W4 – Rational Functions Unit 2

MCV4U

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SOLUTIONS

1) Find the equation of any asymptotes for the following functions. Then, find the one-sided limits approaching the vertical asymptotes.

a) $f(x) = \frac{x+3}{x^2-4} = \frac{x+3}{(x-2)(x+2)}$

HA: $y=0$

VA: $x=2, x=-2$

Tests: $f(1.99) \simeq -125$

$f(2.01) \simeq 125$

$f(-1.99) \simeq -25$

$f(-2.01) \simeq 25$

Limits: $\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to -2^-} f(x) = \infty$

$\lim_{x\to -2^+} f(x) = -\infty$

b) $y = \frac{x^2}{x^2-3x+2} = \frac{x^2}{(x-2)(x-1)}$

HA: $y=1$

VA: $x=2, x=1$

Tests: $y(1.99) \simeq -400$

$y(2.01) \simeq 400$

$y(0.99) \simeq 97$

$y(1.01) \simeq -103$

Limits: $\lim_{x\to 2^+} y = \infty$

$\lim_{x\to 2^-} y = -\infty$

$\lim_{x\to 1^+} y = -\infty$

$\lim_{x\to 1^-} y = \infty$

c) $y = 2x + \frac{1}{x} = \frac{2x^2+1}{x}$

S.A: $y=2x$

VA: $x=0$

Tests: $y(-0.01) \simeq -100$

$y(0.01) \simeq 100$

Limits:

$\lim_{x\to 0^+} y = \infty$

$\lim_{x\to 0^-} y = -\infty$

d) $g(x) = \frac{2x-3}{x^2-6x+9} = \frac{2x-3}{(x-3)^2}$

HA: $y=0$

VA: $x=3$

Tests: $g(2.99) = 29800$

$g(3.01) = 30200$

Limits:

$\lim_{x\to 3^+} g(x) = \infty$

$\lim_{x\to 3^-} g(x) = \infty$

2) Find the derivative of each function. Then, determine whether the function has any local extrema.

a) $f(x) = \frac{2}{x+3}$

$$f'(x) = \frac{0(x+3) - 1(2)}{(x+3)^2}$$

$$f'(x) = \frac{-2}{(x+3)^2}$$

No critical points since $f'(x) \neq 0$ and $x=-3$ is not in the domain of $f(x)$

∴ no local extrema.

b) $h(x) = \frac{-3}{(x-2)^2}$

$$h'(x) = \frac{0(x-2)^2 - 2(x-2)(1)(-3)}{[(x-2)^2]^2}$$

$$h'(x) = \frac{6(x-2)}{(x-2)^4}$$

$$h'(x) = \frac{6}{(x-2)^3}$$

$h'(x) \neq 0$ and $x=2$ is not in the domain of $h(x)$.

∴ no critical points and no local extrema.

3) Consider the function $f(x) = \frac{-2}{(x+1)^2}$ VA: $x=-1$

a) Find the intervals of increase and decrease for $f(x)$.

$$f'(x) = \frac{0(x+1)^2 - 2(x+1)(1)(-2)}{(x+1)^4}$$

$$f'(x) = \frac{4(x+1)}{(x+1)^4}$$

$$f'(x) = \frac{4}{(x+1)^3}$$

	$-\infty$ to -1	-1 to ∞
Test	-2	0
$f'(x)$	$-$	$+$
$f(x)$	dec. ↘	inc. ↗

increasing: $x > -1$

decreasing: $x < -1$

No critical points.

Only use VA as a dividing point when testing.

b) Find the intervals of concavity for $f(x)$.

$$f''(x) = \frac{0(x+1)^3 - 3(x+1)^2(1)(4)}{(x+1)^6}$$

$$f''(x) = \frac{-12(x+1)^2}{(x+1)^6}$$

$$f''(x) = \frac{-12}{(x+1)^4}$$

$f''(x) \neq 0$

only possible change in concavity is at the VA

	$-\infty$ to -1	-1 to ∞
Test	-2	0
$f''(x)$	$-$	$-$
$f(x)$	con. down ⌒	con. down ⌒

concave down: $x<-1$, $x>-1$

concave up: never

4) Consider the function $h(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

a) Write the equations of the asymptotes

HA: $y = 0$

VA: $x = 2$, $x = -2$

b) Make a table showing the increasing and decreasing intervals for the function

$h'(x) = \dfrac{0(x^2-4) - 2x(1)}{(x-2)^2(x+2)^2}$

$h'(x) = \dfrac{-2x}{(x-2)^2(x+2)^2}$

$0 = -2x$

$x = 0$

$h(0) = -0.25$

critical point $(0, -0.25)$

Test:

$-\infty \quad -2 \quad 0 \quad 2 \quad \infty$

Test	-3	-1	1	3
$f'(x)$	+	+	−	−
$F(x)$	inc. ↗	inc. ↗	dec. ↘	dec. ↘

increasing: $x < -2$, $-2 < x < 0$

decreasing: $0 < x < 2$, $x > 2$

c) How can you use the table from part b) to determine the behavior of $f(x)$ near the vertical asymptotes?

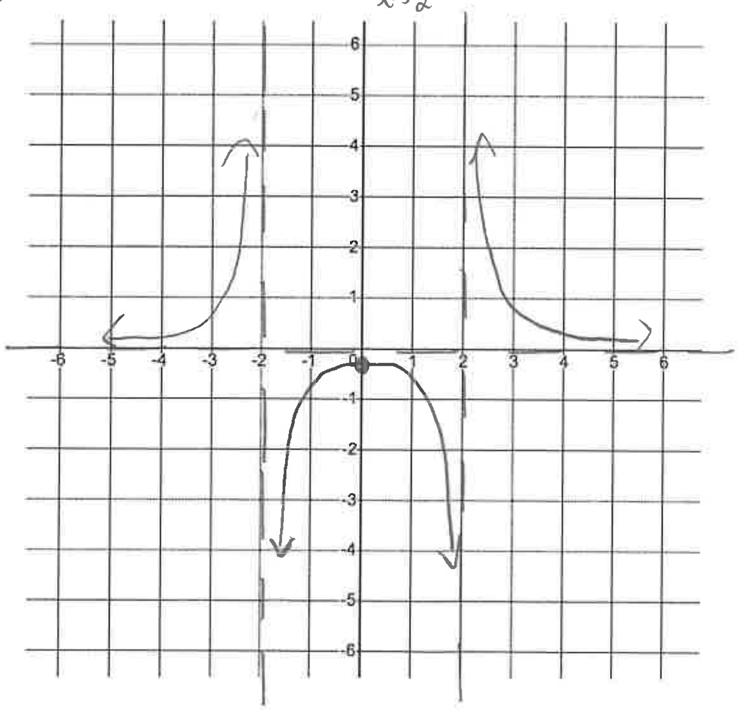
$h(x)$ is increasing to the left of $x = -2$; $\lim_{x \to -2^-} h(x) = \infty$

$h(x)$ is increasing to the right of $x = -2$; $\lim_{x \to 2^+} h(x) = -\infty$

$h(x)$ is decreasing to the left of $x = 2$; $\lim_{x \to 2^-} h(x) = -\infty$

$h(x)$ is decreasing to the right of $x = 2$; $\lim_{x \to 2^+} h(x) = \infty$

d) Sketch a graph of the function.



Answers:

1)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to -2^+} = -\infty$, $\lim_{x\to -2^-} = \infty$

b) VA: $x = 1$ and $x = 2$; HA: $y = 1$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to 1^+} = -\infty$, $\lim_{x\to 1^-} = \infty$

c) VA: $x = 0$; SA: $y = 2x$; $\lim_{x\to 0^+} = \infty$, $\lim_{x\to 0^-} = -\infty$

d) VA: $x = 3$; HA: $y = 0$; $\lim_{x\to 3^+} = \infty$, $\lim_{x\to 3^-} = \infty$

2)a) $f'(x) = \frac{-2}{(x+3)^2}$; no local extrema **b)** $h'(x) = \frac{6}{(x-2)^3}$; no local extrema

3)a) decreasing when $x < -1$, increasing when $x > -1$ **b)** concave down when $x < -1$ or $x > -1$

4)a) VA: $x = 2$ and $x = -2$; HA: $y = 0$
b) increasing when $x < -2$ or $-2 < x < 0$; decreasing when $0 < x < 2$ or $x > 2$
c) Since the curve is increasing to the left of $x = -2$, $\lim_{x\to -2^-} = \infty$
Since the curve is increasing to the right of $x = -2$, $\lim_{x\to -2^+} = -\infty$
Since the curve is decreasing to the left of $x = 2$, $\lim_{x\to 2^-} = -\infty$
Since the curve is decreasing to the right of $x = 2$, $\lim_{x\to 2^+} = \infty$

d)