W2 – The Product Rule Unit 1
MCV4U
Jensen

1) Use the product rule to differentiate each function

a) $f(x) = (5x + 2)(8x - 6)$

$f'(x) = 5(8x-6) + 8(5x+2)$

$f'(x) = 40x - 30 + 40x + 16$

$f'(x) = 80x - 14$

b) $h(t) = (-t + 4)(2t + 1)$

$h'(t) = -1(2t+1) + 2(-t+4)$

$h'(t) = -2t - 1 - 2t + 8$

$h'(t) = -4t + 7$

c) $p(x) = (-2x + 3)(x - 9)$

$p'(x) = -2(x-9) + 1(-2x+3)$

$p'(x) = -2x + 18 - 2x + 3$

$p'(x) = -4x + 21$

d) $g(x) = (x^2 + 2)(4x - 5)$

$g'(x) = 2x(4x-5) + 4(x^2+2)$

$g'(x) = 8x^2 - 10x + 4x^2 + 8$

$g'(x) = 12x^2 - 10x + 8$

e) $f(x) = (1 - x)(x^2 - 5)$

$f'(x) = -1(x^2-5) + 2x(1-x)$

$f'(x) = -x^2 + 5 + 2x - 2x^2$

$f'(x) = -3x^2 + 2x + 5$

f) $h(t) = (t^2 + 3)(3t^2 - 7)$

$h'(t) = 2t(3t^2-7) + 6t(t^2+3)$

$h'(t) = 6t^3 - 14t + 6t^3 + 18t$

$h'(t) = 12t^3 + 4t$

2) Determine $f'(-2)$ for each function.

a) $f(x) = (x^2 - 2x)(3x + 1)$

$f'(x) = (2x-2)(3x+1) + 3(x^2-2x)$

$f'(x) = 6x^2 - 4x - 2 + 3x^2 - 6x$

$f'(x) = 9x^2 - 10x - 2$

$f'(-2) = 9(-2)^2 - 10(-2) - 2$

$f'(-2) = 54$

b) $f(x) = (1 - x^3)(-x^2 + 2)$

$f'(x) = -3x^2(-x^2+2) + (-2x)(1-x^3)$

$f'(x) = 3x^4 - 6x^2 - 2x + 2x^4$

$f'(x) = 5x^4 - 6x^2 - 2x$

$f'(-2) = 5(-2)^4 - 6(-2)^2 - 2(-2)$

$f'(-2) = 60$

3) Determine an equation for the tangent to each curve at the indicated value.

a) $f(x) = (x^2 - 3)(x^2 + 1)$ at $x = -4$

Slope

$f'(x) = 2x(x^2+1) + 2x(x^2-3)$

$f'(x) = 2x^3 + 2x + 2x^3 - 6x$

$f'(x) = 4x^3 - 4x$

$f'(-4) = 4(-4)^3 - 4(-4)$

$f'(-4) = -240$

$m = -240$

Point

$f(-4) = [(-4)^2 - 3][(-4)^2 + 1]$

$f(-4) = 221$

Eqn

$y = mx + b$

$221 = (-240)(-4) + b$

$b = -739$

$\boxed{y = -240x - 739}$

b) $h(x) = (x^4 + 4)(2x^2 - 6)$ at $x = -1$

Slope

$h'(x) = 4x^3(2x^2-6) + 4x(x^4+4)$

$h'(x) = 8x^5 - 24x^3 + 4x^5 + 16x$

$h'(x) = 12x^5 - 24x^3 + 16x$

$h'(-1) = 12(-1)^5 - 24(-1)^3 + 16(-1)$

$h'(-1) = -4$

Point

$h(-1) = [(-1)^4 + 4][2(-1)^2 - 6]$

$h(-1) = -20$

Eqn

$y = mx + b$

$-20 = -4(-1) + b$

$b = -24$

$\boxed{y = -4x - 24}$

4) Determine the point(s) on each curve that correspond to the given slope of the tangent.

a) $y = (-4x+3)(x+3)$, $m = 0$

$y' = -4(x+3) + 1(-4x+3)$

$y' = -4x - 12 - 4x + 3$

$y' = -8x - 9$

$0 = -8x - 9$

$x = -\frac{9}{8}$

$y(-\frac{9}{8}) = [-4(-\frac{9}{8}) + 3][-\frac{9}{8} + 3]$

$y(-\frac{9}{8}) = \frac{225}{16}$

$(-\frac{9}{8}, \frac{225}{16})$

b) $y = (x^2 - 2)(2x + 1)$, $m = -2$

$y' = 2x(2x+1) + 2(x^2 - 2)$

$y' = 4x^2 + 2x + 2x^2 - 4$

$y' = 6x^2 + 2x - 4$

$-2 = 6x^2 + 2x - 4$

$0 = 6x^2 + 2x - 2$

$0 = 3x^2 + x - 1$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{13}}{6}$

$x_1 \simeq 0.43 \quad x_2 \simeq -0.77$

Point 1:

$y(0.43) = -3.38$

$\boxed{(0.43, -3.38)}$

Point 2:

$y(-0.77) = 0.76$

$\boxed{(-0.77, 0.76)}$

5) Differentiate using the product rule.

a) $y = (5x^2 - x + 1)(x + 2)$

$\frac{dy}{dx} = (10x-1)(x+2) + 1(5x^2 - x + 1)$

$= 10x^2 + 19x - 2 + 5x^2 - x + 1$

$= 15x^2 + 18x - 1$

b) $y = \underbrace{-x^2(4x-1)}_{1st}\underbrace{(x^3 + 2x + 3)}_{2nd}$

$\frac{d}{dx}(-x^2)(4x-1) = -2x(4x-1) + 4(-x^2)$

$= -8x^2 + 2x - 4x^2$

$= -12x^2 + 2x$

$\frac{dy}{dx} = (-12x^2 + 2x)(x^3 + 2x + 3) + (3x^2 + 2)(-x^2)(4x-1)$

$= -12x^5 - 24x^3 - 36x^2 + 2x^4 + 4x^2 + 6x + (3x^2+2)(-4x^3 + x^2)$

$= -12x^5 + 2x^4 - 24x^3 - 32x^2 + 6x - 12x^5 + 3x^4 - 8x^3 + 2x^2$

$= -24x^5 + 5x^4 - 32x^3 - 30x^2 + 6x$

6) The owner of a local hair salon is planning to raise the price for a haircut and blow dry. The current rate is \$30 for this service, with the salon averaging 550 clients a month. A survey indicates that the salon will lose 5 clients for every incremental price increase of \$2.50.

a) Write an equation to model the salon's monthly revenue, R, in dollars, as a function of x, where x represents the number of \$2.50 increases in the price.

$$R(x) = (30 + 2.5x)(550 - 5x)$$

b) Use the product rule to determine $R'(x)$

$$R'(x) = 2.5(550-5x)+(-5)(30+2.5x)$$

$$R'(x) = 1375 - 12.5x - 150 - 12.5x$$

$$R'(x) = -25x + 1225$$

c) Evaluate $R'(3)$ and interpret it for this situation.

$$R'(3) = -25(3)+1225$$

$$R'(3) = 1150$$

At an increase of \$7.50, the rate of change of revenue is \$1150

d) Solve $R'(x) = 0$.

$$0 = -25x + 1225$$

$$x = 49$$

e) Explain how the owner can use the result of part d).

$$R(49) = [30+2.5(49)][550-5(49)]$$

$$= (152.50)(305)$$

$$= \$46\,512.50$$

Increasing the price by \$2.50 49 times results in a max revenue of \$46 512.50.

Answers:

1)a) $f'(x) = 80x - 14$ **b)** $h'(t) = -4t + 7$ **c)** $p'(x) = -4x + 21$ **d)** $g'(x) = 12x^2 - 10x + 8$
e) $f'(x) = -3x^2 + 2x + 5$ **f)** $h'(t) = 12t^3 + 4t$
a) 54 **b)** 60
3)a) $y = -240x - 739$ **b)** $y = -4x - 24$
4)a) $\left(-\frac{9}{8}, \frac{225}{16}\right)$ **b)** $(0.43, -3.38)$ and $(-0.77, 0.76)$
5)a) $15x^2 + 18x - 1$ **b)** $-24x^5 + 5x^4 - 32x^3 - 30x^2 + 6x$
6)a) $R(x) = (30 + 2.50x)(550 - 5x)$ **b)** $R'(x) = 1225 - 25x$ **c)** 1150; this is the rate of change of revenue at a \$7.50 increase **d)** $x = 49$ **e)** The owner could maximize the revenue by making 49 increases of \$2.50. A visit to the hair salon would cost \$152.50 and would generate a max revenue of \$46 512.50.