W6 – Applications of Rates of Change Unit 1
MCV4U
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SOLUTIONS

1) The demand function for a DVD player is $p(x) = \frac{575}{\sqrt{x}} - 3$, where x is the number of DVD players sold and p is the price, in dollars. Determine…

a) the revenue function

$R(x) = x \cdot p(x)$

$R(x) = x\left(\frac{575}{\sqrt{x}} - 3\right)$

$R(x) = \frac{575x}{\sqrt{x}} - 3x$

$R(x) = 575\sqrt{x} - 3x$

b) the marginal revenue function

$R'(x) = \frac{1}{2}(575)(x)^{-1/2} - 3$

$R'(x) = \frac{575}{2\sqrt{x}} - 3$

c) the marginal revenue when 200 DVD players are sold

$R'(200) = \frac{575}{2\sqrt{200}} - 3$

$R'(200) = \$17.33$ per DVD player

2) Refer to question 1. If the cost, C, in dollars, of producing x DVD players is $C(x) = 2000 + 150x - 0.002x^2$, determine…

a) the profit function

$P(x) = R(x) - C(x)$

$= 575\sqrt{x} - 3x - (2000 + 150x - 0.002x^2)$

$= 0.002x^2 - 153x + 575\sqrt{x} - 2000$

b) the marginal profit function

$P'(x) = 0.004x - 153 + \frac{575}{2\sqrt{x}}$

c) the marginal profit for the sale of 500 DVD players

$$P'(500) = 0.004(500) - 153 + \frac{575}{2\sqrt{500}}$$

$$= \$ -138.14 \text{ per DVD player}$$

3) A paint store sells 270 cans of paint per month at a price of $32 each. A customer survey indicates that for each $1.20 decrease in price, sales will increase by six cans of paint.

a) Determine the demand, or price, function.

sold $= x = 270 + 6n$ price $= p = 32 - 1.20n$

$n = \frac{x - 270}{6}$

$$p(x) = 32 - 1.2\left(\frac{x-270}{6}\right)$$

$$p(x) = 32 - \left(\frac{x-270}{5}\right)$$

$$p(x) = 32 - \frac{x}{5} + \frac{270}{5}$$

$$p(x) = -0.2x + 86$$

b) Determine the revenue function.

$$R(x) = x \cdot p(x)$$

$$= x(-0.2x + 86)$$

$$= -0.2x^2 + 86x$$

c) Determine the marginal revenue function.

$$R'(x) = -0.4x + 86$$

d) Solve $R'(x) = 0$. Interpret this value for this situation.

$$0 = -0.4x + 86$$

$$x = 215$$

Selling 215 cans per month maximizes the revenue.

e) What price corresponds to the value found in part d)? How can the paint store use this information.

$$p(215) = -0.2(215) + 86$$

$$p(215) = \$43 \text{ per dvd player}$$

Charging $43 per DVD player will maximize the revenue.

4) A yogurt company estimates that the revenue from selling x containers of yogurt is $4.5x$. Its cost, C, in dollars, for producing this number of containers of yogurt is $C(x) = 0.0001x^2 + 2x + 3200$.

a) Determine the marginal cost of producing 4000 containers of yogurt.

$C'(x) = 0.0002x + 2$

$C'(4000) = 0.0002(4000) + 2$

$C'(4000) = \$2.80$ per yogurt container

b) Determine the marginal profit from selling 4000 containers of yogurt.

$P(x) = R(x) - C(x)$

$= 4.5x - (0.0001x^2 + 2x + 3200)$

$= -0.0001x^2 + 2.5x - 3200$

$P'(x) = -0.0002x + 2.5$

$P'(4000) = -0.0002(4000) + 2.5$

$P'(4000) = \$1.70$ per yogurt

c) What is the selling price of a container of yogurt?

$\$4.50$

5) The cost, C, in dollars, of producing x hot tubs can be modelled by the function $C(x) = 3450x - 1.02x^2, 0 \leq x \leq 1500$.

a) Determine the marginal cost at a production level of 750 hot tubs. Explain what this means to the manufacturer.

$C'(x) = 3450 - 2.04x$

$C'(750) = 3450 - 2.04(750)$

$C'(750) = \$1920$

The negative slope of the linear equation shows that the rate of change of cost will decrease as you produce more hot tubs.

b) Find the cost of producing the 751^{st} hot tub.

$C(750) = 3450(750) - 1.02(750)^2$

$= \$2\,013\,750$

$C(751) = 3450(751) - 1.02(751)^2$

$= \$2\,015\,668.98$

Cost of 751^{st} hot tub $= C(751) - C(750)$

$= \$1918.98$

c) Compare and comment on the values you found in parts a) and b).

The marginal cost of producing x items is approximately equal to the cost of producing 1 more item.

d) Each hot tub is sold for \$9200. Write an expression to model the total revenue from the sale of x hot tubs.

$$R(x) = 9200x$$

e) Determine the rate of change of profit for the sale of 750 hot tubs.

$$P(x) = 9200x - (3450x - 1.02x^2)$$
$$P(x) = 5750x + 1.02x^2$$
$$P'(x) = 5750 + 2.04x$$
$$P'(750) = 5750 + 2.04(750)$$
$$P'(750) = \$7280 \text{ per hot tub.}$$

6) The mass, in grams, of the first x meters of a wire can be modelled by the function $f(x) = \sqrt{2x - 1}$.

a) Determine the average linear density of the part of the wire from $x = 1$ to $x = 8$.

$$\text{average LD} = \frac{f(8) - f(1)}{8 - 1}$$
$$= \frac{\sqrt{2(8)-1} - \sqrt{2(1)-1}}{7}$$
$$= \frac{\sqrt{15} - 1}{7}$$
$$\simeq 0.41 \text{ g/m}$$

b) Determine the linear density at $x = 5$ and at $x = 8$, and compare the densities at the two points. What do these values confirm about the wire?

$$f'(x) = \frac{1}{2}(2x-1)^{-1/2}(2)$$
$$f'(x) = \frac{1}{\sqrt{2x-1}}$$

$$f'(5) = \frac{1}{\sqrt{2(5)-1}} = \frac{1}{3} \text{ g/m}$$

$$f'(8) = \frac{1}{\sqrt{2(8)-1}} = \frac{1}{\sqrt{15}} \simeq 0.26 \text{ g/m}$$

The density is decreasing as the the length of the wire increases.

The material is non-homogeneous.