W4 – Vectors in 3-Space Unit 5

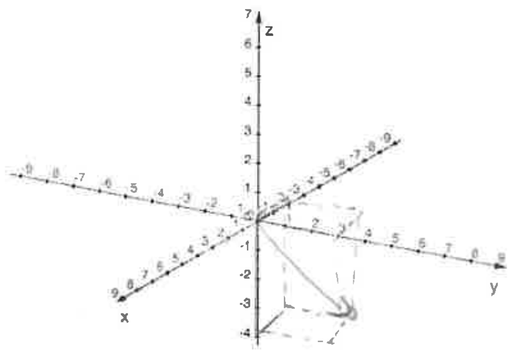
MCV4U

Jensen

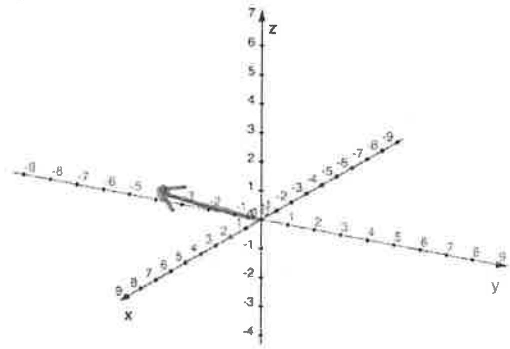
SOLUTIONS

1) Draw the position vectors.

a) [–2, 3, –4]



b) [2, –3, 1]



2) Express each vector as the sum of $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.

a) [2, –1, 7]

$= 2\hat{\imath} - \hat{\jmath} + 7\hat{k}$

b)[–4, –6, 5]

$= -4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

3) Express each vector in the form $[a, b, c]$.

a) $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

$= [3, -4, 5]$

b) $2\hat{\imath} + 3\hat{k}$

$= [2, 0, 3]$

c) $-8\hat{\imath} + 9\hat{\jmath} - 4\hat{k}$

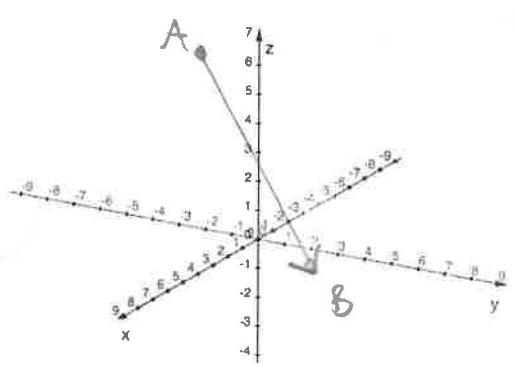
$= [-8, 9, -4]$

d) $-8\hat{\jmath} - 7\hat{k}$

$= [0, -8, -7]$

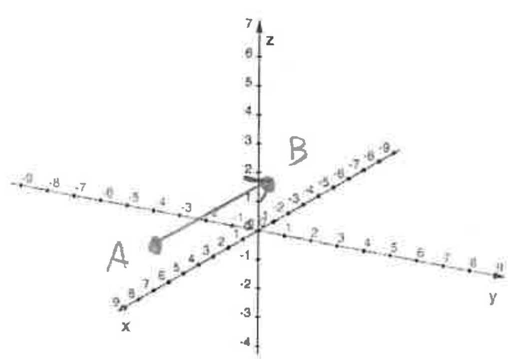
4) Draw vector $\overrightarrow{AB}$ joining each pair of points. Then write the vector in the form $[a, b, c]$.

a) A(2, –1, 7) and B(0, 2, –1)



$\overrightarrow{AB} = [-2, 3, -8]$

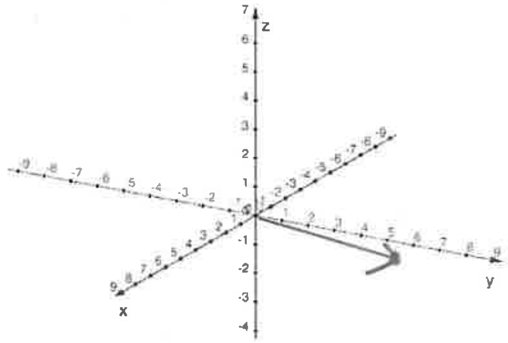
b) A(0, –4, –2) and B(–3, –1, 0)



$\overrightarrow{AB} = [-3, 3, 2]$

5) Draw each position vector. Then find its magnitude.

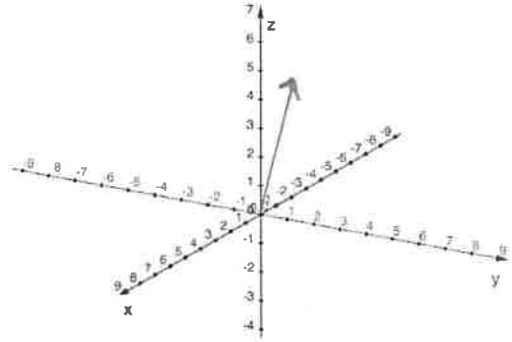
a) $[-1,5,-2]$



Magnitude $= \sqrt{(-1)^2+(5)^2+(-2)^2}$

$= \sqrt{30}$

b) $[-2,0,4]$



Magnitude $= \sqrt{(-2)^2+(0)^2+(4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

6) Find a and b such that $\vec{u} = [a,3,6]$ and $\vec{v} = [-8,12,b]$ are collinear.

x:

$a = -8k$

$a = -8(\frac{1}{4})$

$\boxed{a = -2}$

y:

$3 = 12k$

$k = \frac{1}{4}$

z:

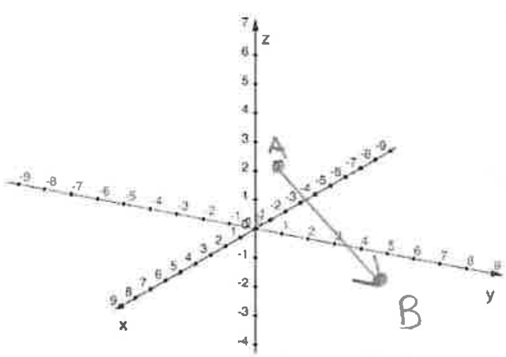
$6 = bk$

$6 = b(\frac{1}{4})$

$\boxed{b = 24}$

7) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Write the vector in the form $[x,y,z]$. Then determine the exact magnitude of the vector.

a) $A(2,1,3)$ and $B(5,7,1)$

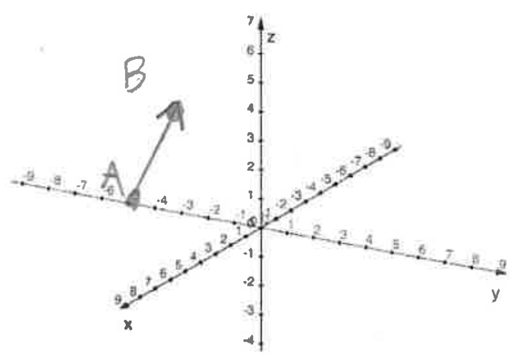


$\overrightarrow{AB} = [3,6,-2]$

$|\overrightarrow{AB}| = \sqrt{(3)^2+(6)^2+(-2)^2}$

$|\overrightarrow{AB}| = 7$

b) $A(3,-4,1)$ and $B(6,-1,5)$



$\overrightarrow{AB} = [3,3,4]$

$|\overrightarrow{AB}| = \sqrt{(3)^2+(3)^2+(4)^2}$

$|\overrightarrow{AB}| = \sqrt{34}$

8) Evaluate each given the vectors $\vec{a}$= [−2, 1, 8], $\vec{b}$= [3, 1, −2], and $\vec{c}$ = [2, −3, 4].

a) $3\vec{b}$

$= 3[3, 1, -2]$

$= [9, 3, -6]$

b) $\vec{b} - \vec{c}$

$= [3, 1, -2] - [2, -3, 4]$

$= [1, 4, -6]$

c) $2\vec{a} - 3\vec{c} + 4\vec{b}$

$= 2[-2, 1, 8] - 3[2, -3, 4] + 4[3, 1, -2]$

$= [-4, 2, 16] - [6, -9, 12] + [12, 4, -8]$

$= [-10, 11, 4] + [12, 4, -8]$

$= [2, 15, -4]$

d) $(\vec{a} + \vec{b}) - (\vec{a} + \vec{c})$

$= ([-2, 1, 8] + [3, 1, -2]) - ([-2, 1, 8] + [2, -3, 4])$

$= [1, 2, 6] - [0, -2, 12]$

$= [1, 4, -6]$

e) $\vec{b} \cdot \vec{c}$

$= 3(2) + 1(-3) + (-2)(4)$

$= 6 - 3 - 8$

$= -5$

f) $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$

$= -2(3) + 1(1) + 8(-2) - (-5)$

$= -6 + 1 - 16 + 5$

$= -16$

Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = 7\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = -2\hat{\imath} + 5\hat{\jmath} + 9\hat{k}$.

a) $(\vec{a} + \vec{b}) \cdot \vec{c}$

$= ([3, -2, 4] + [7, 4, -1]) \cdot [-2, 5, 9]$

$= [10, 2, 3] \cdot [-2, 5, 9]$

$= 10(-2) + 2(5) + 3(9)$

$= 17$

b) $2\vec{a} \cdot (4\vec{b} - 3\vec{c})$

$= 2[3, -2, 4] \cdot (4[7, 4, -1] - 3[-2, 5, 9])$

$= [6, -4, 8] \cdot [34, 1, -31]$

$= 6(34) + (-4)(1) + 8(-31)$

$= -48$

10) Determine the values of k such that $\vec{u}$ and $\vec{v}$ are orthogonal.

a) $\vec{u}$ = [2, k, −1] and $\vec{v}$ = [3, −2, 7]

$\vec{u} \cdot \vec{v} = 0$

$2(3) + k(-2) + (-1)(7) = 0$

$6 - 2k - 7 = 0$

$-2k = 1$

$\boxed{k = \frac{-1}{2}}$

b) $\vec{u}$ = [−3, 1, k] and $\vec{v}$ = [4, −k, k]

$\vec{u} \cdot \vec{v} = 0$

$-3(4) + 1(-k) + k(k) = 0$

$-12 - k + k^2 = 0$

$k^2 - k - 12 = 0$

$(k - 4)(k + 3) = 0$

$\boxed{k_1 = 4 \quad k_2 = -3}$

11) Find a vector orthogonal to each vector.

a) [2, −1, 7]

$[0, 7, 1]$

b) [8, −3, 4]

$[0, 4, 3]$

12) Consider the vectors $\vec{u}$ = [3, −5, 8] and $\vec{v}$ = [3, 1, −2].

a) Find $\vec{u} \cdot \vec{v}$.

$= 3(3) + (-5)(1) + 8(-2)$

$= -12$

b) Calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\cos\theta = \frac{-12}{(\sqrt{98})(\sqrt{14})}$$

$\theta \simeq 108.9°$

13) Determine the projection of $\vec{a}$ on $\vec{b}$.

a) $\vec{a}$ = [2, 1, −3] and $\vec{b}$ = [1, 7, 6]

$$proj_{\vec{b}}\vec{a} = \frac{2(1) + 1(7) + (-3)(6)}{(1)^2 + (7)^2 + (6)^2}[1, 7, 6]$$

$$= \frac{-9}{86}[1, 7, 6]$$

$$= \left[\frac{-9}{86}, \frac{-63}{86}, \frac{-27}{43}\right]$$

b) $\vec{a}$ = [3, 4, 7] and $\vec{b}$ = [2, −1, 1]

formula:

$$proj_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}(\vec{b})$$

$$proj_{\vec{b}}\vec{a} = \frac{3(2) + 4(-1) + 7(1)}{(2)^2 + (-1)^2 + (1)^2}[2, -1, 1]$$

$$= \frac{9}{6}[2, -1, 1]$$

$$= \left[3, -\frac{3}{2}, \frac{3}{2}\right]$$

14) The initial point of vector $\overrightarrow{CD} = [2, -9, 1]$ is $C(-3, 2, 2)$ determine the coordinates of D.

$x - (-3) = 2$

$x + 3 = 2$

$x = -1$

$y - 2 = -9$

$y = -7$

$z - 2 = 1$

$z = 3$

$$\boxed{D(-1, -7, 3)}$$

15) Find 2 unit vectors that are parallel to $\vec{a} = [9, -7, 2]$.

$\pm \frac{1}{|\vec{a}|} \vec{a}$

$|\vec{a}| = \sqrt{(9)^2 + (-7)^2 + (2)^2}$

$|\vec{a}| = \sqrt{134}$

The unit vectors are:

1) $\frac{1}{\sqrt{134}}[9, -7, 2] = \left[\frac{9}{\sqrt{134}}, \frac{-7}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right]$

2) $\frac{-1}{\sqrt{134}}[9, -7, 2] = \left[\frac{-9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, \frac{-2}{\sqrt{134}}\right]$

16) A triangle has vertices at the points $D = (3, -2, -3)$, $E(7, 0, 1)$ and $F(1, 2, 1)$. What type of triangle is ΔDEF? Explain.

$\vec{DE} = [4, 2, 4]$ $\quad |\vec{DE}| = 6$

$\vec{DF} = [-2, 4, 4]$ $\quad |\vec{DF}| = 6$

$\vec{EF} = [-6, 2, 0]$ $\quad |\vec{EF}| = \sqrt{40} = 2\sqrt{10}$

Isosceles

$\vec{DE} \cdot \vec{DF} = 4(-2) + 2(4) + 4(4) = 16$

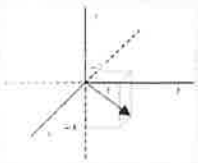
$\vec{DE} \cdot \vec{EF} = 4(-6) + 2(2) + 4(0) = -20$

$\vec{DF} \cdot \vec{EF} = -2(-6) + 4(2) + 4(0) = 20$

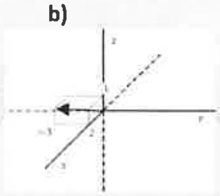
No right angles.

ANSWER KEY:

1. a)



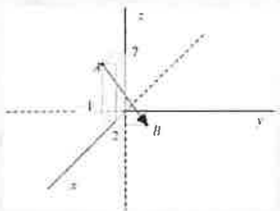
b)



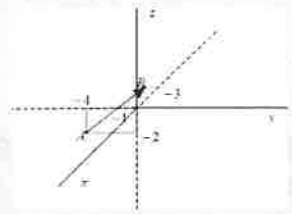
2. a) $2\hat{\imath} - \hat{\jmath} + 7\hat{k}$ **b)** $-4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

3. a) [3, –4, 5] **b)** [2, 0, 3] **c)** [–8, 9, –4] **d)** [0, –8, –7]

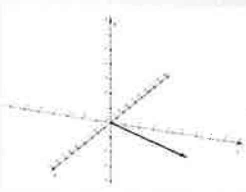
4. a) [–2, 3, –8]



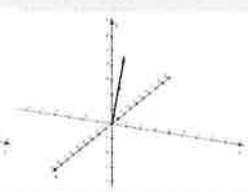
b) [–3, 3, 2]



5)a) $\sqrt{30}$



b) $2\sqrt{5}$



6) $a = -2, b = 24$

7)a) $[3,6,-2], 7$ **b)** $[3,3,4], \sqrt{34}$

8) a) [9, 3, –6] **b)** [1, 4, –6] **c)** [2, 15, –4] **d)** [1, 4, –6] **e)** –5 **f)** –16

9) a) 17 **b)** ~~[illegible]~~ -46

10)a) $k = -0.5$ **b)** $k = 4$, $k = -3$

11)a) [4, 8, 0] **b)** [1,0,–2]

12)a) –12 **b)** 108.9°

13)a) [–0.10, –0.73, –0.63] **b)** [3, –1.5, 1.5]

14) $D(-1, -7, 3)$

15) $[\frac{9}{\sqrt{134}}, -\frac{7}{\sqrt{134}}, \frac{2}{\sqrt{134}}]$ and $[-\frac{9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, -\frac{2}{\sqrt{134}}]$

16) This is a non-right isosceles triangle because 2 sides of the triangle are the same length but no 2 vectors that make up the sides of the triangle dot to 0, this tells us there are no perpendicular vectors and therefore no right angles.