W4 – Derivatives of Exponential Functions Unit 3
MCV4U
Jensen

solutions

1) Determine the derivative with respect to x for each function.

a) $g(x) = 4^x$

$g'(x) = 4^x(\ln 4)$

b) $f(x) = 11^x$

$f'(x) = 11^x[\ln(11)]$

c) $y = \left(\frac{1}{2}\right)^x$

$y' = \left(\frac{1}{2}\right)^x\left[\ln\left(\frac{1}{2}\right)\right]$

d) $N(x) = -3e^x$

$N'(x) = -3e^x$

e) $h(x) = e^x$

$h'(x) = e^x$

f) $y = \pi^x$

$y' = \pi^x[\ln(\pi)]$

2) Find the first, second, and third derivatives of the function $f(x) = e^x$

$f'(x) = f''(x) = f'''(x) = e^x$

3) Calculate the instantaneous rate of change of the function $y = 5^x$ when $x = 2$.

$y' = 5^x[\ln(5)]$

$y'(2) = 5^2[\ln(5)]$

$y'(2) \simeq 40.2$

4) Determine the slope of the graph of $y = \frac{1}{2}e^x$ at $x = 4$.

$y' = \frac{1}{2}e^x$

$y'(4) = \frac{1}{2}e^4$

$y'(4) \simeq 27.3$

5) Determine the equation of the line that is tangent to $y = 8^x$ at the point on the curve where $x = \frac{1}{2}$.

Point

$y(0.5) = 8^{1/2}$

$= 2\sqrt{2}$

$(0.5, 2\sqrt{2})$

Slope

$y' = 8^x[\ln(8)]$

$y'(0.5) = 8^{1/2}[\ln(8)]$

$y'(0.5) = 2\sqrt{2}[\ln(8)]$

$m = 2\sqrt{2}[\ln(8)]$

Eqn:

$y = mx + b$

$2\sqrt{2} = 2\sqrt{2}\ln(8)(\frac{1}{2}) + b$

$2\sqrt{2} = \sqrt{2}\ln(8) + b$

$b = 2\sqrt{2} - \sqrt{2}\ln(8)$

$b = \sqrt{2}[2 - \ln(8)]$

$y = 2\sqrt{2}\ln(8)x + \sqrt{2}[2 - \ln(8)]$

6) A fruit fly infestation is doubling every day. There are 10 flies when the infestation is first discovered.

a) Write an equation that relates the number of flies to time.

$A(t) = 10(2)^t$

b) Determine the number of flies present after 1 week.

$A(7) = 10(2)^7$

$A(7) = 1280$ flies

c) How fast is the fly population increasing after 1 week.

$A'(t) = 10(2)^t \ln(2)$

$A'(7) = 10(2)^7 \ln(2)$

$A'(7) = 887.2$ flies/day

d) How long will it take for the fly population to reach 500?

$500 = 10(2)^t$

$50 = 2^t$

$t = \log_2(50)$

$t \simeq 5.64$ days

e) How fast is the fly population increasing at this point?

$A'(5.64) = 10(2)^{5.64}\ln(2)$

$A'(5.64) \simeq 346$ flies/day

7) Refer to question 6. At which point is the fly population increasing at a rate of

i) 20 flies per day?

$20 = 10(2)^t \ln(2)$

$\frac{2}{\ln(2)} = 2^t$

$\log_2\left[\frac{2}{\ln(2)}\right] = t$

$t \approx 1.53$ days

ii) 2000 flies per day?

$2000 = 10(2)^t \ln(2)$

$\frac{200}{\ln(2)} = 2^t$

$t = \log_2\left[\frac{200}{\ln(2)}\right]$

$t \approx 8.17$ days

8) Determine the equation of the line perpendicular to the tangent line to the function $f(x) = \frac{1}{2}e^x$ at the point on the curve where $x = \ln 3$

Point:

$f(\ln 3) = \frac{1}{2}e^{\ln 3}$

$= \frac{1}{2}(3)$

$= \frac{3}{2}$

$(\ln 3, 1.5)$

Slope

$f'(x) = \frac{1}{2}e^x$

$f'(\ln 3) = \frac{3}{2}$

$m = \frac{3}{2}$

$\perp m = -\frac{2}{3}$

Eqn

$y = mx + b$

$\frac{3}{2} = -\frac{2}{3}\ln 3 + b$

$\frac{3}{2} + \frac{2}{3}\ln 3 = b$

$\boxed{y = -\frac{2}{3}x + \frac{3}{2} + \frac{2}{3}\ln 3}$

Answers:

1)a) $g'(x) = 4^x(\ln 4)$ **b)** $f'(x) = 11^x(\ln 11)$ **c)** $y' = \left(\frac{1}{2}\right)^x\left(\ln\frac{1}{2}\right)$ **d)** $N'(x) = -3e^x$ **e)** $h'(x) = e^x$ **f)** $y' = \pi^x(\ln \pi)$
2) $f'(x) = f''(x) = f'''(x) = e^x$
3) 4.2
4) 27.3
5) $y = 6\sqrt{2}(\ln 2)x + \sqrt{2}(2 - 3\ln 2)$
6)a) $N(t) = 10(2)^t$ **b)** 1280 **c)** 887 flies/day **d)** 5.64 days **e)** 346 flies/day
7)i) 1.53 days **ii)** 8.17 days
8) $y = -\frac{2}{3}x + \frac{2}{3}\ln 3 + \frac{3}{2}$