W5 – The Chain Rule **Unit 1**

MCV4U

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SOLUTIONS

1) Differentiate using the chain rule.

a) $f(x) = (-4x^2)^2$

$f'(x) = 2(-4x^2)(-8x)$

$f'(x) = 64x^3$

b) $f(x) = (16x^2)^{\frac{3}{4}}$

$f'(x) = \frac{3}{4}(16x^2)^{-1/4}(32x)$

$f'(x) = \frac{96x}{4(16x^2)^{1/4}}$

$f'(x) = \frac{96x}{8x^{1/2}}$

$f'(x) = 12\sqrt{x}$

c) $y = (4x + 1)^2$

$y' = 2(4x+1)(4)$

$y' = 8(4x+1)$

OR $y' = 32x+8$

d) $y = (x^3 - x)^{-3}$

$y' = -3(x^3-x)^{-4}(3x^2-1)$

$y' = -\frac{3(3x^2-1)}{(x^3-x)^4}$

OR $y' = \frac{-3(3x^2-1)}{[x(x^2-1)]^4} = \frac{-3(3x^2-1)}{x^4(x^2-1)^4}$

e) $y = \sqrt{2x - 3x^5}$

$y = (2x-3x^5)^{1/2}$

$y' = \frac{1}{2}(2x-3x^5)^{-1/2}(2-15x^4)$

$y^2 = \frac{2-15x^4}{2(2x-3x^5)^{1/2}}$

f) $y = \sqrt[5]{2 + 3x^2 - x^3}$

$y = (2+3x^2-x^3)^{1/5}$

$y' = \frac{1}{5}(2+3x^2-x^3)^{-4/5}(6x-3x^2)$

$y' = \frac{3x(2-x)}{5(2+3x^2-x^3)^{4/5}}$

2) Determine $f'(1)$.

a) $f(x) = (4x^2 - x + 1)^2$

$f'(x) = 2(4x^2 - x+1)(8x-1)$

$f'(1) = 2[4(1)^2 - 1+1)[8(1)-1]$

$f'(1) = 2(4)(7)$

$f'(1) = 56$

b) $f(x) = \frac{5}{\sqrt[3]{2x-x^2}}$

$f'(x) = \frac{0(\sqrt[3]{2x-x^2}) - \frac{1}{3}(2x-x^2)^{-2/3}(2-2x)(5)}{[(2x-x^2)^{1/3}]^2}$

$f'(x) = \frac{-5(2-2x)}{3(2x-x^2)^{2/3}(2x-x^2)^{2/3}}$

$f'(x) = \frac{-5(2-2x)}{3(2x-x^2)^{4/3}}$

$f'(1) = \frac{-5[2-2(1)]}{3[2(1)-(1)^2]^{4/3}}$

$f'(1) = 0$

3) Determine an equation for the tangent to the curve $y = (x^3 - 4x^2)^3$ at $x = 3$

$y' = 3(x^3 - 4x^2)^2(3x^2 - 8x)$

Slope:

$y'(3) = 3[(3)^3 - 4(3)^2]^2[3(3)^2 - 8(3)]$

$y'(3) = 3(-9)^2(3)$

$y'(3) = 729$

Point:

$y(3) = [(3)^3 - 4(3)^2]^3$

$y(3) = (-9)^3$

$y(3) = -729$

Eqⁿ:

$y = mx + b$

$-729 = 729(3) + b$

$b = -2916$

$\boxed{y = 729x - 2916}$

4) Determine the point(s) on the curve $y = x^2(x^3 - x)^2$ where the tangent line is horizontal.

$$y' = 2x(x^3-x)^2 + 2(x^3-x)(3x^2-1)(x^2)$$
$$y' = 2x(x^3-x)\left[(x^3-x) + x(3x^2-1)\right]$$
$$y' = 2x(x^3-x)(4x^3-2x)$$
$$y' = 2x(x)(x^2-1)(2x)(2x^2-1)$$
$$y' = 4x^3(x-1)(x+1)(2x^2-1)$$
$$0 = 4x^3(x-1)(x+1)(2x^2-1)$$

$0 = 4x^3$ $\quad 0 = x-1$ $\quad 0 = x+1$ $\quad 0 = 2x^2-1$

$x_1 = 0$ $\quad x_2 = 1$ $\quad x_3 = -1$ $\quad x^2 = \frac{1}{2}$

$y_1 = 0$ $\quad y_2 = 0$ $\quad y_3 = 0$

$(0,0)$ $\quad (1,0)$ $\quad (-1,0)$ $\quad x_4 = \frac{1}{\sqrt{2}}$ $\quad x_5 = -\frac{1}{\sqrt{2}}$

$y_4 = \frac{1}{16}$ $\quad y_5 = \frac{1}{16}$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$ $\quad \left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5) Differentiate each of the following.

a) $f(x) = (x+4)^3(x-3)^6$

$$f'(x) = 3(x+4)^2(1)(x-3)^6 + 6(x-3)^5(1)(x+4)^3$$
$$f'(x) = 3(x+4)^2(x-3)^5\left[(x-3) + 2(x+4)\right]$$
$$f'(x) = 3(x+4)^2(x-3)^5(3x+5)$$

b) $y = \left(\frac{x^2-3}{x^2+3}\right)^4$

$$y' = 4\left(\frac{x^2-3}{x^2+3}\right)^3\left[\frac{2x(x^2+3) - 2x(x^2-3)}{(x^2+3)^2}\right]$$
$$y' = \frac{4(x^2-3)^3(2x)\left[(x^2+3)-(x^2-3)\right]}{(x^2+3)^3(x^2+3)^2}$$
$$y' = \frac{8x(x^2-3)^3(6)}{(x^2+3)^5}$$
$$y' = \frac{48x(x^2-3)^3}{(x^2+3)^5}$$

Answers:

1a) $f'(x) = 64x^3$ **b)** $f'(x) = \frac{-24x}{(-16x^2)^{\frac{1}{4}}}$ **c)** $y' = 8(4x+1)$ **d)** $y' = \frac{-3(3x^2-1)}{x^4(x^2-1)^4}$ **e)** $\frac{dy}{dx} = \frac{2-15x^4}{2(2x-3x^5)^{\frac{1}{2}}}$ **f)** $\frac{dy}{dx} = \frac{6x-3x^2}{5(2+3x^2-x^3)^{\frac{4}{5}}}$

2)a) 56 **b)** 0

3) $y = 729x - 291$

4) $(-1,0)$, $(1,0)$, $(0,0)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, and $\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5)a) $(x+4)^2(x-3)^5(9x+15)$ **b)** $\frac{48x(x^2-3)^3}{(x^2+3)^5}$