W3 – 7.3 – Product and Quotient Laws of Logarithms
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SOLUTIONS

1) Simplify using laws of logarithms and then evaluate.

a) $\log 9 + \log 6$

$= \log(9 \times 6)$
$= \log(54)$
$= 1.732$

b) $\log 48 - \log 6$

$= \log\left(\frac{48}{6}\right)$
$= \log(8)$
$= 0.903$

c) $\log_3 7 + \log_3 3$

$= \log_3(7 \times 3)$
$= \log_3(21)$
$= \frac{\log(21)}{\log(3)}$
$= 2.771$

2) Simplify each algebraic expression.

a) $\log x + \log y + \log(2z)$

$= \log(2xyz)$

b) $\log_2 a + \log_2(3b) - \log_2(2c)$

$= \log_2\left(\frac{3ab}{2c}\right)$

c) $2\log m + 3\log n - 4\log y$

$= \log m^2 + \log n^3 - \log y^4$
$= \log\left(\frac{m^2 n^3}{y^4}\right)$

3) Evaluate using the product law of logarithms.

a) $\log_6 18 + \log_6 2$

$= \log_6(18 \times 2)$
$= \log_6(36)$
$= \frac{\log 36}{\log 6}$
$= 2$

b) $\log 40 + \log 2.5$

$= \log(40 \times 2.5)$
$= \log(100)$
$= 2$

c) $\log_{12} 8 + \log_{12} 2 + \log_{12} 9$

$= \log_{12}(8 \times 2 \times 9)$
$= \log_{12}(144)$
$= \frac{\log 144}{\log 12}$
$= 2$

4) Evaluate using the quotient law of logarithms.

a) $\log_3 54 - \log_3 2$

$= \log_3\left(\frac{54}{2}\right)$
$= \log_3(27)$
$= 3$

b) $\log 50\,000 - \log 5$

$= \log\left(\frac{50\,000}{5}\right)$
$= \log(10000)$
$= 4$

c) $\log_4 320 - \log_4 5$

$= \log_4\left(\frac{320}{5}\right)$
$= \log_4(64)$
$= 3$

5) Evaluate, using the laws of logarithms

a) $3\log_{16} 2 + 2\log_{16} 8 - \log_{16} 2$

$= \log_{16}(2^3) + \log_{16}(8^2) - \log_{16}(2)$

$= \log_{16}(8) + \log_{16}(64) - \log_{16}(2)$

$= \log_{16}(256)$

$= 2$

b) $\log 20 + \log 2 + \frac{1}{3}\log 125$

$= \log(20\times 2) + \log(125^{1/3})$

$= \log(40) + \log(5)$

$= \log(40\times 5)$

$= \log(200)$

$\doteq 2.301$

6) Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log_7(cd)$

$= \log_7(c) + \log_7(d)$

b) $\log_3\left(\frac{m}{n}\right)$

$= \log_3(m) - \log_3(n)$

c) $\log(uv^3)$

$= \log(u) + \log(v^3)$

$= \log(u) + 3\log(v)$

d) $\log\left(\frac{a\sqrt{b}}{c^2}\right)$

$= \log a + \log(b^{1/2}) - \log(c^2)$

$= \log a + \frac{1}{2}\log b - 2\log c$

e) $\log_2 10$

$= \log_2(2\times 5)$

$= \log_2(2) + \log_2(5)$

$= 1 + \log_2(5)$

7) Simplify

$\log\left(\frac{x^2}{\sqrt{x}}\right) = \log x^2 - \log x^{1/2}$

$= 2\log x - \frac{1}{2}\log x$

$= \frac{3}{2}\log x$

b) $\log\sqrt{k} + \log(\sqrt{k})^3 + \log\sqrt[3]{k^2}$

$= \log(k^{1/2}) + \log(k^{3/2}) + \log(k^{2/3})$

$= \frac{1}{2}\log k + \frac{3}{2}\log k + \frac{2}{3}\log k$

$= \frac{3}{6}\log k + \frac{9}{6}\log k + \frac{4}{6}\log k$

$= \frac{8}{3}\log k$

c) $\log(x^2 - 4) - \log(x - 2)$

$= \log\left(\frac{x^2-4}{x-2}\right)$

$= \log\left[\frac{(x-2)(x+2)}{x-2}\right]$

$= \log(x+2)$

d) $\log(x^2 - x - 6) - \log(2x - 6)$

$= \log\left(\frac{x^2-x-6}{2x-6}\right)$

$= \log\left[\frac{(x-3)(x+2)}{2(x-3)}\right]$

$= \log\left(\frac{x+2}{2}\right)$

ANSWER KEY

1) $\log 54 = 1.732$ **b)** $\log 8 = 0.903$ **c)** $\log_3 21 = 2.771$ **2)a)** $\log(2xyz)$ **b)** $\log_2\left(\frac{3ab}{2c}\right)$ **c)** $\log\left(\frac{m^2n^3}{y^4}\right)$

3)a) 2 **b)** 2 **c)** 2 **4)a)** 3 **b)** 4 **c)** 3 **5)a)** 2 **b)** 2.301

6)a) $\log_7 c + \log_7 d$ **b)** $\log_3 m - \log_3 n$ **c)** $\log u + 3\log v$ **d)** $\log a + \frac{1}{2}\log b - 2\log c$ **e)** $1 + \log_2 5$

7)a) $\frac{3}{2}\log x$ **b)** $\frac{8}{3}\log k$ **c)** $\log(x+2)$ **d)** $\log\left(\frac{x+2}{2}\right)$