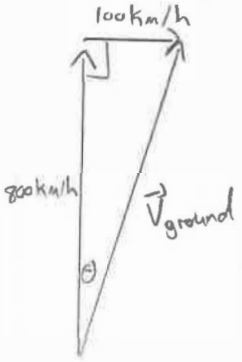
W4 – Applications of Vector Addition
MCV4U
Jensen

Unit 4

1) The velocity of an airplane is 800 km/h north. A wind is blowing due east at 100 km/h. Determine the velocity of the airplane relative to the ground.



$$|\vec{V}_{ground}|^2 = 800^2 + 100^2$$

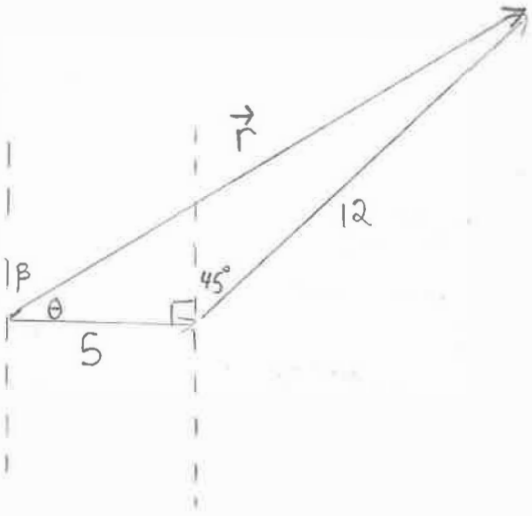
$$|\vec{V}_{ground}| \simeq 806.2 \text{ km/h}$$

$$\tan\theta = \frac{100}{800}$$

$$\theta \simeq 7.1^\circ$$

The ground velocity is 806.2 km/h at a quadrant bearing of $N7.1^\circ E$

2) A particle is displaced 5 units to the East and then displaced 12units in a direction $N45°E$. Find the magnitude and direction of the resultant displacement.



$$|\vec{r}|^2 = 5^2 + 12^2 - 2(5)(12)\cos(135^\circ)$$

$$|\vec{r}| \simeq 15.93 \text{ units}$$

$$\cos\theta = \frac{12^2 - 5^2 - 15.93^2}{-2(5)(15.93)}$$

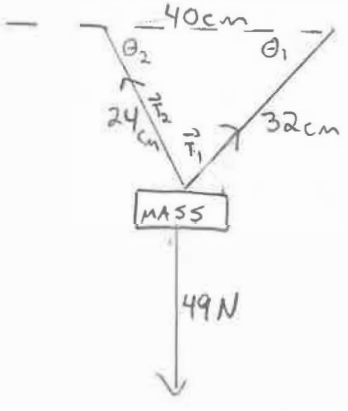
$$\theta \simeq 32.2^\circ$$

$$\beta = 90 - 32.2^\circ$$

$$\beta \simeq 57.8^\circ$$

The resultant is 15.93 units at a quadrant bearing of $N57.8°E$.

3) A mass of 5 kg is suspended by two strings, 24 cm and 32 cm long, from two points that are 40 cm apart and at the same level. Determine the tension in each of the strings.



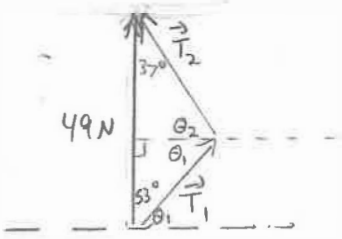
$$\cos\theta_1 = \frac{24^2 - 40^2 - 32^2}{-2(40)(32)}$$

$$\theta_1 \simeq 36.87^\circ$$

$$\cos\theta_2 = \frac{32^2 - 24^2 - 40^2}{-2(24)(40)}$$

$$\theta_2 \simeq 53.13^\circ$$

$$\theta_1 + \theta_2 = 90^\circ$$



$$\cos 53.13 = \frac{\vec{T}_1}{49}$$

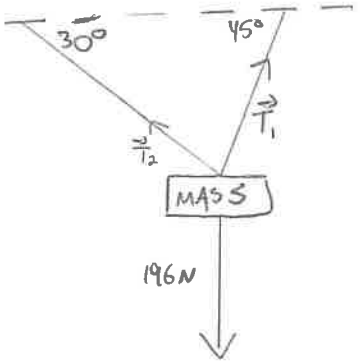
$$\vec{T}_1 \simeq 29.4 \text{ N (32cm string)}$$

$$\sin 53.13 = \frac{\vec{T}_2}{49}$$

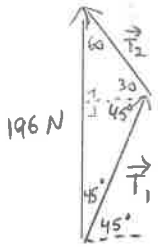
$$\vec{T}_2 \simeq 39.20 \text{ N (24cm string)}$$

4) A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 30° and 45° with the ceiling. Determine the tension in each rope.

Position:



Vector:



$$\frac{196}{\sin 75} = \frac{|\vec{T}_1|}{\sin 60} = \frac{|\vec{T}_2|}{\sin 45}$$

$|\vec{T}_1| \simeq 175.73\ N$ (45° rope)

$|\vec{T}_2| \simeq 143.48\ N$ (30° rope)

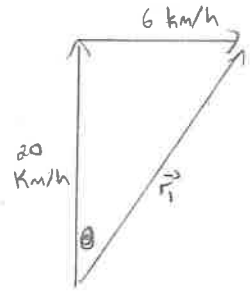
5) A river is 2 km wide and flows at 6 km/h. Anna is driving a motorboat, which has a speed of 20 km/h in still water and she heads out from one bank in a direction perpendicular to the current. A marina lies directly across the river from the starting point on the opposite bank.

a) How far downstream from the marine will the current push the boat?

How long will it take for the boat to cross the river?

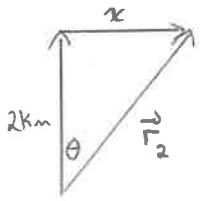
If Anna decides that she wants to end up directly across the river at the marina, in what direction should she head? What is the resultant velocity of the boat?

a)



$\tan\theta = \frac{6}{20}$

$\theta \simeq 16.7°$



$\tan 16.7 = \frac{x}{2}$

$x \simeq 0.6\ km$

b)

$\cos 16.7 = \frac{20}{|\vec{r}_1|}$

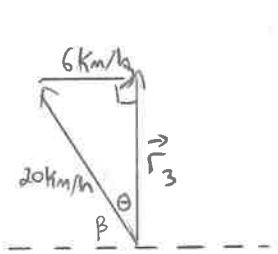
$|\vec{r}_1| \simeq 20.9\ km/h$

$\cos 16.7 = \frac{2}{|\vec{r}_2|}$

$|\vec{r}_2| \simeq 2.1\ km$

Time to cross $= \frac{2.1\ km}{20.9\ km/h} \simeq 0.1$ hours or 6 minutes

c)



$\sin\theta = \frac{6}{20}$

$\theta \simeq 17.5°$

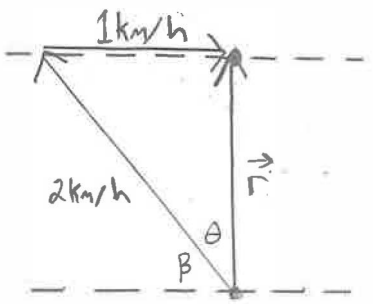
$\beta \simeq 72.5°$

$|\vec{r}_3|^2 = 20^2 - 6^2$

$|r_3| \simeq 19.1\ km/h$

The boat needs to head out at an angle of 72.5° with the shore. The resultant velocity is 19.1 km/h.

6) Adam can swim at the rate of 2 km/h in still water. At what angle to the bank of a river must he head if he wants to swim directly across the river and the current in the river move at the rate of 1 km/h.



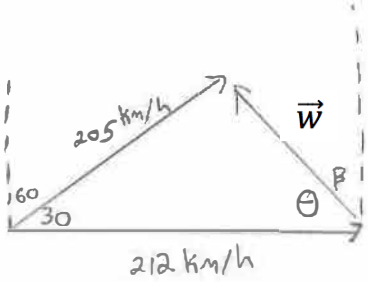
$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\beta = 60°$

60° to the bank of the river.

7) An airplane is travelling $N60°E$ with a resultant ground speed of 205 km/h. The nose of the plane is actually pointing east with an airspeed of 212 km/h. Find the wind speed and direction.



$|\vec{w}|^2 = 205^2 + 212^2 - 2(205)(212)\cos(30)$

$|\vec{w}| \simeq 108.1$ km/h

$\cos\theta = \dfrac{205^2 - 212^2 - 108.1^2}{-2(212)(108.1)}$

$\theta \simeq 71.4°$

$\beta \simeq 18.6°$

The wind speed is 108.1 km/h at N 18.6° W

ANSWER KEY:

1. 806 km/h N 7.1° E **2.** 15.93 units N 57.8 E **3.** 24 cm string: 39.2 N, 32 cm string: 29.4 N

4. 45° rope: 175.73 N 30° rope: 143.48 N **5. a)** 0.6 downstream from the marina **b)** 6 minutes (0.1 hours) **c)** upstream 17.5°, resultant velocity: 19.08 km/h **6.** 30° upstream **7.** 108 km/h $N18.4°W$