

W7 - 5.4 Solve Quadratic Trigonometric Equations

MHF4U

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SOLUTIONS

1) Solve $\sin^2 x - 2 \sin x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

$$(\sin x - 3)(\sin x + 1) = 0$$

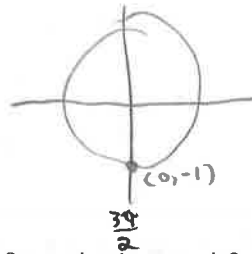
$$\sin x - 3 = 0$$

$$\sin x = 3$$

No solutions

$$\sin x = -1$$

use unit circle where each point is $(\cos x, \sin x)$



$$x = \frac{3\pi}{2}$$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

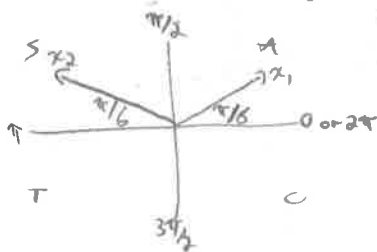
$$(\csc x - 2)(\csc x + 1) = 0$$

$$\csc x - 2 = 0$$

$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$

from Δ ; $\sin \frac{\pi}{6} = \frac{1}{2}$



$$x_1 = \frac{\pi}{6}$$

$$x_2 = \pi - \frac{\pi}{6}$$

$$x_2 = \frac{5\pi}{6}$$

$$\csc x + 1 = 0$$

$$\csc x = -1$$

$$\sin x = -1$$

* refer to part a) *

$$x_3 = \frac{3\pi}{2}$$

3) Solve $2 \sec^2 x - \sec x - 1 = 0$ on the interval $0 \leq x \leq 2\pi$

$$2 \sec^2 x - 2 \sec x + 1 \sec x - 1 = 0$$

$$2 \sec x (\sec x - 1) + 1 (\sec x - 1) = 0$$

$$(\sec x - 1)(2 \sec x + 1) = 0$$

↓

$$\sec x - 1 = 0$$

$$\sec x = 1$$

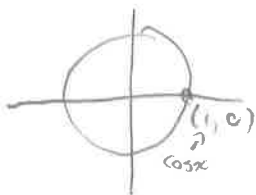
$$\cos x = 1$$

$$2 \sec x + 1 = 0$$

$$\sec x = -\frac{1}{2}$$

$$\cos x = -2$$

No solutions



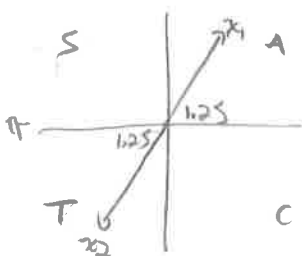
$$x_1 = 0$$

$$x_2 = 2\pi$$

4) Solve $\tan^2 x - \tan x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$. Round answers to the nearest hundredth of a radian.

$$(\tan x - 3)(\tan x + 2) = 0$$

$$\tan x = 3$$



$$x_1 = \tan^{-1}(3)$$

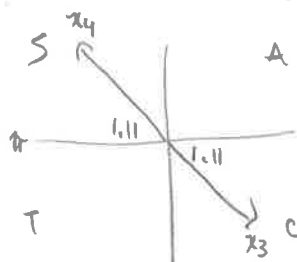
$$x_1 = 1.25$$

$$x_2 = \pi + 1.25$$

$$x_2 = 4.39$$

$$\tan x + 2 = 0$$

$$\tan x = -2$$



$$x_3 = \tan^{-1}(-2)$$

$$x_3 = -1.107 + 2\pi$$

$$x_3 = 5.18$$

$$x_4 = \pi - 1.11$$

$$x_4 = 2.03$$

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$

$$6\cos^2 x + 9\cos x - 4\cos x - 6 = 0$$

$$3\cos x(2\cos x + 3) - 2(2\cos x + 3) = 0$$

$$(2\cos x + 3)(3\cos x - 2) = 0$$

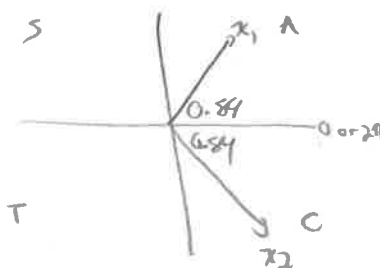
$$2\cos x + 3 = 0$$

$$\cos x = -\frac{3}{2}$$

No solutions

$$3\cos x - 2 = 0$$

$$\cos x = \frac{2}{3}$$



$$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$$

$$x_1 = 0.84$$

$$x_2 = 2\pi - 0.84$$

$$x_2 = 5.44$$

6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

$$3\csc^2 x - 6\csc x + 1\csc x - 2 = 0$$

$$3\csc x(\csc x - 2) + 1(\csc x - 2) = 0$$

$$(\csc x - 2)(3\csc x + 1) = 0$$

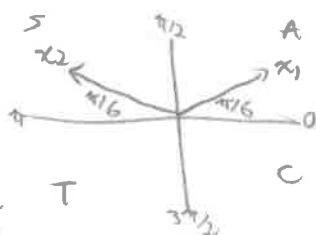
$$\csc x - 2 = 0$$

$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$

From Δ ; $\sin \frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2



$$x_1 = \frac{\pi}{6}$$

$$x_2 = \frac{5\pi}{6}$$

$$3\csc x + 1 = 0$$

$$\csc x = -\frac{1}{3}$$

$$\sin x = -3$$

No solutions

7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

$$2\tan^2 x - 6\tan x + 4\tan x - 3 = 0$$

$$2\tan x (\tan x - 3) + 1(\tan x - 3) = 0$$

$$(\tan x - 3)(2\tan x + 1) = 0$$

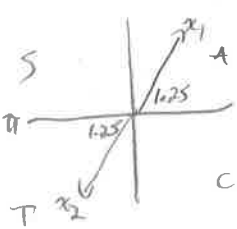
$$\tan x = 3$$

$$x_1 = \tan^{-1}(3)$$

$$x_1 = 1.1071$$

$$x_2 = \pi + 1.1071$$

$$x_2 = 4.3395$$



$$2\tan x + 1 = 0$$

$$\tan x = -\frac{1}{2}$$

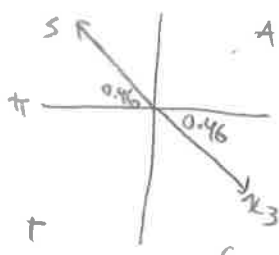
$$x_3 = \tan^{-1}\left(-\frac{1}{2}\right)$$

$$x_3 = -0.463547609 + 2\pi$$

$$x_3 = 5.82$$

$$x_4 = \pi - 0.46$$

$$x_4 = 2.68$$



8) Solve $\cot x \csc^2 x = 2 \cot x$ on the interval $0 \leq x \leq 2\pi$

$$\left(\frac{\cos x}{\sin x}\right) \left(\frac{1}{\sin^2 x}\right) = 2 \left(\frac{\cos x}{\sin x}\right)$$

$$\frac{\cos x}{\sin^3 x} = 2 \cos x$$

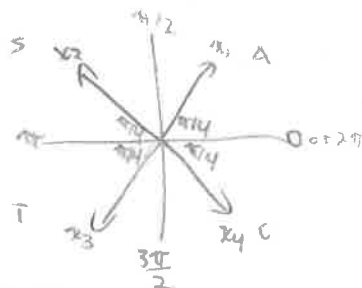
$$\frac{\cos x}{\cos x} = 2 \sin^2 x$$

$$\frac{1}{2} = \sin^2 x$$

$$\sin x = \pm \frac{1}{\sqrt{2}}$$

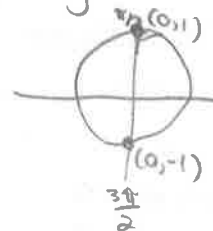
from Δ ; $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Place in all 4 quadrants



* Notice in 1st line of solution; if $\cos x = 0$, then LHS = RHS as well.

or using unit circle where a point is $(\cos x, \sin x)$



$$x_1 = \frac{\pi}{4}, x_2 = \frac{3\pi}{4}, x_3 = \frac{5\pi}{4}, x_4 = \frac{7\pi}{4}, x_5 = \frac{\pi}{2}, x_6 = \frac{3\pi}{2}$$

9) Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$

a) $3\tan^2 \theta - 2\tan \theta = 1$

$$3\tan^2 \theta - 2\tan \theta - 1 = 0$$

$$3\tan^2 \theta - 3\tan \theta + 1\tan \theta - 1 = 0$$

$$3\tan \theta (\tan \theta - 1) + 1(\tan \theta - 1) = 0$$

$$(\tan \theta - 1)(3\tan \theta + 1) = 0$$

$$\tan \theta - 1 = 0$$

$$\tan \theta = 1$$

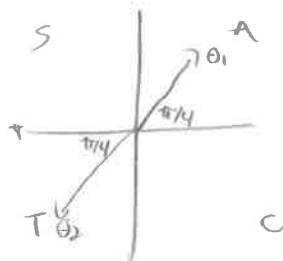
from Δ ; $\tan \frac{\pi}{4} = 1$

Place in Q1 + Q3

$$\theta_1 = \frac{\pi}{4}$$

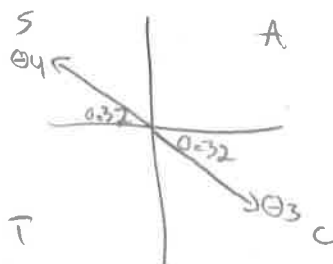
$$\theta_2 = \pi + \frac{\pi}{4}$$

$$\theta_2 = \frac{5\pi}{4}$$



$$3\tan \theta + 1 = 0$$

$$\tan \theta = -\frac{1}{3}$$



$$\theta_3 = \tan^{-1}\left(-\frac{1}{3}\right)$$

$$\theta_3 = -0.32175 + 2\pi$$

$$\theta_3 = 5.96$$

$$\theta_4 = \pi - 0.32$$

$$\theta_4 = 2.82$$

b) $12 \sin^2 \theta + \sin \theta - 6 = 0$

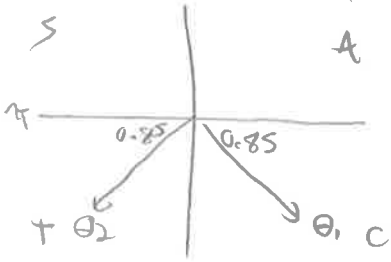
$12 \sin^2 \theta + 9 \sin \theta - 8 \sin \theta - 6 = 0$

$3 \sin \theta (4 \sin \theta + 3) - 2(4 \sin \theta + 3) = 0$

$(4 \sin \theta + 3)(3 \sin \theta - 2) = 0$

$4 \sin \theta + 3 = 0$

$\sin \theta = -\frac{3}{4}$



$\theta_1 = \sin^{-1}(-\frac{3}{4})$

$\theta_1 = -0.848062 + 2\pi$

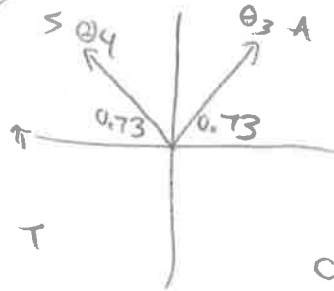
$\theta_1 = 5.44$

$\theta_2 = \pi + 0.85$

$\theta_2 = 3.99$

$3 \sin \theta - 2 = 0$

$\sin \theta = \frac{2}{3}$



$\theta_3 = \sin^{-1}(\frac{2}{3})$

$\theta_3 = 0.73$

$\theta_4 = \pi - 0.73$

$\theta_4 = 2.41$

c) $5 \cos(2\theta) - \cos \theta + 3 = 0$

$5(2 \cos^2 \theta - 1) - \cos \theta + 3 = 0$

$10 \cos^2 \theta - 5 - \cos \theta + 3 = 0$

$10 \cos^2 \theta - \cos \theta - 2 = 0$

$10 \cos^2 \theta - 5 \cos \theta + 4 \cos \theta - 2 = 0$

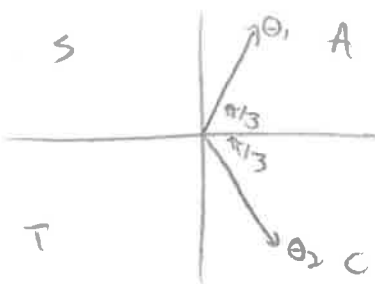
$5 \cos \theta (2 \cos \theta - 1) + 2(2 \cos \theta - 1) = 0$

$(2 \cos \theta - 1)(5 \cos \theta + 2) = 0$

$\cos \theta = \frac{1}{2}$

From 4; $\cos \frac{\pi}{3} = \frac{1}{2}$

place in Q1 + Q4

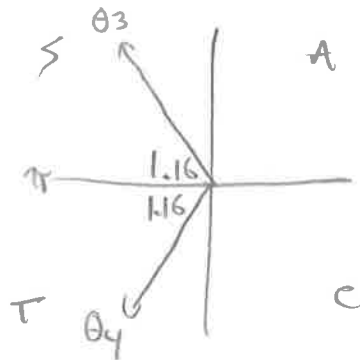


$\theta_1 = \frac{\pi}{3}$

$\theta_2 = 2\pi - \frac{\pi}{3}$

$\theta_2 = \frac{5\pi}{3}$

$\cos \theta = -\frac{2}{5}$



$\theta_3 = \cos^{-1}(-\frac{2}{5})$

$\theta_3 = 1.98$

$\theta_4 = \pi + 1.16$

$\theta_4 = 4.3$

Answer Key

- 1) $\frac{3\pi}{2}$ 2) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ 3) π 4) 1.11, 1.89, 4.25, 5.03 5) 0.84, 5.44 6) $\frac{\pi}{6}, \frac{5\pi}{6}$ 7) 1.25, 2.68, 4.39, 5.82
 8) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ 9)a) $\frac{\pi}{4}, 2.82, \frac{5\pi}{4}, 5.96$ b) 0.73, 2.41, 3.99, 5.44 c) $\frac{\pi}{3}, 1.98, 4.3, \frac{5\pi}{3}$