W5 – 5.4 Solve Linear Trigonometric Equations
MHF4U
Jensen

SOLUTIONS

1) Determine approximate solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

$\sin x = \frac{1}{4}$

$x_1 = \sin^{-1}\left(\frac{1}{4}\right)$

≈ 0.25

$x_2 = \pi - 0.25$

≈ 2.89

$\sin 0.25 = \sin 2.89 = \frac{1}{4}$

b) $\cos x + 0.75 = 0$

$\cos x = -0.75$

$x_1 = \cos^{-1}(-0.75)$

≈ 2.42

$\beta = \pi - 2.42$

$= 0.72$

$x_2 = \pi + \beta$

$= \pi + 0.72$

$= 3.86$

$\cos 0.72 = \cos 3.86 = -0.75$

c) $\tan x - 5 = 0$

$\tan x = 5$

$x_1 = \tan^{-1}(5)$

$= 1.37$

$x_2 = \pi + 1.37$

$= 4.51$

$\tan 1.37 = \tan 4.51 = 5$

d) $\sec x - 4 = 0$

$\sec x = 4$

$\cos x = \frac{1}{4}$

$x_1 = \cos^{-1}\left(\frac{1}{4}\right)$

$= 1.32$

$x_2 = 2\pi - 1.32$

$x_2 = 4.96$

$\sec 1.32 = \sec 4.96 = 4$

e) $3 \cot x + 2 = 0$

$3 \cot x = -2$

$\cot x = -\frac{2}{3}$

$\tan x = -\frac{3}{2}$

$x_1 = \tan^{-1}\left(-\frac{3}{2}\right)$

$= -0.98 + 2\pi$

$= 5.3$

$x_2 = \pi - 0.98$

$= 2.16$

$\cot 5.3 = \cot 2.16 = -\frac{2}{3}$

f) $2 \csc x + 5 = 0$

$2 \csc x = -5$

$\csc x = -\frac{5}{2}$

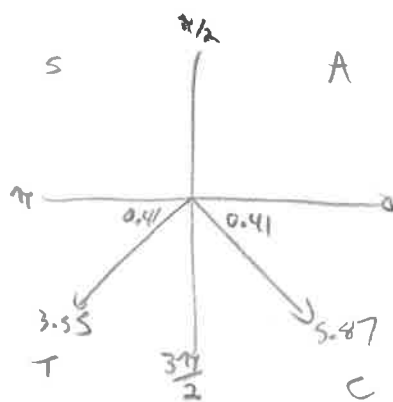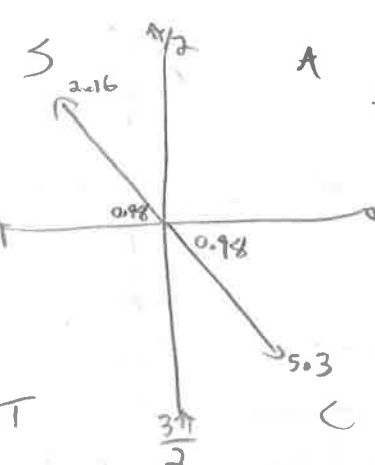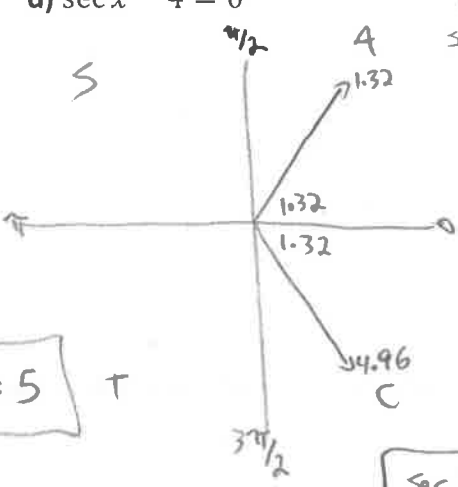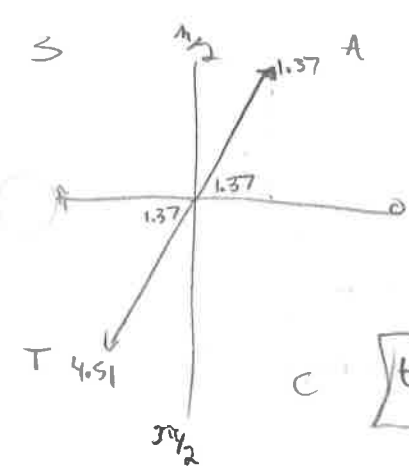$\sin x = -\frac{2}{5}$

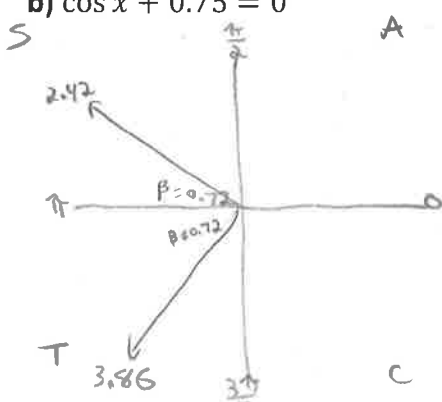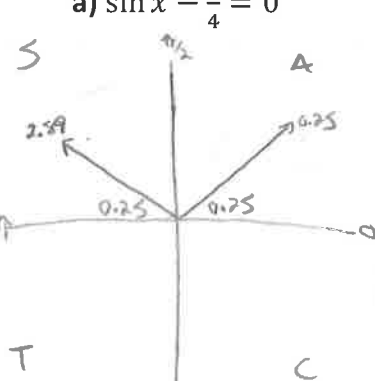$x_1 = \sin^{-1}\left(-\frac{2}{5}\right)$

$= -0.41 + 2\pi$

$= 5.87$

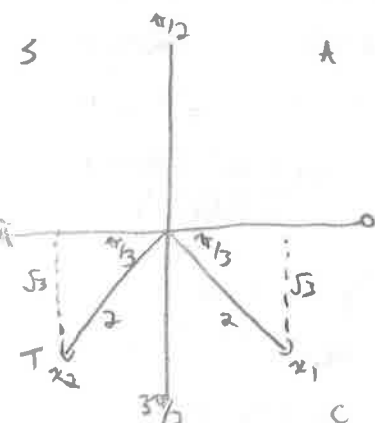$x_2 = \pi + 0.41$

$= 3.55$

$\csc 5.87 = \csc 3.55 = -\frac{5}{2}$

2) Determine exact solutions for each equation in the interval $0 \le x \le 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$ $\qquad \sin x = -\frac{\sqrt{3}}{2}$

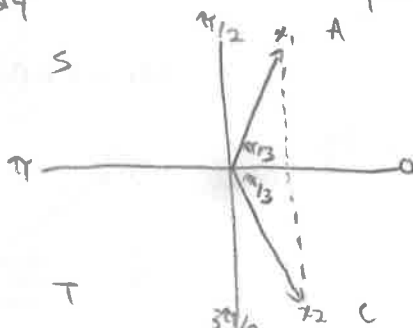from special $\triangle$: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3 + Q4

$x_1 = 2\pi - \frac{\pi}{3}$

$\boxed{x_1 = \frac{5\pi}{3}}$

$x_2 = \pi + \frac{\pi}{3}$

$\boxed{x_2 = \frac{4\pi}{3}}$

b) $\cos x - 0.5 = 0$ $\qquad \cos x = \frac{1}{2}$

from special $\triangle$; $\cos\frac{\pi}{3} = \frac{1}{2}$

place $\frac{\pi}{3}$ in Q1 + Q4

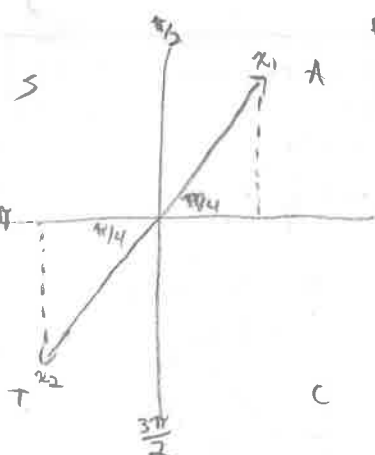$\boxed{x_1 = \frac{\pi}{3}}$

$x_2 = 2\pi - \frac{\pi}{3}$

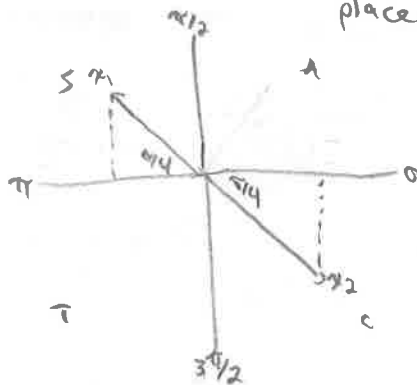$\boxed{x_2 = \frac{5\pi}{3}}$

c) $\tan x - 1 = 0$

$\tan x = 1$

from special $\triangle$; $\tan\frac{\pi}{4} = 1$

place in Q1 + Q3

$\boxed{x_1 = \frac{\pi}{4}}$

$x_2 = \pi + \frac{\pi}{4}$

$\boxed{x_2 = \frac{5\pi}{4}}$

d) $\cot x + 1 = 0$

$\cot x = -1$

$\tan x = -1$

from special $\triangle$; $\tan\frac{\pi}{4} = 1$

place $\frac{\pi}{4}$ in Q2 + Q4

$x_1 = \pi - \frac{\pi}{4}$

$\boxed{x_1 = \frac{3\pi}{4}}$

$x_2 = 2\pi - \frac{\pi}{4}$

$\boxed{x_2 = \frac{7\pi}{4}}$

3) Determine approximate solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian.

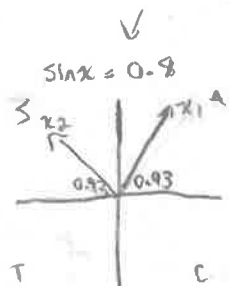**a)** $\sin^2 x - 0.64 = 0$

$\sin^2 x = 0.64$

$\sin x = \pm\sqrt{0.64}$

$\sin x = \pm 0.8$

$\sin x = 0.8 \qquad\qquad \sin x = -0.8$
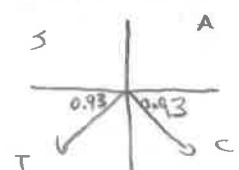
$x_1 = \sin^{-1}(0.8)$

$\boxed{x_1 = 0.93}$

$x_2 = \pi - 0.93$

$\boxed{x_2 = 2.21}$

$x_3 = \sin^{-1}(-0.8)$

$x_3 = -0.93 + 2\pi$

$\boxed{x_3 = 5.36}$

$x_4 = \pi + 0.93$

$\boxed{x_4 = 4.07}$

b) $\cos^2 x - \frac{4}{9} = 0$

$\cos^2 x = \frac{4}{9}$

$\cos x = \pm\sqrt{\frac{4}{9}}$

$\cos x = \pm\frac{2}{3}$

$\cos x = \frac{2}{3} \qquad\qquad \cos x = -\frac{2}{3}$
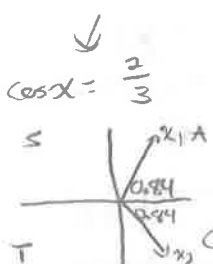
$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$

$\boxed{x_1 = 0.84}$

$x_2 = 2\pi - 0.84$

$\boxed{x_2 = 5.44}$
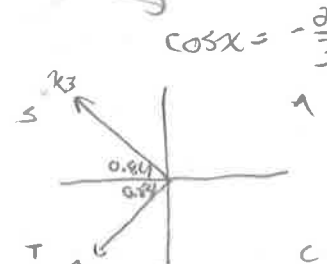
$x_3 = \cos^{-1}\left(-\frac{2}{3}\right)$

$\boxed{x_3 = 2.30}$

$x_4 = \pi + 0.84$

$\boxed{x_4 = 3.98}$

c) $\tan^2 x - 1.44 = 0$

$$\tan^2 x = 1.44$$
$$\tan x = \pm \sqrt{1.44}$$
$$\tan x = \pm 1.2$$

$\tan x = 1.2$

$x_1 = \tan^{-1}(1.2)$

$\boxed{x_1 = 0.88}$

$x_2 = \pi + 0.88$

$\boxed{x_2 = 4.02}$

$\tan x = -1.2$
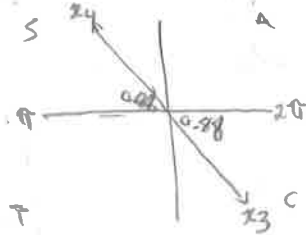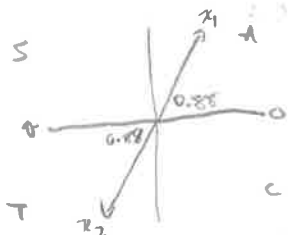
$x_3 = \tan^{-1}(-1.2)$

$x_3 = -0.88 + 2\pi$

$\boxed{x_3 = 5.4}$

$x_4 = \pi - 0.88$

$\boxed{x_4 = 2.26}$

d) $\sec^2 x - 2.5 = 0$

$$\sec^2 x = 2.5$$
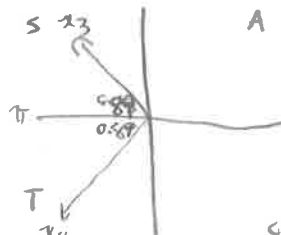$$\cos^2 x = \frac{1}{2.5}$$
$$\cos x = \pm \sqrt{\frac{1}{2.5}}$$
$$\cos x = \pm 0.63$$

$\cos x = 0.63$

$x_1 = \cos^{-1}(0.63)$

$\boxed{x_1 = 0.89}$

$x_2 = 2\pi - 0.89$

$\boxed{x_2 = 5.39}$

$\cos x = -0.63$

$x_3 = \cos^{-1}(-0.63)$

$\boxed{x_3 = 2.25}$

$x_4 = \pi + 0.89$

$\boxed{x_4 = 4.03}$

Determine exact solutions for each equation in the interval $0 \le x \le 2\pi$.

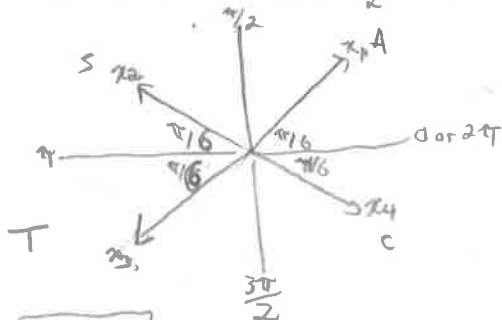**a)** $\sin^2 x - \frac{1}{4} = 0$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm \sqrt{\frac{1}{4}}$

$\sin x = \pm \frac{1}{2}$

from special $\triangle$; $\sin \frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2 for $\sin x = \frac{1}{2}$
Place in Q3 + Q4 for $\sin x = -\frac{1}{2}$

$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = \pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{5\pi}{6}}$

$x_3 = \pi + \frac{\pi}{6}$

$\boxed{x_3 = \frac{7\pi}{6}}$

$x_4 = 2\pi - \frac{\pi}{6}$

$\boxed{x_4 = \frac{11\pi}{6}}$

b) $\cos^2 x - \frac{3}{4} = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm \sqrt{\frac{3}{4}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

from special $\triangle$; $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{6}$ in Q1 + Q4 for $\cos x = \frac{\sqrt{3}}{2}$
Place $\frac{\pi}{6}$ in Q2 + Q3 for $\cos x = -\frac{\sqrt{3}}{2}$

$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{11\pi}{6}}$

$x_3 = \pi - \frac{\pi}{6}$
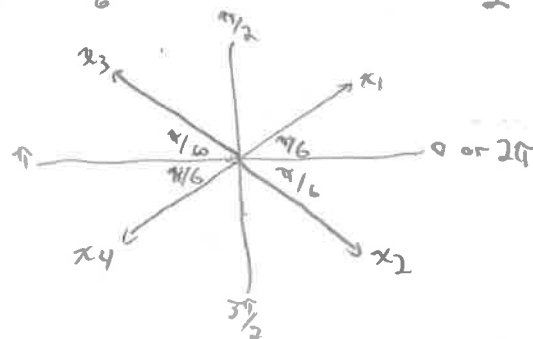
$\boxed{x_3 = \frac{5\pi}{6}}$
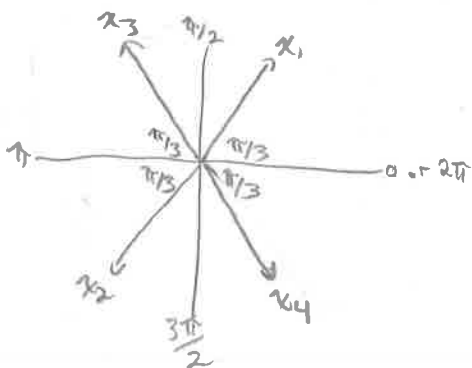
$x_4 = \pi + \frac{\pi}{6}$

$\boxed{x_4 = \frac{7\pi}{6}}$

c) $\tan^2 x - 3 = 0$ $\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

From special $\triangle$; $\tan\frac{\pi}{3} = \sqrt{3}$

place $\frac{\pi}{3}$ in Q1 + Q3 for $\tan x = \sqrt{3}$

place $\frac{\pi}{3}$ in Q2 + Q4 for $\tan x = -\sqrt{3}$

$\boxed{x_1 = \frac{\pi}{3}}$ $x_3 = \pi - \frac{\pi}{3}$

$x_2 = \pi + \frac{\pi}{3}$ $\boxed{x_3 = \frac{2\pi}{3}}$

$\boxed{x_2 = \frac{4\pi}{3}}$ $x_4 = 2\pi - \frac{\pi}{3}$

$\boxed{x_4 = \frac{5\pi}{3}}$

d) $3\csc^2 x - 4 = 0$ $\csc^2 x = \frac{4}{3}$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

From special $\triangle$; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q1 + Q2 for $\sin x = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3 + Q4 for $\sin x = -\frac{\sqrt{3}}{2}$

$\boxed{x_1 = \frac{\pi}{3} \qquad x_3 = \frac{4\pi}{3} \\ x_2 = \frac{2\pi}{3} \qquad x_4 = \frac{5\pi}{3}}$

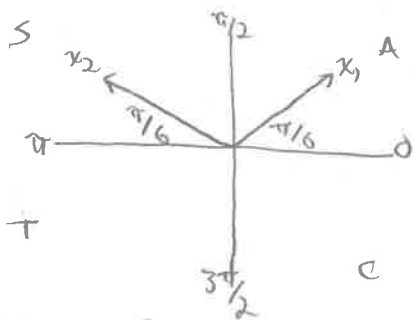**5)** Determine solutions for each equation in the interval $0 \le x \le 2\pi$.

a) $3\sin x = \sin x + 1$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

From special $\triangle$; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2

$\boxed{x_1 = \frac{\pi}{6}}$ $x_2 = \pi - \frac{\pi}{6}$
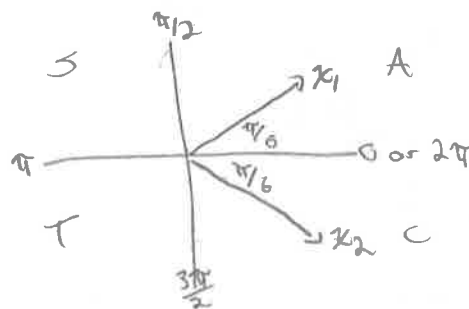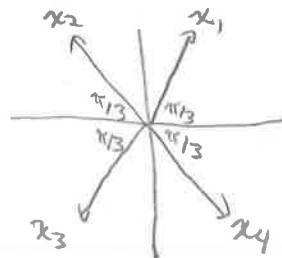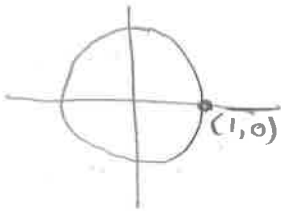
$\boxed{x_2 = \frac{5\pi}{6}}$

b) $5\cos x - \sqrt{3} = 3\cos x$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

From special $\triangle$; $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place in Q1 + Q4

$\boxed{x_1 = \frac{\pi}{6}}$ $x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{11\pi}{6}}$

c) $7 \sec x = 7$ $\sec x = 1$

$\cos x = 1$

use unit circle
where each point is $(\cos x, \sin x)$
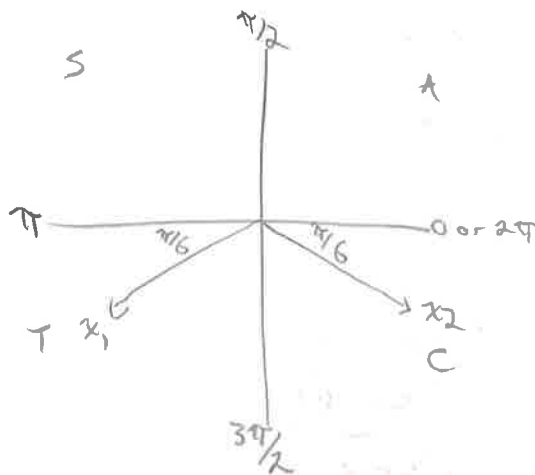
$(1,0)$

$x_1 = 0$

$x_2 = 2\pi$

d) $2 \csc x + 17 = 15 + \csc x$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

from special $\triangle$; $\sin \frac{\pi}{6} = \frac{1}{2}$

Place $\frac{\pi}{6}$ in Q3 + Q4

$\pi/2$

S   A

π 0 or 2π

$\pi/6$ $\pi/6$

T x_1 x_2

C

$3\pi/2$

$x_1 = \pi + \frac{\pi}{6}$ $x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_1 = \frac{7\pi}{6}}$ $\boxed{x_2 = \frac{11\pi}{6}}$