**W4 – 7.1/7.2 – Solving Exponential Equations**
**MHF4U**
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**1)** Write each expression with base 2.

**a)** $4^6$

$= (2^2)^6$

$= 2^{12}$

**b)** $8^3$

$= (2^3)^3$

$= 2^9$

**c)** $\left(\frac{1}{8}\right)^2$

$= \left(\frac{1}{2^3}\right)^2$

$= (2^{-3})^2$

$= 2^{-6}$

**d)** $14$

$2^x = 14$

$x = \log_2(14)$

$\boxed{2^{\log_2(14)}}$

**2)** Write each expression as a power of 4.

**a)** $(\sqrt{16})^3$

$= 4^3$

**b)** $\sqrt[3]{16}$

$= (16)^{1/3}$

$= (4^2)^{1/3}$

$= 4^{2/3}$

**c)** $\sqrt{64} \times (\sqrt[4]{128})^3$

$= (64)^{1/2} \times (128)^{3/4}$

$= (4^3)^{1/2} \times \left[(4^{1/2})^7\right]^{3/4}$

$= 4^{3/2} \times 4^{21/8}$

$= 4^{12/8} \times 4^{21/8}$

$= 4^{33/8}$

**3)** Solve each equation

**a)** $2^{4x} = 4^{x+3}$

$2^{4x} = (2^2)^{x+3}$

$2^{4x} = 2^{2x+6}$

$4x = 2x + 6$

$2x = 6$

$\boxed{x = 3}$

**b)** $3^{w+1} = 9^{w-1}$

$3^{w+1} = (3^2)^{w-1}$

$3^{w+1} = 3^{2w-2}$

$w + 1 = 2w - 2$

$\boxed{3 = w}$

**c)** $4^{3x} = 8^{x-3}$

$(2^2)^{3x} = (2^3)^{x-3}$

$2^{6x} = 2^{3x-9}$

$6x = 3x - 9$

$3x = -9$

$\boxed{x = -3}$

**d)** $125^{2y-1} = 25^{y+4}$

$(5^3)^{2y-1} = (5^2)^{y+4}$

$5^{6y-3} = 5^{2y+8}$

$6y - 3 = 2y + 8$

$4y = 11$

$\boxed{y = \frac{11}{4}}$

**4)** Consider the equation $10^{2x} = 100^{2x-5}$

**a)** Solve this equation by expressing both sides as powers of a common base.

$10^{2x} = (10^2)^{2x-5}$

$10^{2x} = 10^{4x-10}$

$2x = 4x - 10$

$10 = 2x$

$\boxed{x = 5}$

**b)** Solve the same equation by taking the common logarithm of both sides.

$\log(10^{2x}) = \log(100^{2x-5})$

$2x\log(10) = (2x-5)\log(100)$

$2x(1) = (2x-5)(2)$

$2x = 4x - 10$

$10 = 2x$

$\boxed{x = 5}$

**5)** Solve $2^{3x} > 4^{x+1}$

$2^{3x} > (2^2)^{x+1}$

$2^{3x} > 2^{2x+2}$

$3x > 2x + 2$

$x > 2$

**6)** Solve for $t$. Round answers to 2 decimal places.

**a)** $2 = 1.07^t$

$t = \log_{1.07} 2$

$t \simeq 10.24$

**b)** $100 = 10(1.04)^t$

$10 = 1.04^t$

$t = \log_{1.04}(10)$

$t \simeq 58.71$

**c)** $15 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\frac{t}{4} = \log_{1/2}(15)$

$t = 4\log_{1/2}(15)$

$t \simeq -15.63$

**7)** Solve each equation. Round answers to 3 decimal places.

**a)** $2^x = 3^{x-1}$

$\log(2^x) = \log(3^{x-1})$

$x\log(2) = (x-1)\log(3)$

$x\log(2) = x\log(3) - \log(3)$

$\log(3) = x\log(3) - x\log(2)$

$\log(3) = x[\log(3) - \log(2)]$

$\frac{\log(3)}{\log(\frac{3}{2})} = x$

$x = \log_{3/2}(3)$

$x \simeq 2.71$

**b)** $5^{x-2} = 4^x$

$\log(5^{x-2}) = \log(4^x)$

$(x-2)\log(5) = x\log(4)$

$x\log(5) - 2\log(5) = x\log(4)$

$x\log(5) - x\log(4) = 2\log(5)$

$x[\log(5) - \log(4)] = \log(25)$

$x = \frac{\log(25)}{\log(\frac{5}{4})}$

$x = \log_{5/4}(25)$

$x \simeq 14.425$

**c)** $7^{2x+1} = 4^{x-2}$

$\log(7^{2x+1}) = \log(4^{x-2})$

$(2x+1)\log(7) = (x-2)\log(4)$

$2x\log(7) + \log(7) = x\log(4) - 2\log(4)$

$\log(7) + 2\log(4) = x\log(4) - 2x\log(7)$

$\log(7) + \log(16) = x[\log(4) - 2\log(7)]$

$\log(112) = x\log(\frac{4}{49})$

$x = \frac{\log(112)}{\log(\frac{4}{49})}$

$x = \log_{4/49}(112)$

$x \simeq -1.883$

**8)** Solve $2^{2x} + 2^x - 6 = 0$ using the quadratic formula (or by factoring). Clearly identify any extraneous roots.

$(2^x)^2 + (2^x) - 6 = 0$

Let $k = 2^x$

$k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$k = -3 \qquad k = 2$

$2^x = -3 \qquad 2^x = 2^1$

$x = \log_2(-3) \qquad \boxed{x = 1}$

No Real solution

**9)** Solve $8^{2x} - 2(8^x) - 5 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$(8^x)^2 - 2(8^x) - 5 = 0$

let $k = 8^x$

$k^2 - 2k - 5 = 0$

$k = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$

$k = \frac{2 \pm \sqrt{24}}{2}$

$k = \frac{2 \pm 2\sqrt{6}}{2}$

$k = \frac{\cancel{2}(1 \pm \sqrt{6})}{\cancel{2}}$

$k = 1 \pm \sqrt{6}$

$k = 1 + \sqrt{6}$

$8^x = 1 + \sqrt{6}$

$x = \log_8(1 + \sqrt{6})$

$\boxed{x \simeq 0.595}$

$k = 1 - \sqrt{6}$

$8^x = 1 - \sqrt{6}$

$x = \log_8(1 - \sqrt{6})$

NRS

**10)** Use the decay equation for polonium-218, $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{3.1}}$, $A$ is the amount remaining after $t$ minutes and $A_0$ is the initial amount.

**a)** How much will remain after 90 seconds from an initial sample of 50 mg?

$A(1.5) = 50\left(\frac{1}{2}\right)^{1.5/3.1}$

$\simeq 35.75$ mg

**b)** How long will it take for this sample to decay to 10% of its initial amount of 50 mg?

$5 = 50\left(\frac{1}{2}\right)^{t/3.1}$

$0.1 = (0.5)^{t/3.1}$

$\frac{t}{3.1} = \log_{0.5}(0.1)$

$t = 3.1 \log_{0.5}(0.1)$

$t \simeq 10.3$ minutes

**11)** A 20-mg sample of thorium-233 decays to 17 mg after 5 minutes.

**a)** What is the half-life of thorium-233?

$17 = 20\left(\frac{1}{2}\right)^{5/H}$

$0.85 = 0.5^{5/H}$

$\frac{5}{H} = \log_{0.5}(0.85)$

$H = \frac{5}{\log_{0.5}(0.85)}$

$H \simeq 21.33$ minutes

**b)** How long will it take this sample to decay to 1 mg?

$1 = 20\left(\frac{1}{2}\right)^{t/21.33}$

$0.05 = 0.5^{t/21.33}$

$\frac{t}{21.33} = \log_{0.5}(0.05)$

$t = 21.33 \log_{0.5}(0.05)$

$t \simeq 92.2$ minutes

<u>**ANSWER KEY**</u>

**1)a)** $2^{12}$ **b)** $2^9$ **c)** $2^{-6}$ **d)** $2^{\frac{\log 14}{\log 2}}$ **2)a)** $4^3$ **b)** $4^{\frac{2}{3}}$ **c)** $4^{\frac{33}{8}}$ **3)a)** 3 **b)** 3 **c)** -3 **d)** $\frac{11}{4}$

**4)a)** 5 **b)** 5 **5)** $x > 2$ **6)a)** 10.24 **b)** 58.71 **c)** -15.63 **7)a)** 2.710 **b)** 14.425 **c)** -1.883

**8)** $x = 1$ is the only solution; $2^x = -3$ or $x = \frac{\log(-3)}{\log 2}$ is an extraneous root

**9)** $x = \frac{\log(1+\sqrt{6})}{\log 8} \cong 0.6$ is the only solution; $8^x = 1 - \sqrt{6}$ or $x = \frac{\log(1-\sqrt{6})}{\log 8}$ is an extraneous root

**10)a)** 35.75 mg **b)** 10.3 min **11)a)** 21.3 min **b)** 92.06 min