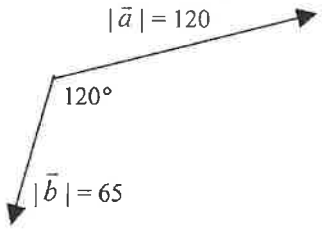
W2 – Dot Product Unit 5
MCV4U
Jensen

1) Calculate the dot product for each pair.

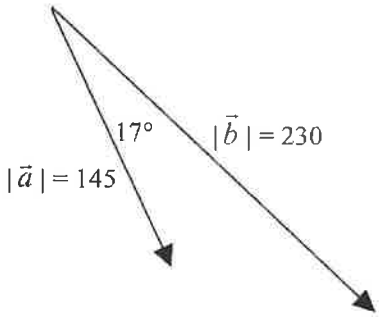
a)



$\vec{a} \cdot \vec{b} = 120(65)\cos(120)$

$= -3900$

b)



$\vec{a} \cdot \vec{b} = 145(230)\cos(17)$

$\simeq 31892.76$

2) Calculate the dot product for each pair of vectors. θ is the angle between the vectors when they are placed tail to tail.

a) $|\vec{u}| = 7$, $|\vec{v}| = 12$, and $\theta = 47°$

$\vec{u} \cdot \vec{v} = 7(12)\cos(47)$

$\simeq 57.29$

b) $|\vec{s}| = 520$, $|\vec{t}| = 745$, and $\theta = 135°$

$\vec{s} \cdot \vec{t} = 520(745)\cos(135)$

$\simeq -273933.17$

3) Calculate the dot product of each pair of vectors.

a) $\vec{a} = [5, 8]$, $\vec{b} = [-2, 1]$

$\vec{a} \cdot \vec{b} = 5(-2) + 8(1)$

$= -2$

b) $\vec{c} = [-1, 8]$, $\vec{d} = [3, -3]$

$\vec{c} \cdot \vec{d} = -1(3) + 8(-3)$

$= -27$

c) $\vec{l} = 2\hat{i} - 3\hat{j}$, $\vec{m} = -9\hat{i} + 4\hat{j}$

$l = [2, -3]$ $m = [-9, 4]$

$\vec{l} \cdot \vec{m} = 2(-9) + (-3)(4)$

$= -30$

d) $\vec{u} = -6\hat{i} + 7\hat{j}$, $\vec{v} = 3\hat{i} - 2\hat{j}$

$\vec{u} = [-6, 7]$ $\vec{v} = [3, -2]$

$\vec{u} \cdot \vec{v} = -6(3) + 7(-2)$

$= -32$

4) Decide whether the following expressions have meaning or not. If not, explain why.

a) $\vec{u} \cdot (\vec{v} \cdot \vec{w})$ NO — can't dot vector with scalar

b) $|\vec{u} \cdot \vec{v}|$ YES

c) $\vec{u}(\vec{v} \cdot \vec{w})$ YES

d) $|\vec{u}|^2$ YES

e) $\vec{v}^2$ NO — can't multiply vectors

f) $(\vec{u} \cdot \vec{v})^2$ YES

5) Let $\vec{a}$ = [1, –2], $\vec{b}$ = [2, 5], and $\vec{c}$ = [4, –1]. Evaluate the following if possible. If not possible, explain why not.

a) $\vec{a} \cdot (\vec{b} + \vec{c})$

$= [1,-2] \cdot ([2,5] + [4,-1])$

$= [1,-2] \cdot [6,4]$

$= 1(6) + (-2)(4)$

$= -2$

b) $(\vec{a} + \vec{b}) \cdot \vec{c}$

$= ([1,-2] + [2,5]) \cdot [4,-1]$

$= [3,3] \cdot [4,-1]$

$= 3(4) + 3(-1)$

$= 9$

c) $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{c})$

$= ([1,-2] + [2,5]) \cdot ([1,-2] + [4,-1])$

$= [3,3] \cdot [5,-3]$

$= 3(5) + 3(-3)$

$= 6$

d) $(3\vec{a} + 2\vec{b}) \cdot (4\vec{a} - \vec{b})$

$= (3[1,-2] + 2[2,5]) \cdot (4[1,-2] - [2,5])$

$= ([3,-6] + [4,10]) \cdot ([4,-8] - [2,5])$

$= [7,4] \cdot [2,-13]$

$= 7(2) + 4(-13)$

$= -38$

e) $\vec{a} \cdot \vec{b} \cdot \vec{c}$

Not possible.

$\vec{a} \cdot \vec{b}$ is a scalar.

Can't do $\vec{c}$ with a scalar.

f) $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

$= [1,-2] \cdot [2,5] + [1,-2] \cdot [4,-1]$

$= 1(2) + (-2)(5) + 1(4) + (-2)(-1)$

$= -2$

g) $4\vec{b} \cdot (-2\vec{c})$

$= 4[2,5] \cdot (-2[4,-1])$

$= [8,20] \cdot [-8,2]$

$= 8(-8) + 20(2)$

$= -24$

h) $(\vec{a} + \vec{b}) \cdot \vec{c}$

$= [1,-2] + [2,5] \cdot [4,-1]$

$= [3,3] \cdot [4,-1]$

$= 3(4) + 3(-1)$

$= 9$

6) Determine a value of t so that $\vec{u} = [9, t]$ and $\vec{v} = [-16, t]$ are perpendicular.

If perpendicular, $\vec{u} \cdot \vec{v} = 0$

$[9,t] \cdot [-16,t] = 0$

$9(-16) + t(t) = 0$

$t^2 = 144$

$t = \pm 12$

7) Find a vector that is perpendicular to $\vec{a} = [3, -1]$. Verify that the vectors are perpendicular.

$\vec{a} = [3,-1]$ $\vec{b} = [5,k]$

$\vec{a} \cdot \vec{b} = 0$

$3(5) + (-1)(k) = 0$

$k = 15$

$[5,15]$

Shortcut:

$[3,-1]$

$= [-1,3]$ change 1 sign

$= [1,3]$

check $[1,3] \cdot [3,-1]$

$= 3 + (-3)$

$= 0$

8) Which of the following is a right-angled triangle? Identify the right angle in that triangle.

- ΔABC for $A(3,1)$, $B(-2,3)$, and $C(5,6)$
- ΔSTU for $S(4,6)$, $T(-3,7)$, and $U(-5,-4)$

$\vec{AB} = [-5, 2]$

$\vec{BC} = [7, 3]$

$\vec{AC} = [2, 5]$

$\vec{AB} \cdot \vec{BC} = -5(7) + 2(3) = -30$

$\vec{AB} \cdot \vec{AC} = -5(2) + 2(5) = 0$

$\therefore \vec{AB}$ and $\vec{AC}$ are perpendicular.

$\angle A = 90°$

$\vec{ST} = [-7, 1]$

$\vec{SU} = [-9, -10]$

$\vec{TU} = [-2, -11]$

$\vec{ST} \cdot \vec{SU} = -7(-9) + 1(-10) = 53$

$\vec{ST} \cdot \vec{TU} = -7(-2) + 1(-11) = 3$

$\vec{SU} \cdot \vec{TU} = -9(-2) + (-10)(-11) = 128$

$\therefore$ NOT a right triangle.

ANSWER KEY:

1)a) −3900 **b)** 31892.76

2)a) 57.28 **b)** −273 933.16

3)a) −2 **b)** −27 **c)** −30 **d)** −32

4)a) no, you cannot dot a vector with a scalar **b)** yes **c)** yes **d)** yes **e)** no, you cannot multiply vectors **f)** yes

5)a) −2 **b)** 9 **c)** 6 **d)** −38 **e)** not possible- you cannot dot a vector with a scalar **f)** −2 **g)** −24 **h)** 9

6) $t = 12, -12$

7) Answers may vary: $[-1, -3]$, $[1, 3]$, check using the dot product

8) ΔABC is a right triangle; the right angle is $\angle BAC$