

W5 - 7.4 - Solving Logarithmic Equations

MHF4U

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1) Find the roots of each equation

a) $2 = \log(x + 25)$

$$10^2 = x + 25$$

$$100 = x + 25$$

$$x = 75$$

b) $1 - \log(w - 7) = 0$

$$1 = \log(w - 7)$$

$$10^1 = w - 7$$

$$10 = w - 7$$

$$w = 17$$

c) $6 - 3 \log(2n) = 0$

$$6 = 3 \log(2n)$$

$$2 = \log(2n)$$

$$10^2 = 2n$$

$$100 = 2n$$

$$n = 50$$

2) Solve

a) $5 = \log_2(2x - 10)$

$$2^5 = 2x - 10$$

$$32 = 2x - 10$$

$$42 = 2x$$

$$x = 21$$

b) $9 = \log_5(x + 100) + 6$

$$3 = \log_5(x + 100)$$

$$125 = x + 100$$

$$x = 25$$

c) $\log_3(n^2 - 3n + 5) = 2$

$$3^2 = n^2 - 3n + 5$$

$$9 = n^2 - 3n + 5$$

$$0 = n^2 - 3n - 4$$

$$0 = (n - 4)(n + 1)$$

$$n = 4$$

$$n = -1$$

3) Solve. Make sure to reject any extraneous roots.

a) $\log x + \log(x - 4) = 1$

$$\log[x(x - 4)] = 1$$

$$10^1 = x(x - 4)$$

$$10 = x^2 - 4x$$

$$0 = x^2 - 4x - 10$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{56}}{2}$$

$$x = \frac{4 \pm 2\sqrt{14}}{2}$$

$$x = \frac{2(2 \pm \sqrt{14})}{2}$$

$$x = 2 \pm \sqrt{14}$$

Reject $2 - \sqrt{14}$

$$x = 2 + \sqrt{14}$$

b) $\log x^3 - \log 2 = \log(2x^2)$

$$\log x^3 - \log(2x^2) = \log 2$$

$$\log\left(\frac{x^3}{2x^2}\right) = \log 2$$

$$\log\left(\frac{x}{2}\right) = \log 2$$

$$\frac{x}{2} = 2$$

$$x = 4$$

c) $\log(v-1) = 2 + \log(v-16)$

$$\log(v-1) - \log(v-16) = 2$$

$$\log\left(\frac{v-1}{v-16}\right) = 2$$

$$10^2 = \frac{v-1}{v-16}$$

$$100(v-16) = v-1$$

$$100v - 1600 = v - 1$$

$$99v = 1599$$

$$v = \frac{533}{33}$$

4) Solve. Check for extraneous roots.

a) $\log\sqrt{x^2-3x} = \frac{1}{2}$

$$\log(x^2-3x)^{1/2} = \frac{1}{2}$$

$$\frac{1}{2} \log(x^2-3x) = \frac{1}{2}$$

$$\log(x^2-3x) = 1$$

$$10^1 = x^2-3x$$

$$0 = x^2-3x-10$$

$$0 = (x-5)(x+2)$$

$$x = 5 \text{ or } x = -2$$

5) Solve. Check for extraneous roots.

a) $\log_2(x+5) - \log_2(2x) = 8$

$$\log_2\left(\frac{x+5}{2x}\right) = 8$$

$$2^8 = \frac{x+5}{2x}$$

$$256 = \frac{x+5}{2x}$$

$$512x = x+5$$

$$511x = 5$$

$$x = \frac{5}{511}$$

ANSWER KEY

1)a) 75 b) 17 c) 50 2)a) 21 b) 25 c) 4, -1 3)a) $2 + \sqrt{14}$ b) 4 c) $\frac{533}{33}$ d) 3

4)a) 5, -2 b) -50, 2 5)a) $\frac{5}{511}$ b) $\frac{1}{2}$

d) $\log(k+2) + \log(k-1) = 1$

$$\log[(k+2)(k-1)] = 1$$

$$10^1 = (k+2)(k-1)$$

$$10 = k^2 - k + 2k - 2$$

$$0 = k^2 + k - 12$$

$$0 = (k+4)(k-3)$$

$$k = -4$$

Reject

$$k = 3$$

b) $\log\sqrt{x^2+48x} = 1$

$$\log(x^2+48x)^{1/2} = 1$$

$$\frac{1}{2} \log(x^2+48x) = 1$$

$$\log(x^2+48x) = 2$$

$$10^2 = x^2+48x$$

$$0 = x^2+48x-100$$

$$0 = (x+50)(x-2)$$

$$x = -50$$

$$x = 2$$

b) $\log(2k+4) = 1 + \log k$

$$\log(2k+4) - \log(k) = 1$$

$$\log\left(\frac{2k+4}{k}\right) = 1$$

$$10^1 = \frac{2k+4}{k}$$

$$10k = 2k+4$$

$$8k = 4$$

$$k = \frac{1}{2}$$