W4 – Rational Functions Unit 2

MCV4U

Jensen

1) Find the equation of any asymptotes for the following functions. Then, find the one-sided limits approaching the vertical asymptotes.

a) $f(x) = \frac{x+3}{x^2-4}$

b) $y = \frac{x^2}{x^2-3x+2}$

c) $y = 2x + \frac{1}{x}$

d) $g(x) = \frac{2x-3}{x^2-6x+9}$

2) Find the derivative of each function. Then, determine whether the function has any local extrema.

a) $f(x) = \frac{2}{x+3}$

b) $h(x) = \frac{-3}{(x-2)^2}$

3) Consider the function $f(x) = \frac{-2}{(x+1)^2}$

a) Find the intervals of increase and decrease for $f(x)$.

b) Find the intervals of concavity for $f(x)$.

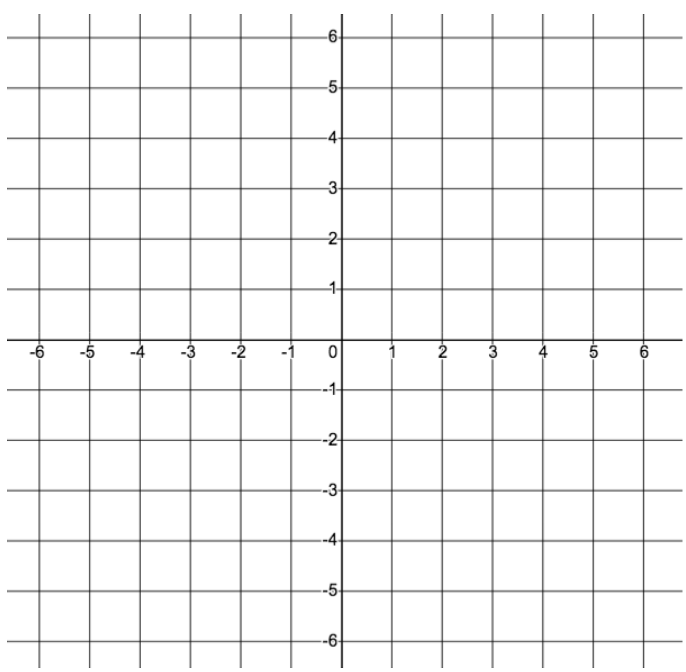
4) Consider the function $h(x) = \frac{1}{x^2-4}$

a) Write the equations of the asymptotes

b) Make a table showing the increasing and decreasing intervals for the function

c) How can you use the table from part b) to determine the behavior of $f(x)$ near the vertical asymptotes?

d) Sketch a graph of the function.



Answers:

1)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to -2^+} = -\infty$, $\lim_{x\to -2^-} = \infty$

b) VA: $x = 1$ and $x = 2$; HA: $y = 1$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to 1^+} = -\infty$, $\lim_{x\to 1^-} = \infty$

c) VA: $x = 0$; SA: $y = 2x$; $\lim_{x\to 0^+} = \infty$, $\lim_{x\to 0^-} = -\infty$

d) VA: $x = 3$; HA: $y = 0$; $\lim_{x\to 3^+} = \infty$, $\lim_{x\to 3^-} = \infty$

2)a) $f'(x) = \frac{-2}{(x+3)^2}$; no local extrema **b)** $h'(x) = \frac{6}{(x-2)^3}$; no local extrema

3)a) decreasing when $x < -1$, increasing when $x > -1$ **b)** concave down when $x < -1$ or $x > -1$

4)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$
b) increasing when $x < -2$ or $-2 < x < 0$; decreasing when $0 < x < 2$ or $x > 2$
c) Since the curve is increasing to the left of $x = -2$, $\lim_{x\to -2^-} = \infty$
Since the curve is increasing to the right of $x = -2$, $\lim_{x\to -2^+} = -\infty$
Since the curve is decreasing to the left of $x = 2$, $\lim_{x\to 2^-} = -\infty$
Since the curve is decreasing to the right of $x = 2$, $\lim_{x\to 2^+} = \infty$

d)