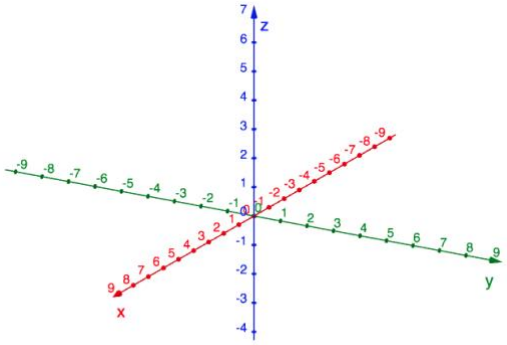
W4 – Vectors in 3-Space Unit 5

MCV4U

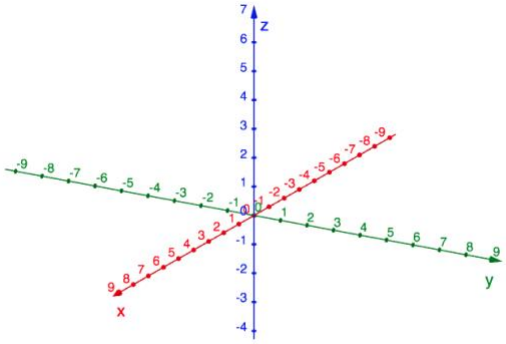
Jensen

1) Draw the position vectors.

a) [–2, 3, –4]



b) [2, –3, 1]



2) Express each vector as the sum of $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.

a) [2, –1, 7]

b)[–4, –6, 5]

3) Express each vector in the form [*a*, *b*, *c*].

a) $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

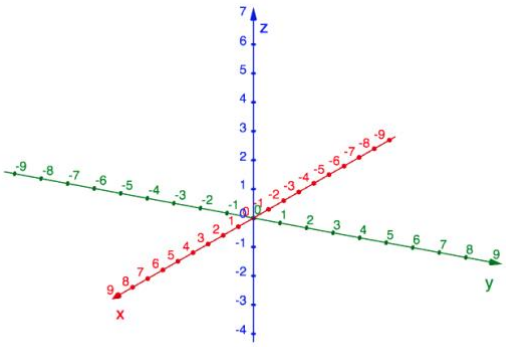
b) $2\hat{\imath} + 3\hat{k}$

c) $-8\hat{\imath} + 9\hat{\jmath} - 4\hat{k}$

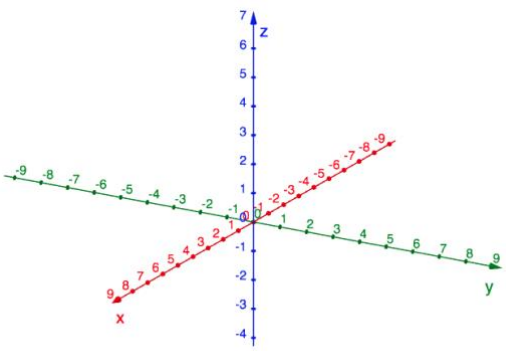
d) $-8\hat{\jmath} - 7\hat{k}$

4) Draw vector $\overrightarrow{AB}$ joining each pair of points. Then write the vector in the form [*a*, *b*, *c*].

a) A(2, –1, 7) and B(0, 2, –1)

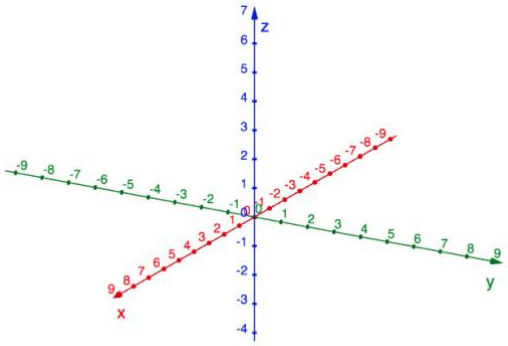


b) A(0, –4, –2) and B(–3, –1, 0)

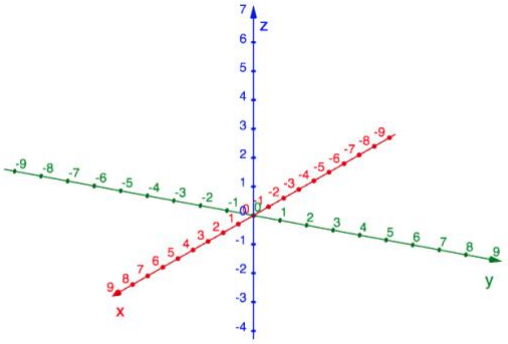


5) Draw each position vector. Then find its magnitude.

a) $[-1,5,-2]$



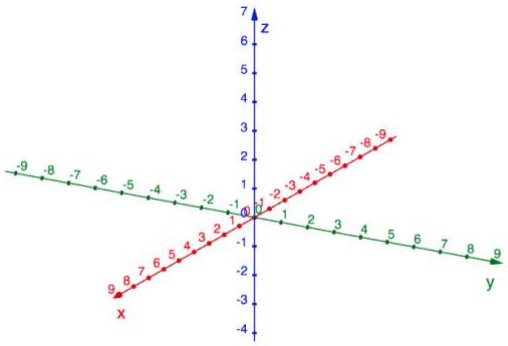
b) $[-2,0,4]$



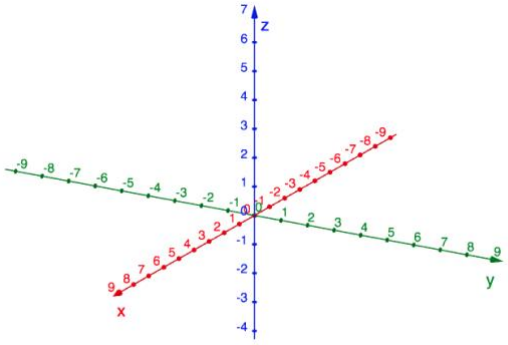
6) Find a and b such that $\vec{u} = [a, 3,6]$ and $\vec{v} = [-8,12, b]$ are collinear.

7) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Write the vector in the form $[x, y, z]$. Then determine the exact magnitude of the vector.

a) $A(2,1,3)$ and $B(5,7,1)$



b) $A(3,-4,1)$ and $B(6,-1,5)$



8) Evaluate each given the vectors $\vec{a}$= [–2, 1, 8], $\vec{b}$= [3, 1, –2], and $\vec{c}$ = [2, –3, 4].

a) $3\vec{b}$ **b)** $\vec{b} - \vec{c}$ **c)** $2\vec{a} - 3\vec{c} + 4\vec{b}$

d) $(\vec{a} + \vec{b}) - (\vec{a} + \vec{c})$ **e)** $\vec{b} \cdot \vec{c}$ **f)** $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$

9) Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = 7\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = -2\hat{\imath} + 5\hat{\jmath} + 9\hat{k}$.

a) $(\vec{a} + \vec{b}) \cdot \vec{c}$ **b)** $2\vec{a} \cdot (4\vec{b} - 3\vec{c})$

10) Determine the values of *k* such that $\vec{u}$ and $\vec{v}$ are orthogonal.

a) $\vec{u}$ = [2, *k*, –1] and $\vec{v}$ = [3, –2, 7] **b)** $\vec{u}$ = [–3, 1, *k*] and $\vec{v}$ = [4, –*k*, *k*]

11) Find a vector orthogonal to each vector.

a) [2, –1, 7]

b) [8, –3, 4]

12) Consider the vectors $\vec{u}$ = [3, –5, 8] and $\vec{v}$ = [3, 1, –2].

a) Find $\vec{u} \cdot \vec{v}$.

b) Calculate the angle between $\vec{u}$ and $\vec{v}$.

13) Determine the projection of $\vec{a}$ on $\vec{b}$.

a) $\vec{a}$ = [2, 1, –3] and $\vec{b}$ = [1, 7, 6]

b) $\vec{a}$ = [3, 4, 7] and $\vec{b}$ = [2, –1, 1]

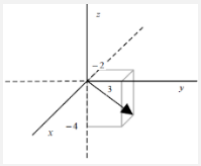
14) The initial point of vector $\overrightarrow{CD} = [2, -9, 1]$ is $C(-3, 2, 2)$ determine the coordinates of D.

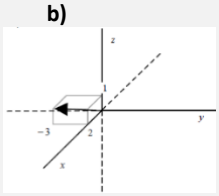
15) Find 2 unit vectors that are parallel to $\vec{a} = [9, -7, 2]$.

16) A triangle has vertices at the points $D = (3, -2, -3)$, $E(7, 0, 1)$ and $F(1, 2, 1)$. What type of triangle is $\triangle DEF$? Explain.

ANSWER KEY:

1. a) **b)**

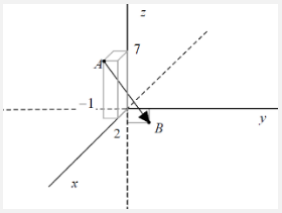


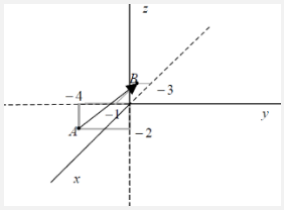


2. a) $2\hat{\imath} - \hat{\jmath} + 7\hat{k}$ **b)** $-4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

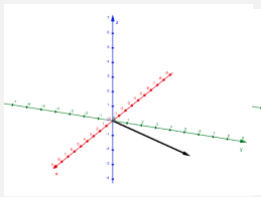
3. a) [3, –4, 5] **b)** [2, 0, 3] **c)** [–8, 9, –4] **d)** [0, –8, –7]

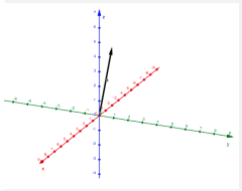
4. a) [–2, 3, –8] **b)** [–3, 3, 2]





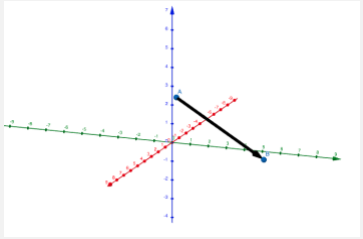
5)a) $\sqrt{30}$ **b)** $2\sqrt{5}$





6) $a = -2, b = 24$

7)a) $[3,6,-2], 7$ **b)** $[3,3,4], \sqrt{34}$



8) a) [9, 3, –6] **b)** [1, 4, –6] **c)** [2, 15, –4] **d)** [1, 4, –6] **e)** –5 **f)** –16

9) a) 17 **b)** -48

10)a) $k = -0.5$ **b)** $k = 4, k = -3$

11)a) [4, 8, 0] **b)** [1,0,–2]

12)a) –12 **b)** 108.9°

13)a) $\left[\frac{-9}{86}, \frac{-63}{86}, \frac{-27}{43}\right]$ **b)** $\left[3, \frac{-3}{2}, \frac{3}{2}\right]$

14) $D(-1, -7, 3)$

15) $\left[\frac{9}{\sqrt{134}}, -\frac{7}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right]$ and $\left[-\frac{9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, -\frac{2}{\sqrt{134}}\right]$

16) This is a non-right isosceles triangle because 2 sides of the triangle are the same length but no 2 vectors that make up the sides of the triangle dot to 0, this tells us there are no perpendicular vectors and therefore no right angles.