W3 – Newton Quotient
MHF4U
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SOLUTIONS

1. Find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when $x = 4$ and $x = -1$.

a) $f(x) = 3x - 8$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h) - 8 - (3x-8)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3x + 3h - 8 - 3x + 8}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3h}{h}$$

$$\boxed{f'(x) = 3}$$

$$\boxed{\begin{array}{l} f'(4) = 3 \\ f'(-1) = 3 \end{array}}$$

b) $y = 20x + x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{20(x+h) + (x+h)^2 - (20x + x^2)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{20x} + 20h + \cancel{x^2} + 2xh + h^2 - \cancel{20x} - \cancel{x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{20h + 2xh + h^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(20 + 2x + h)}{\cancel{h}}$$

$$f'(x) = 20 + 2x + 0$$

$$\boxed{f'(x) = 2x + 20}$$

$$f'(4) = 2(4) + 20$$

$$\boxed{f'(4) = 28}$$

$$f'(-1) = 2(-1) + 20$$

$$\boxed{f'(-1) = 18}$$

c) $y = 2x^3 + 4$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^3 + 4 - (2x^3 + 4)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)(x^2 + 2xh + h^2) + 4 - 2x^3 - 4}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3) - 2x^3}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 - \cancel{2x^3}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(6x^2 + 6xh + 2h^2)}{\cancel{h}}$$

$$f'(x) = 6x^2 + 6x(0) + 2(0)^2$$

$$\boxed{f'(x) = 6x^2}$$

$$\boxed{\begin{array}{l} f'(4) = 6(4)^2 \\ = 96 \end{array}}$$

$$\boxed{\begin{array}{l} f'(-1) = 6(-1)^2 \\ = 6 \end{array}}$$

d) $f(x) = x^2 - 9x + 17$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 9(x+h) + 17 - (x^2 - 9x + 17)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{9x} - 9h + \cancel{17} - \cancel{x^2} + \cancel{9x} - \cancel{17}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - 9h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 9)}{\cancel{h}}$$

$$f'(x) = 2x + 0 - 9$$

$$\boxed{f'(x) = 2x - 9}$$

$$\boxed{\begin{array}{l} f'(4) = 2(4) - 9 \\ = -1 \end{array}}$$

$$\boxed{\begin{array}{l} f'(-1) = 2(-1) - 9 \\ = -11 \end{array}}$$

e) $f(x) = \frac{x(x+1)}{2}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{(x+h)(x+h+1)}{2} - \frac{x(x+1)}{2}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^2} + xh + \cancel{x} + xh + h^2 + h - \cancel{x^2} - \cancel{x}}{2h}$$

$$f'(x) = \lim_{h\to 0} \frac{2xh + h^2 + h}{2h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(2x+h+1)}{2\cancel{h}}$$

$$f'(x) = \frac{2x+0+1}{2}$$

$$f'(x) = \frac{2x+1}{2}$$

$$\boxed{f'(x) = x + \frac{1}{2}}$$

$$\boxed{f'(4) = 4 + \frac{1}{2} = \frac{9}{2}}$$

$$\boxed{f'(-1) = -1 + \frac{1}{2} = \frac{-1}{2}}$$

f) $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x - 1(x+h)}{x(x+h)}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h\to 0} \frac{-1\cancel{h}}{\cancel{h}(x)(x+h)}$$

$$f'(x) = \frac{-1}{x(x+0)}$$

$$f'(x) = \frac{-1}{x^2}$$

2) State whether the functions are increasing, decreasing, or neither when $x = 4$ for each function in #1. How do you know?

a, b, c, e are increasing since $f'(4)$ is positive

d, f are decreasing since $f'(4)$ is negative

3)a) State the derivative of $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2) - x^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^3} + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 - \cancel{x^3}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$f'(x) = 3x^2 + 3x(0) + (0)^2$$

$$\boxed{f'(x) = 3x^2}$$

b) Evaluate $f'(-6)$ $= 3(-6)^2$

$= 108$

c) Determine the equation of the tangent line at $x = 6$

$f(6) = (6)^3$
$= 216$

$\therefore$ point $(6, 216)$ is on the tangent line

$f'(6) = 3(6)^2$
$= 108$

$\therefore$ slope of the tangent line is 108

$y = mx + b$

$216 = 108(6) + b$

$b = -432$

$\boxed{y = 108x - 432}$

<u>**Answer Key**</u>

1)a) $f'(x) = 3, f'(4) = 3, f'(-1) = 3$ **b)** $f'(x) = 20 + 2x, f'(4) = 28, f'(-1) = 18$

c) $f'(x) = 6x^2, f'(4) = 96, f'(-1) = 6$ **d)** $f'(x) = 2x - 9, f'(4) = -1, f'(-1) = -11$

e) $f'(x) = x + \frac{1}{2}, f'(4) = \frac{9}{2}, f'(-1) = -\frac{1}{2}$ **f)** $f'(x) = -\frac{1}{x^2}, f'(4) = -\frac{1}{16}, f'(-1) = -1$

2) a,b,c and e are increasing functions when $x = 4$ since the instantaneous rate of change is positive

nd f are decreasing when $x = 4$

3)a) $f'(x) = 3x^2$ **b)** 108 **c)** $y = 108x - 432$