{10\pi}{9} - \frac{5\pi}{18}\right)$

$= \cos\left(\frac{15\pi}{18}\right)$

$= \cos\left(\frac{5\pi}{6}\right)$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$ $= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

c) $\cos\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$ $= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} + \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1+\sqrt{3}}{2\sqrt{2}}$

d) $\sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$ $= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \left(-\cos\frac{\pi}{3}\right)\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

e) $\tan\left(\frac{\pi}{4} + \pi\right) = \dfrac{\tan\frac{\pi}{4} + \tan\pi}{1 - \tan\frac{\pi}{4}\tan\pi}$

$= \dfrac{1+0}{1-1(0)}$

$= \dfrac{1}{1}$

$= 1$

f) $\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

$= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{1+1}$

$= \dfrac{\left(\frac{2}{\sqrt{3}}\right)}{2}$

$\Bigg\rbrace = \dfrac{1}{\sqrt{3}}\dfrac{(\sqrt{3})}{(\sqrt{3})}$

$= \dfrac{\sqrt{3}}{3}$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$

b) $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \dfrac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) + \dfrac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

c) $\cos\frac{11\pi}{12}$

$= \cos\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3}$

$= \dfrac{1}{\sqrt{2}}\left(\frac{-1}{2}\right) - \dfrac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{-1-\sqrt{3}}{2\sqrt{2}}$

d) $\cos\frac{5\pi}{12} = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \dfrac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \dfrac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$
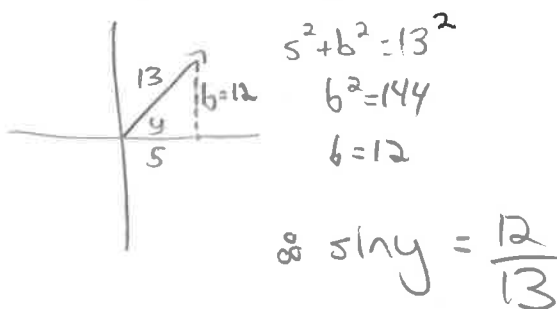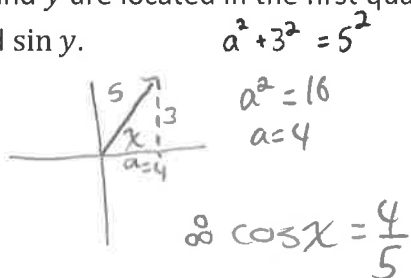
$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

e) $\sin\frac{13\pi}{12} = \sin\left(\frac{4\pi}{12} + \frac{9\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{3\pi}{4} + \cos\frac{\pi}{3}\sin\frac{3\pi}{4}$

$= \dfrac{\sqrt{3}}{2}\left(-\frac{1}{\sqrt{2}}\right) + \dfrac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{-\sqrt{3}+1}{2\sqrt{2}}$

f) $\cos\frac{17\pi}{12} = \cos\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

g) $\sin\frac{19\pi}{12} = \sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$

$= \sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$

$= \sin\frac{5\pi}{6}\cos\frac{3\pi}{4} + \cos\frac{5\pi}{6}\sin\frac{3\pi}{4}$

$= \frac{1}{2}\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{-1-\sqrt{3}}{2\sqrt{2}}$

h) $\cos\frac{23\pi}{12} = \cos\left(\frac{8\pi}{12} + \frac{15\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3} + \frac{5\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{5\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{5\pi}{4}$

$= \left(\frac{-1}{2}\right)\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

$a^2 + 3^2 = 5^2$

$a^2 = 16$

$a = 4$

$\therefore \cos x = \frac{4}{5}$

$5^2 + b^2 = 13^2$

$b^2 = 144$

$b = 12$

$\therefore \sin y = \frac{12}{13}$

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x+y)$

$= \sin x \cos y + \cos x \sin y$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{3}{13} + \frac{48}{65}$

$= \frac{63}{65}$

b) $\sin(x-y) = \sin x \cos y - \cos x \sin y$

$= \frac{3}{13} - \frac{48}{65}$

$= \dfrac{-33}{65}$

c) $\cos(x+y)$

$= \cos x \cos y - \sin x \sin y$

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \frac{3}{5}\left(\frac{12}{13}\right)$

$= \frac{4}{13} - \frac{36}{65}$

$= \dfrac{-16}{65}$

d) $\cos(x-y)$

$= \cos x \cos y + \sin x \sin y$

$= \frac{4}{13} + \frac{36}{65}$

$= \dfrac{56}{65}$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

$$\cos(2x) = \cos(x+x)$$
$$= \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x$$

Answer Key

1)a) $\sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ b) $\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$
c) $\cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$ d) $\cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
e) $\cos\left(\frac{2\pi}{9} + \frac{5\pi}{18}\right) = \cos\frac{\pi}{2} = 0$ f) $\cos\left(\frac{10\pi}{9} - \frac{5\pi}{18}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

3)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ b) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ c) $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ e) 1 f) $\frac{\sqrt{3}}{3}$

4)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ c) $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ d) $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ e) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ f) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ g) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ h) $\frac{1+\sqrt{3}}{2\sqrt{2}}$

5) $\cos x = \frac{4}{5}$ and $\sin y = \frac{12}{13}$

6)a) $\frac{63}{65}$ b) $-\frac{33}{65}$ c) $-\frac{16}{65}$ d) $\frac{56}{65}$