W4 – 4.5 Prove Trig Identities
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SOLUTIONS

Prove each identity using the space on the following pages.

a) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

b) $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

c) $\sin(2x) = 2\sin x \cos x$

d) $\cos(2x) = \cos^2 x - \sin^2 x$

e) $\cot\theta - \tan\theta = 2\cot(2\theta)$

f) $\frac{\sin(2\theta)}{1-\cos(2\theta)} = \cot\theta$

g) $\sin x \sec x = \tan x$

h) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

i) $\frac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

j) $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$

k) $\frac{1-\sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$

l) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

m) $\cot\theta - \tan\theta = 2\cot(2\theta)$

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

o) $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

p) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$

q) $\cos^4 x - \sin^4 x = \cos(2x)$

r) $\csc(2x) + \cot(2x) = \cot x$

s) $\cos(2x) = 2\cos^2 x - 1$

t) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

u) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

v) $\cot x + \tan x = 2\csc(2x)$

a)

LS	RS
$= \sin(x+y)$	$= \sin x \cos y + \cos x \sin y$
$= \cos\left[\frac{\pi}{2} - (x+y)\right]$	
$= \cos\left[\left(\frac{\pi}{2} - x\right) - y\right]$	
$= \cos\left(\frac{\pi}{2} - x\right)\cos y + \sin\left(\frac{\pi}{2} - x\right)\sin y$	
$= \sin x \cos y + \cos x \sin y$	

LS = RS

b)

LS	RS
$= \tan(x-y)$	$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$
$= \frac{\sin(x-y)}{\cos(x-y)}$	
$= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y + \sin x \sin y} \left(\frac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}}\right)$	
$= \frac{\frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} - \frac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y}}{\frac{\cancel{\cos x \cos y}}{\cancel{\cos x \cos y}} + \frac{\sin x \sin y}{\cos x \cos y}}$	
$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$	

c)

LS	RS
$= \sin(2x)$	$= 2\sin x\cos x$
$= \sin(x+x)$	
$= \sin x\cos x + \cos x\sin x$	
$= 2\sin x\cos x$	

LS = RS

d)

LS	RS
$= \cos(2x)$	$= \cos^2 x - \sin^2 x$
$= \cos(x+x)$	
$= \cos x\cos x - \sin x\sin x$	
$= \cos^2 x - \sin^2 x$	

LS = RS

e)

LS	RS
$= \cot\theta - \tan\theta$	$= 2\cot(2\theta)$
$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$	$= \frac{2\cos(2\theta)}{\sin(2\theta)}$
$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$	$= \frac{\cancel{2}(\cos^2\theta - \sin^2\theta)}{\cancel{2}\sin\theta\cos\theta}$
	$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$

LS = RS

f)

LS	RS
$= \frac{\sin(2\theta)}{1-\cos(2\theta)}$	$= \cot\theta$
$= \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}$	
$= \frac{2\sin\theta\cos\theta}{1-1+2\sin^2\theta}$	
$= \frac{\cancel{2\sin\theta}\cos\theta}{\cancel{2}\sin^{\cancel{2}}\theta}$	
$= \frac{\cos\theta}{\sin\theta}$	
$= \cot\theta$	

LS = RS

g)

LS	RS
$= \sin x \sec x$	$= \tan x$
$= \sin x \left(\frac{1}{\cos x}\right)$	$= \frac{\sin x}{\cos x}$
$= \frac{\sin x}{\cos x}$	

LS=RS

h)

LS	RS
$= \frac{1 - \sin x}{\cos x}$	$= \frac{\cos x}{1+\sin x} \frac{(1-\sin x)}{(1-\sin x)}$
	$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$
	$= \frac{\cancel{\cos x}(1-\sin x)}{\cos^{\cancel{2}} x}$
	$= \frac{1-\sin x}{\cos x}$

LS=RS

i)

LS	RS
$= \frac{\sec\theta - 1}{1-\cos\theta}$	$= \sec\theta$
$= \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{1-\cos\theta}$	$= \frac{1}{\cos\theta}$
$= \frac{\left(\frac{1-\cos\theta}{\cos\theta}\right)}{1-\cos\theta}$	
$= \left(\frac{\cancel{1-\cos\theta}}{\cos\theta}\right)\left(\frac{1}{\cancel{1-\cos\theta}}\right)$	
$= \frac{1}{\cos\theta}$	

LS=RS

j)

LS	RS
$= \frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x}$	$= \sec x \csc x$
$= \frac{\sin x(\sin x - \cos x) + \cos x(\sin x + \cos x)}{\cos x \sin x}$	$= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$
$= \frac{\sin^2 x - \sin x\cos x + \cos x \sin x + \cos^2 x}{\cos x \sin x}$	$= \frac{1}{\cos x \sin x}$
$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$	
$= \frac{1}{\cos x \sin x}$	

LS=RS

k) LS | RS

LS:

$$= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$$

RS:

$$= \tan^4 x + \tan^2 x + 1$$

$$= \frac{\sin^4 x}{\cos^4 x} + \frac{\sin^2 x}{\cos^2 x}\frac{(\cos^2 x)}{(\cos^2 x)} + \frac{\cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}$$

$$= \frac{(\sin^2 x)^2 + \sin^2 x \cos^2 x + (\cos^2 x)^2}{\cos^4 x}$$

$$= \frac{\sin^2 x(1 - \cos^2 x) + \sin^2 x \cos^2 x + \cos^2 x(1 - \sin^2 x)}{\cos^4 x}$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x + \sin^2 x \cos^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\cos^4 x}$$

$$= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$$

LS = RS

l)

LS	RS
$=\dfrac{\cos(2x)+1}{\sin(2x)}$	$=\cot x$
	$=\dfrac{\cos x}{\sin x}$
$=\dfrac{2\cos^2 x-1+1}{2\sin x\cos x}$	
$=\dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}}$	
$=\dfrac{\cos x}{\sin x}$	

LS=RS

m)

LS	RS
$=\cot\theta-\tan\theta$	$=2\cot(2\theta)$
$=\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}$	$=\dfrac{2\cos(2\theta)}{\sin(2\theta)}$
$=\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$	$=\dfrac{\cancel{2}\cos(2\theta)}{\cancel{2}\sin\theta\cos\theta}$
$=\dfrac{\cos(2\theta)}{\sin\theta\cos\theta}$	$=\dfrac{\cos(2\theta)}{\sin\theta\cos\theta}$

LS=RS

n)

LS	RS
$=(\sin x+\cos x)^2$	$=1+\sin(2x)$
$=(\sin x+\cos x)(\sin x+\cos x)$	$=1+2\sin x\cos x$
$=\sin^2 x+2\sin x\cos x+\cos^2 x$	
$=1+2\sin x\cos x$	

LS=RS

o)

LS	RS
$=\dfrac{2\tan x}{1+\tan^2 x}$	$=\sin(2x)$
$=\dfrac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}}$	$=2\sin x\cos x$
$=\dfrac{\left(\frac{2\sin x}{\cos x}\right)}{\left(\frac{1}{\cos^2 x}\right)}$	
$=\left(\dfrac{2\sin x}{\cancel{\cos x}}\right)\left(\dfrac{\cos^{\cancel{2}} x}{1}\right)$	
$=2\sin x\cos x$	

LS=RS

p)

LS	RS
$= \sin\left(\frac{\pi}{4}+x\right) + \sin\left(\frac{\pi}{4}-x\right)$	$= \sqrt{2}\cos x$

$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$

$= \frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}$

$= \frac{\cos x + \cos x}{\sqrt{2}}$

$= \frac{2\cos x}{\sqrt{2}} \frac{(\sqrt{2})}{(\sqrt{2})}$

$= \frac{\cancel{2}\sqrt{2}\cos x}{\cancel{2}}$

$= \sqrt{2}\cos x$

LS=RS

q)

LS	RS
$= \cos^4 x - \sin^4 x$	$= \cos(2x)$
$= (\cos^2 x)^2 - (\sin^2 x)^2$	$= \cos^2 x - \sin^2 x$

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos^2 x - \sin^2 x$

LS=RS

r)

LS	RS
$= \csc(2x) + \cot(2x)$	$= \cot x$
$= \frac{1}{\sin(2x)} + \frac{\cos(2x)}{\sin(2x)}$	$= \frac{\cos x}{\sin x}$

$= \frac{1 + \cos(2x)}{\sin(2x)}$

$= \frac{1 + 2\cos^2 x - 1}{2\sin x\cos x}$

$= \frac{\cancel{2}\cos^2 x}{\cancel{2}\sin x\cos x}$

$= \frac{\cos x}{\sin x}$

LS=RS

s)

LS	RS
$= \cos(2x)$	$= 2\cos^2 x - 1$

$= \cos(x+x)$

$= \cos x\cos x - \sin x\sin x$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1-\cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

LS=RS

t)

LS	RS
$= \sin\left(\frac{3\pi}{2} - x\right)$	$= -\cos x$
$= \sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin x$	
$= (-1)\cos x - 0\sin x$	
$= -\cos x$	

LS=RS

u)

LS	RS
$= \dfrac{\cos(2x)+1}{\sin(2x)}$	$= \cot x$
$= \dfrac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$	$= \dfrac{\cos x}{\sin x}$
$= \dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\cancel{\cos x}}$	
$= \dfrac{\cos x}{\sin x}$	

LS=RS

v)

LS	RS
$= \cot x + \tan x$	$= 2\csc(2x)$
$= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$	$= \dfrac{2}{\sin(2x)}$
$= \dfrac{\cos^2 x + \sin^2 x}{\sin x\cos x}$	$= \dfrac{2}{2\sin x\cos x}$
$= \dfrac{1}{\sin x\cos x}$	$= \dfrac{1}{\sin x\cos x}$

LS=RS