W3 – Applications of the Dot Product Unit 5

MCV4U

Jensen

1) Determine the work done by each force $\vec{F}$, in Newtons, for each object moving along $\vec{s}$.

a) $\vec{F} = [3, -2]$, $\vec{s} = [1, 8]$

$W = \vec{F} \cdot \vec{s}$

$= 3(1) + (-2)(8)$

$= -13$ Joules

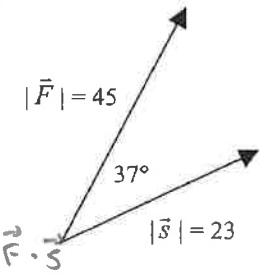
b) $\vec{F} = [8, -9]$, $\vec{s} = [-3, 7]$

$W = 8(-3) + (-9)(7)$

$= -87$ Joules

2) Determine the work done by the force $\vec{F}$, in Newtons, for each object moving along $\vec{s}$.

a)



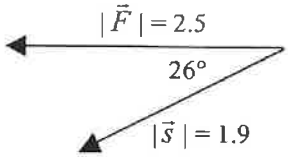
$W = \vec{F} \cdot \vec{s}$

$W = |\vec{F}||\vec{s}|\cos(37)$

$= 45(23)\cos(37)$

$\simeq 826.59$ Joules

b)



$W = \vec{F} \cdot \vec{s}$

$W = |\vec{F}||\vec{s}|\cos(26)$

$W = 2.5(1.9)\cos(26)$

$W \simeq 4.27$ Joules.

3) Determine the angle between the vectors in each pair.

a) $\vec{p} = [6, 7]$ and $\vec{q} = [3, 2]$

$\cos\theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}$

$\cos\theta = \frac{6(3) + 7(2)}{(\sqrt{6^2+7^2})(\sqrt{3^2+2^2})}$

$\cos\theta = \frac{32}{\sqrt{85}\sqrt{13}}$

$\theta \simeq 15.71°$

b) $\vec{r} = [-1, -7]$ and $\vec{s} = [5, 4]$

$\cos\theta = \frac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|}$

$\cos\theta = \frac{-1(5) + (-7)(4)}{(\sqrt{(-1)^2+(-7)^2})(\sqrt{5^2+4^2})}$

$\cos\theta = \frac{-33}{\sqrt{50}\sqrt{41}}$

$\theta \simeq 136.79°$

4) Determine the projection of the first vector on the second.

a) $\vec{a} = [6, -1]$, $\vec{b} = [3, -4]$

$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}(\vec{b})$

$= \frac{6(3) + (-1)(-4)}{3^2 + (-4)^2}[3, -4]$

$= \frac{22}{25}[3, -4]$

$= \left[\frac{66}{25}, \frac{-88}{25}\right]$

b) $\vec{c} = [6, 7]$, $\vec{d} = [3, 2]$

$\text{proj}_{\vec{d}}\vec{c} = \frac{\vec{c} \cdot \vec{d}}{\vec{d} \cdot \vec{d}}\vec{d}$

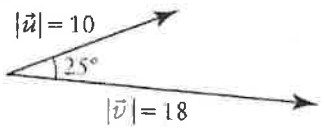
$= \frac{6(3) + 7(2)}{3^2 + 2^2}[3, 2]$

$= \frac{32}{13}[3, 2]$

$= \left[\frac{96}{13}, \frac{64}{13}\right]$

5) Determine the projection of $\vec{u}$ on $\vec{v}$

a)

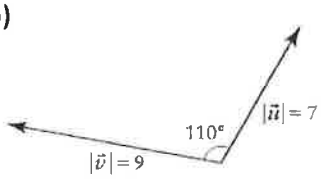


$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,(\hat{v})$

$= 10\cos(25)\,\hat{v}$

$\simeq 9.06\hat{v}$

b)

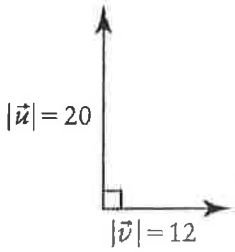


$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,(\hat{v})$

$= 7\cos(110)\hat{v}$

$\simeq -2.39\hat{v}$

c)



$= \vec{0}$

6) For each of the following, find the magnitude of the projection of $\vec{x}$ on $\vec{y}$ and also the vector projection of $\vec{x}$ on $\vec{y}$.

a) $\vec{x} = [1,1], \vec{y} = [1,-1]$

$|\text{proj}_{\vec{y}}\vec{x}| = \left|\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}\right|$

$= \left|\frac{1(1)+1(-1)}{\sqrt{(1)^2+(-1)^2}}\right|$

$= 0$

$\text{proj}_{\vec{y}}\vec{x} = 0\left(\frac{\vec{y}}{|\vec{y}|}\right)$

$= \vec{0}$

b) $\vec{x} = [2,5], \vec{y} = [-5,12]$

$|\text{proj}_{\vec{y}}\vec{x}| = \left|\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}\right|$

$= \frac{2(-5)+5(12)}{\sqrt{(-5)^2+(12)^2}}$

$= \frac{50}{13}$

$\text{proj}_{\vec{y}}\vec{x} = \frac{50}{13}\left(\frac{\vec{y}}{|\vec{y}|}\right)$

$= \frac{50}{13}\left(\frac{[-5,12]}{13}\right)$

$= \frac{50}{169}[-5,12]$

$= \left[\frac{-250}{169}, \frac{600}{169}\right]$

7) ΔDEF has vertices D(–3, 5), E(2, 3), and F(6, 7). Calculate ∠DEF.

$\vec{ED} = [-5,2]$

$\vec{EF} = [4,4]$

$\cos\theta = \frac{\vec{ED}\cdot\vec{EF}}{|\vec{ED}||\vec{EF}|}$

$\cos\theta = \frac{-5(4)+(2)(4)}{(\sqrt{29})(\sqrt{32})}$

$\cos\theta = \frac{-12}{\sqrt{928}}$

$\theta \simeq 113.2^\circ$

against gravity

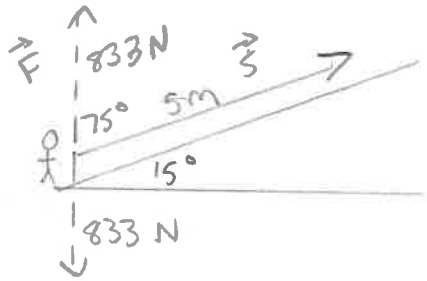
8) How much work is done by the orderly pushing an 85 kg person up a 5 m ramp inclined at an angle of 15° to the horizontal?

$\vec{F} = 85(9.8) = 833$ N

$W = \vec{F} \cdot \vec{s}$

$W = 833(5)\cos(75)$

$W \approx 1077.98$ J



9) A stage lamp is dragged 15 m along level ground by a 120 N force applied at an angle of 35° to the ground. It is then dragged up a 12m ramp onto a stage by the same force. If the ramp is inclined at 15° to the ground. Find the total work done.

$W = 120(15)\cos(35°) + 120(12)\cos(20°)$

$W \approx 2827.63$ J

10) A box on a wagon pulled a distance of 35 m by a 27 N force applied at an angle of 40° to the ground. The box is then lifted a distance of 1.5 m and placed on a table by exerting a force of 37 N. Find the total work done.

$W = 27(35)\cos(40°) + 37(1.5)\cos(0)$

$W \approx 779.4$ J

ANSWER KEY

1)a) −13 **b)** −87

2)a) 826.58 **b)** 4.269

3)a) $\theta = 15.7°$ **b)** $\theta = 136.8°$

4)a) [2.64, −3.52] **b)** [7.38, 4.92]

5)a) $9.1\hat{v}$ **b)** $-2.4\hat{v}$ **c)** $\vec{0}$

6) magnitude = 0, vector projection: $\vec{0}$ b) magnitude = $\frac{50}{13}$, vector projection: $\left[\frac{-250}{169}, \frac{600}{169}\right]$

7) 113.2°

8) 1077.98 J

9) 2865.4 J

10) 779.4 J