W3 – 5.1/5.2 Graphing Trig Functions
MHF4U
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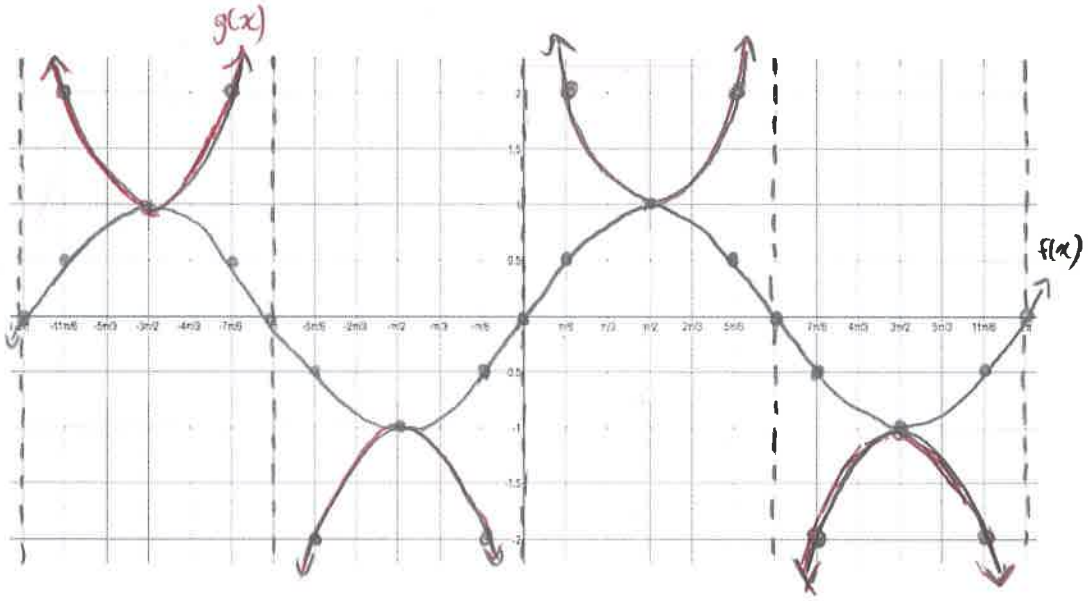
SOLUTIONS

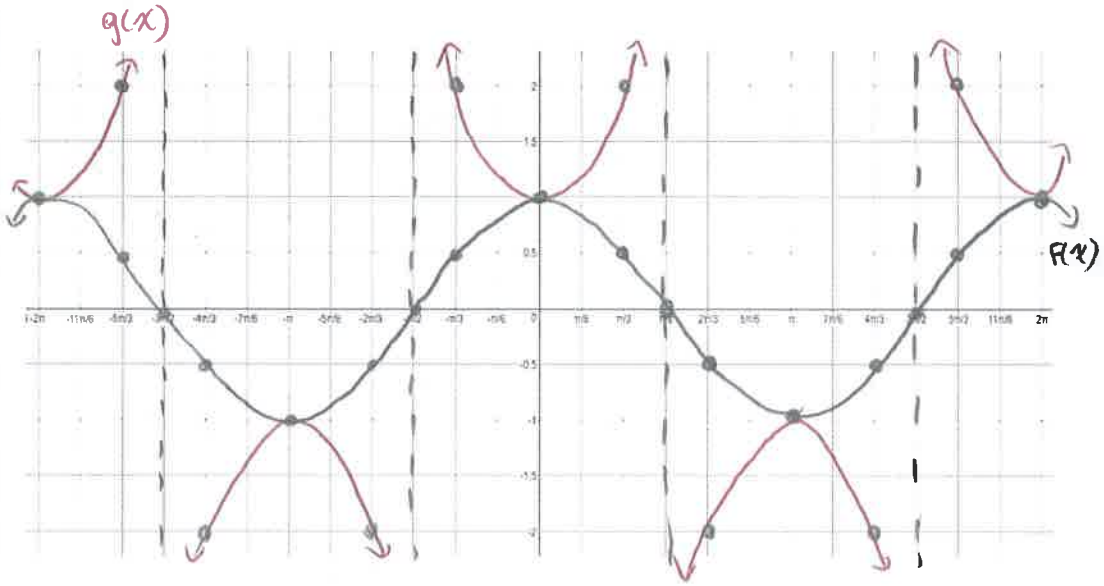
1) Complete the following table of values for the function $f(x) = \sin(x)$ and $g(x) = \csc(x)$. Use special triangles, the unit circle, or a calculator to find values for the function. Then graph both functions on the same grid. Draw asymptotes where necessary.

x	$f(x)$	$g(x)$
0	0	und
$\frac{\pi}{6}$	$\frac{1}{2}$	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	2
$\frac{6\pi}{6} = \pi$	0	und
$\frac{7\pi}{6}$	$-\frac{1}{2}$	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	-2
$\frac{12\pi}{6} = 2\pi$	0	und



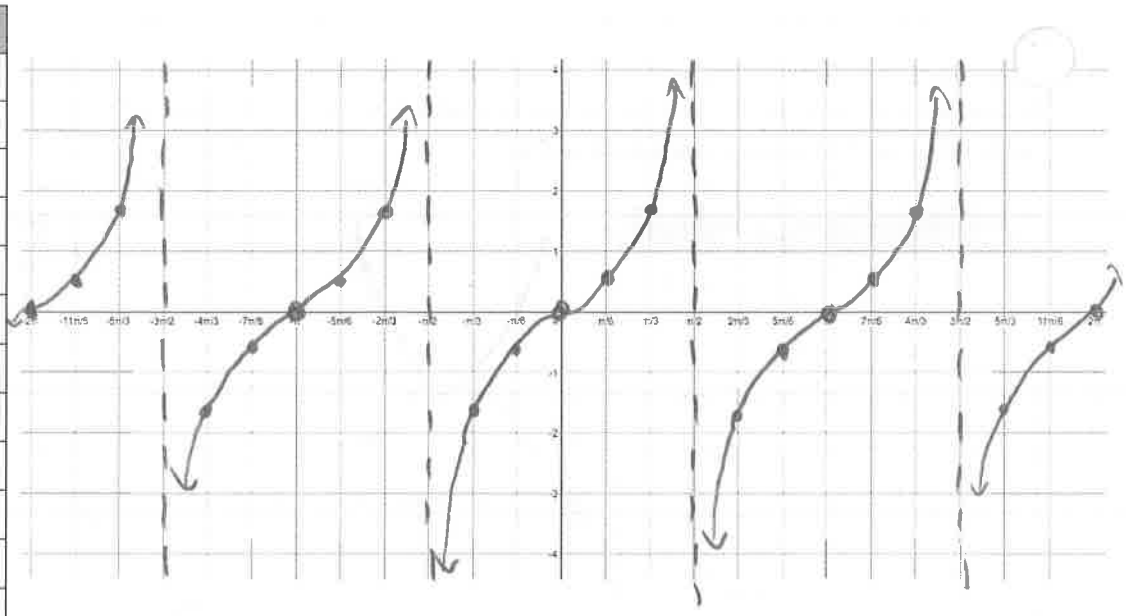
2) Complete the following table of values for the function $f(x) = \cos(x)$ and $g(x) = \sec(x)$. Use special triangles, the unit circle, or a calculator to find values for the function. Then graph both functions on the same grid. Draw asymptotes where necessary.

x	$f(x)$	$g(x)$
0	1	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{2}$	2
$\frac{3\pi}{6} = \frac{\pi}{2}$	0	und
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{2}$	-2
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$
$\frac{6\pi}{6} = \pi$	-1	-1
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{1}{2}$	-2
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0	und
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$\frac{1}{2}$	2
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$
$\frac{12\pi}{6} = 2\pi$	1	1



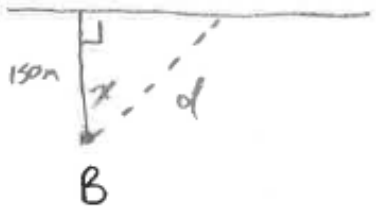
3) Complete the following table of values for the function $f(x) = \tan(x)$. Use the quotient identity to find y-values.

x	$f(x)$
0	0
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} \approx 0.58$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\sqrt{3} \approx 1.73$
$\frac{3\pi}{6} = \frac{\pi}{2}$	und
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\sqrt{3}$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}}$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\sqrt{3}$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	und
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\sqrt{3}$
$\frac{11\pi}{6}$	$-\frac{1}{\sqrt{3}}$
$\frac{12\pi}{6} = 2\pi$	0



4) A boat is in the water 150 meters from a straight shoreline. There is a rotating beam on the boat.

a) Determine a reciprocal trigonometric relation for the distance, d, from the boat to where the light hits the shoreline in terms of the angle of rotation x.



$\cos x = \frac{150}{d}$

$\sec x = \frac{d}{150}$

$d = 150 \sec x$

b) Determine an exact expression for the distance when $x = \frac{\pi}{6}$

$d = 150 \sec\left(\frac{\pi}{6}\right)$

$d = \frac{150}{\cos\left(\frac{\pi}{6}\right)}$

$d = \frac{150}{\left(\frac{\sqrt{3}}{2}\right)}$

$\rightarrow d = \frac{300}{\sqrt{3}}$ m.

c) Determine an approximate value, to the nearest tenth of a meter, for the distance.

$d \approx 173.2$ m.

5) A variant on the carousel at a theme park is the swing ride. Swings are suspended from a rotating platform and move outward to form an angle x with the vertical as the ride rotates. The angle is related to the radial distance, in meters, from the center of rotation; the acceleration, $g = 9.8$ m/s^2, due to gravity; and the speed, v, in meters per second, of the swing, according to the formula

$$\cot x = \frac{rg}{v^2}$$

Determine the angle x for a swing located 3.5 meters from the center of rotations and moving at 5.4 m/s, to the nearest hundredth of a radian.

$\cot x = \frac{3.5(9.8)}{(5.4)^2}$

$\tan x = \frac{(5.4)^2}{3.5(9.8)}$

$\tan x = 0.850146$

$x = \tan^{-1}(0.850146)$

$x \approx 0.7$

Explain the difference between $\csc\frac{1}{\sqrt{2}}$ and $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\csc\frac{1}{\sqrt{2}} = \frac{1}{\sin\left(\frac{1}{\sqrt{2}}\right)}$; it is the reciprocal of the sine function

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$; the -1 is not an exponent; it means "inverse of sine" not reciprocal of sine.

Answer Key

See posted solutions for #1-3

4)a) $d = 150\sec x$ **b)** $\frac{300}{\sqrt{3}}$ m **c)** 173.2 m

5) 0.70

6) The cosecant function is the reciprocal of the sine function. For $\sin^{-1}$, the -1 is NOT an exponent but instead a notation meaning the opposite operation of sine. The sine function takes an angle for an input and gives a ratio as an output. $\sin^{-1}$ takes a ratio for an input and gives the angle as an output.