W2 – MORE Derivatives of Sine and Cosine Unit 3

MCV4U

Jensen

SOLUTIONS

1) Determine the derivative of each function.

a) $y = \sin(4x)$

$y' = \cos(4x)(4)$

$y' = 4\cos(4x)$

b) $f(x) = \sin(2x + \pi)$

$f'(x) = \cos(2x+\pi)(2)$

$f'(x) = 2\cos(2x+\pi)$

c) $y = -2\sin(3\theta)$

$y' = -2\cos(3\theta)(3)$

$y' = -6\cos(3\theta)$

d) $y = \sin^2 x$

$y = (\sin x)^2$

$y' = 2(\sin x)(\cos x)$

$y' = 2\sin x \cos x$

e) $f(x) = \cos^2 x - \sin^2 x$

$f(x) = (\cos x)^2 - (\sin x)^2$

$f'(x) = 2\cos x(-\sin x) - 2\sin x(\cos x)$

$f'(x) = -2\cos x \sin x - 2\sin x \cos x$

$f'(x) = -2(\cos x \sin x + \sin x \cos x)$

$f'(x) = -2(2\sin x \cos x)$

$f'(x) = -2\sin(2x)$

f) $y = 3\sin^2(2t-4) - 2\cos^2(3t+1)$

$y = 3[\sin(2t-4)]^2 - 2[\cos(3t+1)]^2$

$y' = 3(2)[\sin(2t-4)]\cos(2t-4)(2) -$

$2(2)[\cos(3t+1)][-\sin(3t+1)](3)$

$y' = 12\sin(2t-4)\cos(2t-4)$

$+ 12\cos(3t+1)\sin(3t+1)$

$y' = 6\sin[2(2t-4)] + 6\sin[2(3t+1)]$

$y' = 6\sin(4t-8) + 6\sin(6t+2)$

g) $f(t) = \sin^2(\cos t)$

$P(t) = [\sin(\cos t)]^2$

$f'(t) = 2[\sin(\cos t)][\cos(\cos t)](-\sin t)$

$f'(t) = -2\sin(t)\sin(\cos t)\cos(\cos t)$

h) $f(x) = -x^2 \sin(3x - \pi)$

$f'(x) = -2x\sin(3x-\pi) + \cos(3x-\pi)(3)(-x^2)$

$f'(x) = -2x(3x-\pi) - 3x^2\cos(3x-\pi)$

i) $f(\theta) = \sin^2\theta \cos^2\theta$

$f(\theta) = (\sin\theta)^2(\cos\theta)^2$

$f'(\theta) = 2\sin\theta\cos\theta\cos^2\theta + 2\cos\theta(-\sin\theta)\sin^2\theta$

$f'(\theta) = 2\sin\theta\cos^3\theta - 2\cos\theta\sin^3\theta$

j) $y = x^{-1}\cos^2 x$

$y = x^{-1}(\cos x)^2$

$y' = -1x^{-2}(\cos^2 x) + 2\cos x(-\sin x)x^{-1}$

$y' = \frac{-\cos^2 x}{x^2} - \frac{2\cos x \sin x}{x}$

k) $y = 2\tan x - \tan(2x)$

$y' = 2\sec^2 x - \sec^2(2x)(2)$

$y' = 2\sec^2 x - 2\sec^2(2x)$

l) $y = (\tan x + \cos x)^2$

$y' = 2(\tan x + \cos x)(\sec^2 x - \sin x)$

2) Find the slope of the function $y = 2\cos x \sin(2x)$ at $x = \frac{\pi}{2}$.

$$y' = -2\sin x[\sin(2x)] + \cos(2x)(2)(2\cos x)$$

$$y' = -2\sin x[\sin(2x)] + 4\cos(2x)\cos x$$

$$y'\left(\tfrac{\pi}{2}\right) = -2\sin\left(\tfrac{\pi}{2}\right)[\sin(\pi)] + 4\cos(\pi)\cos\left(\tfrac{\pi}{2}\right)$$

$$y'\left(\tfrac{\pi}{2}\right) = -2(1)(0) + 4(-1)(0)$$

$$y'\left(\tfrac{\pi}{2}\right) = 0$$

$$\boxed{m = 0}$$

3) Find the equation of the line that is tangent to $y = x^2\sin(2x)$ at $x = -\pi$.

Slope:

$$y' = 2x\sin(2x) + \cos(2x)(2)x^2$$

$$y' = 2x\sin(2x) + 2x^2\cdot\cos(2x)$$

$$y'(-\pi) = 2(-\pi)\sin(-2\pi) + 2(-\pi)^2\cos(-2\pi)$$

$$y'(-\pi) = -2\pi(0) + 2\pi^2(1)$$

$$y'(-\pi) = 2\pi^2$$

Point:

$$y(-\pi) = (-\pi)^2\sin(-2\pi)$$

$$y(-\pi) = 0$$

$$(-\pi, 0)$$

Eqn: $y = mx + b$

$$0 = 2\pi^2(-\pi) + b$$

$$b = 2\pi^3$$

$$\boxed{y = 2\pi^2 x + 2\pi^3}$$

4) Determine $\frac{d^2y}{dx^2}$ for $y = x^2\cos x$.

$$\frac{dy}{dx} = 2x\cos x + (-\sin x)(x^2)$$

$$\frac{dy}{dx} = 2x\cos x - x^2\sin x$$

$$\frac{d^2y}{dx^2} = 2\cos x + (-\sin x)(2x) - \left[2x\sin x + \cos x\,(x^2)\right]$$

$$= 2\cos x - 2x\sin x - 2x\sin x - x^2\cos x$$

$$= 2\cos x - 4x\sin x - x^2\cos x$$

5)a) Write $y = \csc x$ in terms of $\sin x$ as a reciprocal function.

$y = \frac{1}{\sin x}$

b) Write the function in terms of a negative power of $\sin x$

$y = (\sin x)^{-1}$

c) Use the power rule and chain rule to find the derivative of $y = \csc x$

$y' = -1(\sin x)^{-2}(\cos x)$

$y' = \frac{-\cos x}{\sin^2 x}$

$y' = \frac{-\cos x}{(\sin x)(\sin x)}$

$y = -\cot x \left(\frac{1}{\sin x}\right)$

$y = -\cot x \csc x$

Answers:

1)a) $\frac{dy}{dx} = 4\cos(4x)$ **b)** $f'(x) = 2\cos(2x + \pi)$ **c)** $\frac{dy}{d\theta} = -6\cos(3\theta)$ **d)** $\frac{dy}{dx} = 2\sin x \cos x$

e) $f'(x) = -2\sin(2x)$ **f)** $\frac{dy}{dx} = 12\sin(2t - 4)\cos(2t - 4) + 12\cos(3t - 1)\sin(3t - 1)$

g) $f'(t) = -2\sin(\cos t)\cos(\cos t)\sin t$ **h)** $f'(x) = -3x^2\cos(3x - \pi) - 2x\sin(3x - \pi)$

i) $f'(\theta) = -2\sin^3\theta\cos^3\theta + 2\sin\theta\cos^3\theta$ **j)** $\frac{dy}{dx} = \frac{-2}{x}\cos x \sin x - \frac{\cos^2 x}{x^2}$

k) $y' = 2\sec^2 x - 2\sec(2x)$ **l)** $y' = 2(\tan x + \cos x)(\sec^2 x - \sin x)$

2) 0

3) $y = 2\pi^2 x + 2\pi^3$

4) $\frac{d^2y}{dx^2} = -x^2\cos x - 4x\sin x + 2\cos x$

5)a) $y = \frac{1}{\sin x}$ **b)** $y = (\sin x)^{-1}$ **c)** $\frac{dy}{dx} = -\csc x \cot x$