Prove each identity using the space on the following pages.

a) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

b) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

c) $\sin(2x) = 2 \sin x \cos x$

d) $\cos(2x) = \cos^2 x - \sin^2 x$

e) $\cot \theta - \tan \theta = 2 \cot(2\theta)$

f) $\frac{\sin(2\theta)}{1 - \cos(2\theta)} = \cot \theta$

g) $\sin x \sec x = \tan x$

h) $\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

i) $\frac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

j) $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$

k) $\frac{1 - \sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$

l) $\frac{\cos(2x) + 1}{\sin(2x)} = \cot x$

m) $\cot \theta - \tan \theta = 2 \cot(2\theta)$

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

o) $\frac{2 \tan x}{1 + \tan^2 x} = \sin(2x)$

p) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$

q) $\cos^4 x - \sin^4 x = \cos(2x)$

r) $\csc(2x) + \cot(2x) = \cot x$

s) $\cos(2x) = 2 \cos^2 x - 1$

t) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

u) $\frac{\cos(2x) + 1}{\sin(2x)} = \cot x$

v) $\cot x + \tan x = 2 \csc(2x)$