W4 – The Quotient Rule Unit 1

MCV4U

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1) Use the quotient rule to differentiate each function

a) $h(x) = \frac{x}{x+1}$

$h'(x) = \frac{1(x+1) - 1(x)}{(x+1)^2}$

$h'(x) = \frac{x+1-x}{(x+1)^2}$

$h'(x) = \frac{1}{(x+1)^2}$

b) $h(t) = \frac{2t-3}{t+5}$

$h'(t) = \frac{2(t+5) - 1(2t-3)}{(t+5)^2}$

$h'(t) = \frac{2t+10-2t+3}{(t+5)^2}$

$h'(t) = \frac{13}{(t+5)^2}$

c) $h(x) = \frac{x^3}{2x^2-1}$

$h'(x) = \frac{3x^2(2x^2-1) - 4x(x^3)}{(2x^2-1)^2}$

$h'(x) = \frac{6x^4 - 3x^2 - 4x^4}{(2x^2-1)^2}$

$h'(x) = \frac{2x^4 - 3x^2}{(2x^2-1)^2}$

d) $h(x) = \frac{1}{x^2+3}$

$h'(x) = \frac{0(x^2+3) - 2x(1)}{(x^2+3)^2}$

$h'(x) = \frac{-2x}{(x^2+3)^2}$

e) $y = \frac{x(3x+5)}{1-x^2} = \frac{3x^2+5x}{1-x^2}$

$y' = \frac{(6x+5)(1-x^2) - (-2x)(3x^2+5x)}{(1-x^2)^2}$

$y' = \frac{6x - 6x^3 + 5 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2}$

$y' = \frac{5x^2+6x+5}{(1-x^2)^2}$

f) $y = \frac{x^2-x+1}{x^2+3}$

$y' = \frac{(2x-1)(x^2+3) - (2x)(x^2-x+1)}{(x^2+3)^2}$

$y' = \frac{2x^3 + 6x - x^2 - 3 - 2x^3 + 2x^2 - 2x}{(x^2+3)^2}$

$y' = \frac{x^2+4x-3}{(x^2+3)^2}$

2) Determine $\frac{dy}{dx}$ at each given value of x.

a) $y = \frac{3x+2}{x+5}$ at $x = -3$

$$\frac{dy}{dx} = \frac{3(x+5) - 1(3x+2)}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{3x+15-3x-2}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{13}{(x+5)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=-3} = \frac{13}{(-3+5)^2} = \frac{13}{4}$$

b) $y = \frac{x^3}{x^2+9}$ at $x = 1$

$$\frac{dy}{dx} = \frac{3x^2(x^2+9) - 2x(x^3)}{(x^2+9)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3(1)^2[(1)^2+9] - 2(1)[(1)^3]}{[(1)^2+9]^2}$$

$$= \frac{30-2}{100}$$

$$= \frac{7}{25}$$

3) Find the point(s) at which the tangent to the curve is horizontal.

a) $y = \frac{2x^2}{x-4}$

$$y' = \frac{4x(x-4) - 1(2x^2)}{(x-4)^2}$$

$$y' = \frac{4x^2 - 16x - 2x^2}{(x-4)^2}$$

$$0 = 2x^2 - 16x$$

$$0 = 2x(x-8)$$

$x_1 = 0$, $y_1 = \frac{2(0)^2}{0-4}$, $y_1 = 0$, $(0, 0)$

$x_2 = 8$, $y_2 = \frac{2(8)^2}{8-4}$, $y_2 = 32$, $(8, 32)$

b) $y = \frac{x^2-1}{x^2+x-2}$

Note: could have simplified first

$y = \frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$; $x \neq -2, 1$

VA hole

$$y' = \frac{2x(x^2+x-2) - (2x+1)(x^2-1)}{(x^2+x-2)^2}$$

$$y' = \frac{2x^3+2x^2-4x - (2x^3-2x+x^2-1)}{(x^2+x-2)^2}$$

$$y' = \frac{x^2-2x+1}{(x^2+x-2)^2}$$

$$y' = \frac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}(x+2)^2}$$

$$0 = \frac{1}{(x+2)^2}$$

$\therefore$ No Solutions

Note: hole in graph when $x = 1$

4) Determine the equation of the tangent to the curve $y = \frac{x^2-1}{3x}$ at $x = 2$.

$$y' = \frac{2x(3x) - 3(x^2-1)}{(3x)^2}$$

$$y' = \frac{6x^2 - 3x^2 + 3}{9x^2}$$

$$y' = \frac{3x^2+3}{9x^2}$$

$$y' = \frac{x^2+1}{3x^2}$$

Slope:

$$y'(2) = \frac{(2)^2+1}{3(2)^2}$$

$$= \frac{5}{12}$$

Point:

$$y(2) = \frac{(2)^2-1}{3(2)}$$

$$y(2) = \frac{1}{2}$$

Eq^n:

$$y = mx + b$$

$$\frac{1}{2} = \frac{5}{12}(2) + b$$

$$\frac{1}{2} = \frac{5}{6} + b$$

$$\frac{3}{6} - \frac{5}{6} = b$$

$$-\frac{1}{3} = b$$

$$\boxed{y = \frac{5}{12}x - \frac{1}{3}}$$

Answers:

1a) $h'(x) = \frac{1}{(x+1)^2}$ **b)** $h'(t) = \frac{13}{(t+5)^2}$ **c)** $h'(x) = \frac{2x^4-3x^2}{(2x^2-1)^2}$ **d)** $h'(x) = \frac{-2x}{(x^2+3)^2}$ **e)** $y' = \frac{5x^2+6x+5}{(1-x^2)^2}$ **f)** $\frac{dy}{dx} = \frac{x^2+4x-3}{(x^2+3)^2}$

2)a) $\frac{13}{4}$ **b)** $\frac{7}{25}$

3)a) (0,0) and (8,32) **b)** no horizontal tangents

4) $y = \frac{5}{12}x - \frac{1}{3}$