W6 – Applications of Dot and Cross Product Unit 5

MCV4U

Jensen

SOLUTIONS

1) Given $\vec{a} = [2, 4, -5]$, $\vec{b} = [-1, 3, 7]$, and $\vec{c} = [-2, 7, 3]$, evaluate each expression.

a) $\vec{a} \times \vec{b} \cdot \vec{c}$

$\vec{a} \times \vec{b} = [4(7) - (-5)(3), -5(-1) - 2(7), 2(3) - 4(-1)]$

$= [43, -9, 10]$

$\vec{a} \times \vec{b} \cdot \vec{c} = [43, -9, 10] \cdot [-2, 7, 3]$

$= 43(-2) + (-9)(7) + 10(3)$

$= -119$

b) $\vec{a} \times \vec{c} \cdot \vec{b}$

$\vec{a} \times \vec{c} = [4(3) - (-5)(7), -5(-2) - 2(3), 2(7) - 4(-2)]$

$= [47, 4, 22]$

$\vec{a} \times \vec{c} \cdot \vec{b} = [47, 4, 22] \cdot [-1, 3, 7]$

$= 47(-1) + 4(3) + 22(7)$

$= 119$

2) Determine the projection, and its magnitude of $\vec{u}$ on $\vec{v}$.

a) $\vec{u} = [2, 1, 7]$, $\vec{v} = [-7, 2, 6]$

$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} (\vec{v})$

$= \frac{2(-7) + 1(2) + 7(6)}{(-7)^2 + (2)^2 + (6)^2} [-7, 2, 6]$

$= \frac{30}{89} [-7, 2, 6]$

$|\text{proj}_{\vec{v}} \vec{u}| = \frac{30}{89} \sqrt{89} = \frac{30}{\sqrt{89}}$

b) $\vec{u} = 7\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$, $\vec{v} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$= [7, -6, 5]$ $= [3, -2, 1]$

$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} (\vec{v})$

$= \frac{7(3) + (-6)(-2) + 5(1)}{(3)^2 + (-2)^2 + (1)^2} [3, -2, 1]$

$= \frac{38}{14} [3, -2, 1]$

$|\text{proj}_{\vec{v}} \vec{u}| = \frac{38}{14} \sqrt{14} = \frac{38}{\sqrt{14}}$

3) Determine the work done in the direction of travel.

a) $\vec{F} = [200, 150, 75]$, $\vec{s} = [2, -1, 8]$

$W = \vec{F} \cdot \vec{s}$

$W = 200(2) + 150(-1) + 75(8)$

$W = 850 \text{ J}$.

b) $\vec{F} = -3\hat{\imath} + 9\hat{\jmath} + 5\hat{k}$, $\vec{s} = 2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$

$= [-3, 9, 5]$ $= [2, 5, 3]$

$W = \vec{F} \cdot \vec{s}$

$W = -3(2) + 9(5) + 5(3)$

$W = 54 \text{ J}$.

4) Find the area of the parallelogram with sides consisting of the vectors.

a) $\vec{a} = [-4, 5, -8]$, $\vec{b} = [1, -2, 3]$

$\vec{a} \times \vec{b} = [5(3) - (-8)(-2), -8(1) - (-4)(3), -4(-2) - 5(1)]$

$= [-1, 4, 3]$

$\text{Area} = |\vec{a} \times \vec{b}| = \sqrt{26}$ units^2

b) $\vec{a} = [9, -5, 7]$, $\vec{b} = [3, -2, 5]$

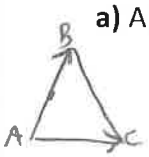
$\vec{a} \times \vec{b} = [-5(5) - 7(-2), 7(3) - 9(5), 9(-2) - (-5)(3)]$

$= [-11, -24, -3]$

$\text{Area} = |\vec{a} \times \vec{b}| = \sqrt{706}$ units^2

5) Find the area of the triangle with the given vertices.

a) A(0, 2, 4), B(3, –2, 1), C(4, –2, 5)



$\vec{AB} = [3, -4, -3]$ $\quad AC = [4, -4, 1]$

$\vec{AB} \times \vec{AC} = [-4(1) - (-3)(-4), -3(4) - 3(1), 3(-4) - (-4)(4)]$

$= [-16, -15, 4]$

$\text{Area} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{497}$ units^2

b) A(–2, 4, 5), B(1, 4, 2), C(7, 4, 9)

$\vec{AB} = [3, 0, -3]$ $\quad \vec{AC} = [9, 0, 4]$

$\vec{AB} \times \vec{AC} = [0(4) - (-3)(0), -3(9) - 3(4), 3(0) - 0(9)]$

$= [0, -39, 0]$

$\text{Area} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}(39)$

$= \frac{39}{2}$ units^2

6) Determine the volume of the parallelepiped determined by the vectors.

a) $\vec{a} = [2, 5, -8]$, $\vec{b} = [7, -2, 3]$, and $\vec{c} = [8, 2, -1]$

$\vec{a} \times \vec{b} = [5(3) - (-8)(-2), -8(7) - 2(3), 2(-2) - 5(7)]$

$= [-1, -62, -39]$

$\text{Volume} = |\vec{a} \times \vec{b} \cdot \vec{c}|$

$= |[-1, -62, -39] \cdot [8, 2, -1]|$

$= |-1(8) + (-62)(2) + (-39)(-1)|$

$= |-93|$

$= 93$ units^3

b) $\vec{a} = [1, -5, 9]$, $\vec{b} = [3, 4, -7]$, and $\vec{c} = [1, 0, 2]$

$\vec{a} \times \vec{b} = [-5(-7) - 9(4), 9(3) - 1(-7), 1(4) - (-5)(3)]$

$= [-1, 34, 19]$

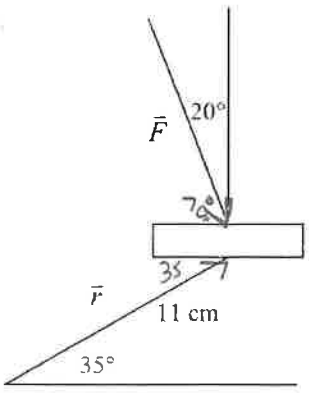
$\text{Volume} = |\vec{a} \times \vec{b} \cdot \vec{c}|$

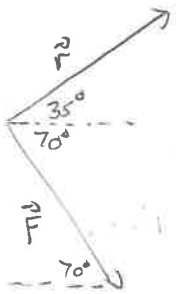
$= |[-1, 34, 19] \cdot [1, 0, 2]|$

$= |-1(1) + 34(0) + 19(2)|$

$= 37$ units^3

7) Find the torque produced by a cyclist exerting a force of 85 N on the pedal in the position shown in the diagram, if the shaft of the petal is 11 cm long.





$\tau = \vec{r} \times \vec{F}$

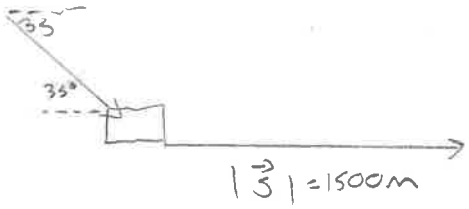
$\tau = [|\vec{r}||\vec{F}| \sin\theta](-\hat{n})$

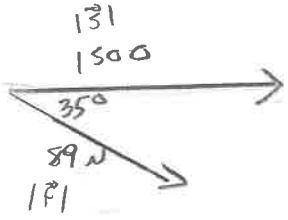
$\tau = [0.11(85)\sin 105](-\hat{n})$

$\tau = -9.03\hat{n}$ N·m

9.03 N·m in to the bike.

8) A woman pushes her baby stroller a distance of 1500 m by a force of 89 N applied at an angle of 35° to the roadway. Calculate the work done.



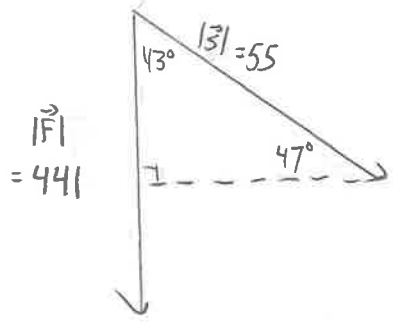


$W = \vec{F} \cdot \vec{s}$

$W = 89(1500)\cos(35)$

$W \approx 109\,356.8$ N·m

9) Determine the work done by gravity in causing a 45 kg child to slide down a 55 m slope, which has an angle of 47° to the horizontal.



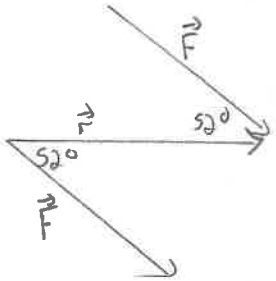
$W = \vec{F} \cdot \vec{s}$

$W = 441(55)\cos(47)$

$W = 17\,738.98$ N·m

10) A force of 75 N is applied to a wrench in a clockwise direction at 52° to the handle, 17 cm from the centre of the bolt.

a) Calculate the magnitude of the torque.



$|\tau| = \vec{r} \times \vec{F}$

$|\tau| = |\vec{r}||\vec{F}|\sin\theta$

$|\tau| = 0.17(75)\sin(52)$

$|\tau| = 10.05\ N \cdot m$

b) In what direction does the bolt move?

In to the material.

ANSWER KEY:

1. a) −119 b) 119
2. a) $\frac{30}{89}[-7, 2, 6]; \frac{30}{\sqrt{89}}$ b) $\frac{38}{14}[3, -2, 1]; \frac{38}{\sqrt{14}}$
3. a) 850 J b) 54 J
4. a) $\sqrt{26}$ units^2 b) $\sqrt{706}$ units^2
5. a) $\frac{\sqrt{261}\sqrt{47}}{2}$ units^2 b) $\frac{39}{2}$ units^2
6. a) 93 units^3 b) 37 units^3
7. 9.03 N•m
8. 109 356.8 J
9. 17 738.98 J
10. a) 10.05 N•m b) The bolt is being tightened into the material