

W4 – Limits

MHF4U

Jensen

SOLUTIONS

1) Evaluate each limit

$$\text{a) } \lim_{x \rightarrow 2} \frac{3x}{x^2+2} = \frac{3(2)}{(2)^2+2} = \frac{6}{6} = 1$$

$$\text{b) } \lim_{x \rightarrow -1} (x^4 + x^3 + x^2) = (-1)^4 + (-1)^3 + (-1)^2 = 1 - 1 + 1 = 1$$

$$\text{c) } \lim_{x \rightarrow 9} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = \left(\sqrt{9} + \frac{1}{\sqrt{9}} \right)^2 = \left(3 + \frac{1}{3} \right)^2 = \left(\frac{10}{3} \right)^2 = \frac{100}{9}$$

2) Evaluate the limit of each

$$\text{a) } \lim_{x \rightarrow 2} \frac{4-x^2}{2-x} = \lim_{x \rightarrow 2} \frac{(2-x)(2+x)}{2-x} = 2+2 = 4$$

$$\text{b) } \lim_{x \rightarrow -1} \frac{2x^2+5x+3}{x+1} = \lim_{x \rightarrow -1} \frac{(x+1)(2x+3)}{x+1} = 2(-1)+3 = 1$$

$$\text{c) } \lim_{x \rightarrow 3} \frac{x^3-27}{x-3} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2+3x+9)}{x-3} = (3)^2 + 3(3) + 9 = 27$$

$$\text{d) } \lim_{x \rightarrow 4} \frac{16-x^2}{x^3+64} = \lim_{x \rightarrow 4} \frac{(4-x)(4+x)}{(x+4)(x^2-4x+16)} = \frac{4-4}{(4)^2 - 4(4) + 16} = \frac{0}{16} = 0$$

$$\text{e) } \lim_{x \rightarrow 4} \frac{x^2-16}{x^2-5x+6} = \lim_{x \rightarrow 4} \frac{(x-4)(x+4)}{(x-2)(x-3)} = \frac{(4-4)(4+4)}{(4-2)(4-3)} = \frac{0(8)}{2(1)} = 0$$

$$\text{f) } \lim_{x \rightarrow -1} \frac{x^2+x}{x+1} = \lim_{x \rightarrow -1} \frac{x(x+1)}{x+1} = -1$$

3) Complete the following table and use results to estimate $\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x-2}{x^2-x-2}$	0.3448	0.3844	0.3334	0.3332	0.3322	0.3226

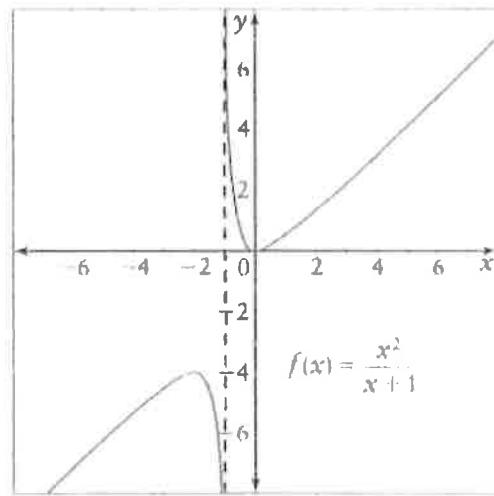
$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2} \approx \frac{1}{3}$$

4) Use the graph to find the following limits:

a) $\lim_{x \rightarrow -1^+} \frac{x^2}{x+1} = \infty$

b) $\lim_{x \rightarrow -1^-} \frac{x^2}{x+1} = -\infty$

c) $\lim_{x \rightarrow -1} \frac{x^2}{x+1}$ Does not exist



5) Use the graph to determine the following limits

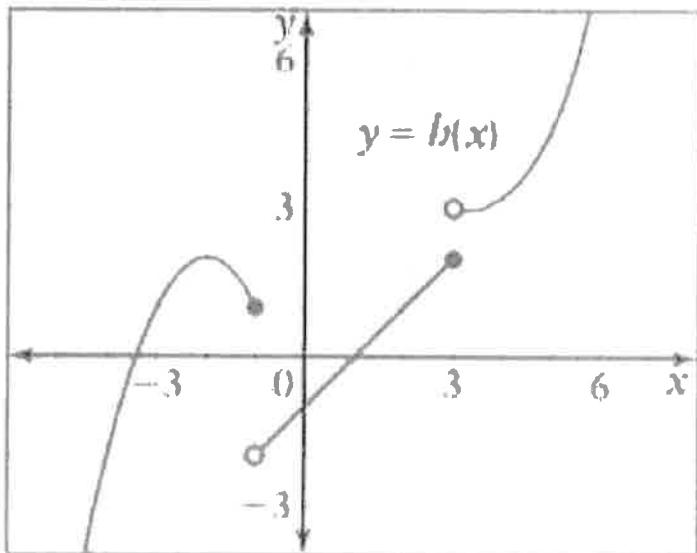
a) $\lim_{x \rightarrow -1^+} h(x) = -2$

b) $\lim_{x \rightarrow -1^-} h(x) = 1$

c) $\lim_{x \rightarrow -1} h(x)$ Does not exist

d) $\lim_{x \rightarrow 3^+} h(x) = 3$

e) $\lim_{x \rightarrow 3^-} h(x) = 2$



f) $\lim_{x \rightarrow 3} h(x)$ Does not exist

Answer Key

4) a) 1 b) 1 c) $\frac{100}{9}$ 2) a) 4 b) 1 c) 27 d) 0 e) 0 f) -1 3) $\frac{1}{3}$ 4) a) ∞ b) $-\infty$ c) does not exist

5) a) -2 b) 1 c) does not exist d) 3 e) 2 f) does not exist

