Unit 5 – Solving Quadratic Equations

Lessons

MPM2D

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

L1 –Solving Quadratics by Factoring Unit 5
MPM2D
Jensen

You will be able to solve some quadratic equations by:

1) writing them in the form $ax^2 + bx + c = 0$
2) factoring the quadratic
3) setting each factor to zero and solving

Setting each factor to ________________ and solving allows you to find what makes the entire product equal zero because of the zero product rule.

The zero product rule states that the product of factors is zero if one or more of the factors are zero.

For example, if $ab = 0$, then $a = 0$ or $b = 0$ (or both)

For the quadratic equation $x^2 + 2x - 3 = 0$, it factors to $(x + 3)(x - 1) = 0$. Then using the zero product rule, we can see that the product would be zero if either $x + 3 = 0$ or if $x - 1 = 0$. The values of x make those factors become zero are ______________________________. Therefore, those are the solutions to the equation. They can be checked in the original equation.

When trying to factor a quadratic, remember to always check for a common factor. Also, remember that if the leading coefficient in a quadratic trinomial is not equal to 1, you must factor by decomposition.

Example 1: Solve each of the following quadratic equations

a) $x^2 + 8x + 15 = 0$

b) $x^2 - 7x = 0$

c) $x^2 + 4x = 12$

Tip: Always start by setting the quadratic equal to zero.

d) $2x^2 - 22x + 48 = 0$

Tip: Always check for a common factor!

e) $0 = 2x^2 + 5x - 3$

f) $3x^2 + 17x = 20$

g) $2x^2 - 6 = 0$

h) $3x^2 + 1 = 0$

Example 2: A picture that measures 10 cm by 5 cm is to be surrounded by a mat before being framed. The width of the mat is to be the same on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?

L2 –Solving Quadratics by Completing the Square Unit 5
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Part 1: Investigation

Example 1: For the quadratic $y = x^2 + 6x + 8$

a) Determine the x-intercepts by factoring

b) Determine the vertex using the x-intercepts

c) Convert to vertex form by completing the square

d) Solve for the x-intercepts by rearranging the vertex form equation to isolate for x.

A quadratic equation in standard form, $0 = ax^2 + bx + c$, can be solved for x by first converting into vertex form, $0 = a(x - h)^2 + k$, by completing the square, then rearranging to isolate for x.

Part 2: Rational Solutions

Example 2: Solve each of the following equations by completing the square.

a) $0 = x^2 - 12x + 20$

b) $10 = 2x^2 - x$

Notice for each of the equations in this question there were ________________ solutions. That means that each of the equations could have been solved by ________________.

Part 3: Irrational Solutions

Example 3: Solve each of the following quadratic equations by completing the square.

a) $-3x^2 + 6x + 7 = 0$

b) $x^2 - 6x - 8 = 0$

Notice for each of the equations in this question there were only ________________ solutions. That means that each of the equations could **NOT** have been solved by ________________. This is why solving by completing the square is a useful strategy.

Part 4: Non-Real Solutions

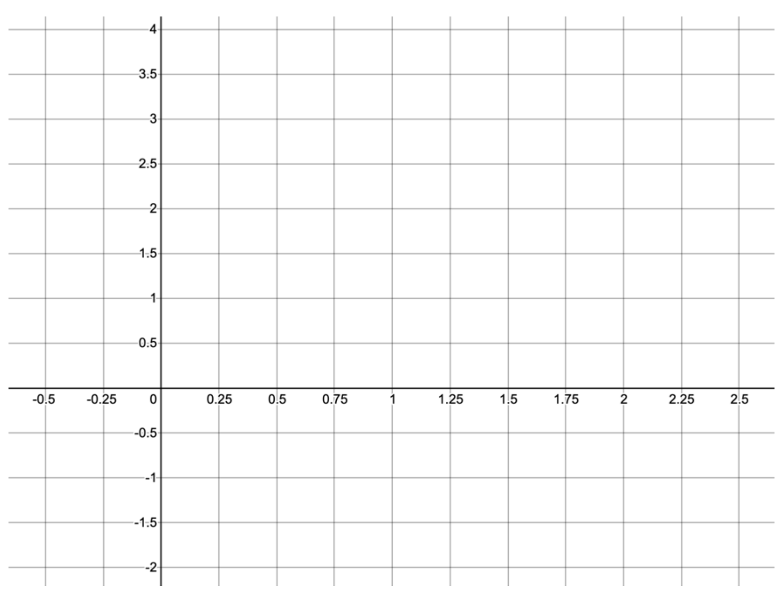
Example 4: Solve the following quadratic equation

$$x^2 - 6x + 10 = 0$$

Notice that not all quadratics have _________________.

Part 5: Application

Example 5: For the quadratic $y = 2x^2 - 5x + 3$, determine the vertex, x-intercepts, and then sketch a graph of the function.



Part 1: Proof of Quadratic Formula

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is a formula that can be used to find rational, irrational, and non-real solutions to any quadratic equation that is in the form $ax^2 + bx + c = 0$. It is a formula that is derived from completing the square of a general standard form quadratic and then rearranging to isolate for x. Using the formula is just a shortcut to avoid having to complete the square and rearrange like we did last lesson.

Proof:

Part 2: Rational Solutions

Example 2: Solve each of the following equations using the Quadratic Formula.

a) $0 = x^2 - 7x + 12$

b) $0 = 3x^2 + 11x - 4$

c) $-9 = x^2 - 6x$

Notice for each of the equations in this question there were ___________________ solutions. That means that each of the equations could have been solved by factoring. But quadratic formula also works!

Part 3: Irrational Solutions

Example 3: Solve each of the following quadratic equations

a) $0 = 3x^2 + 8x - 5$

b) $-5 = -x^2 - 10x$

Notice for each of the equations in this question there were only ___________________ solutions. That means that each of the equations could NOT have been solved by factoring. This is why the Quadratic formula is so useful!

Part 4: Non-Real Solutions

Example 4: Solve each of the following quadratic equations

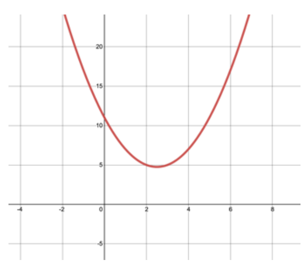
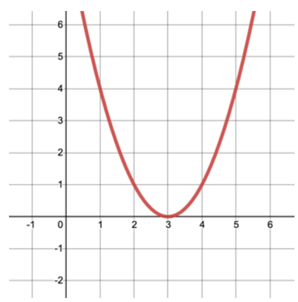
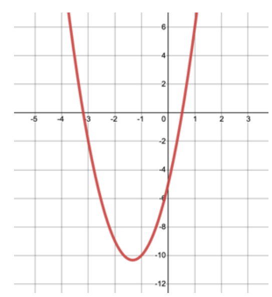
a) $0 = x^2 - 5x + 11$

b) $-7 = 2x^2 - 4x$

There are no real solutions when under the square root (the discriminant) is ___________________. You can't square root a negative!

Part 5: The Discriminant

Looking back through the previous parts of the lesson, you can see that quadratic equations can have either zero, one, or two solutions. The discriminant, $b^2 - 4ac$, is the part of the equation that tells you how many solutions a quadratic equation will have.

# of solutions	0	1	2
Discriminant	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$
Example Equation	$0 = x^2 - 5x + 11$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(11)}}{2(1)}$ $x = \frac{5 \pm \sqrt{-19}}{2}$ No real solutions Notice $b^2 - 4ac = -19$ which is less than zero	$0 = x^2 - 6x + 9$ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$ $x = \frac{6 \pm \sqrt{0}}{2(1)}$ $x = 3$ Notice $b^2 - 4ac = 0$	$0 = 3x^2 + 8x - 5$ $x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5)}}{2(3)}$ $x = \frac{-8 \pm \sqrt{124}}{6}$ $x = -3.19, 0.52$ Notice $b^2 - 4ac = 124$ which is greater than zero
Example Graph	 Notice the graph never crosses the x-axis	 Notice the vertex is ON the x-axis	 Notice the graph crosses through the x-axis twice

Example 5: How many real solutions does each quadratic equation have?

a) $0 = x^2 + 10x + 25$ **b)** $0 = -2x^2 + 2x + 7$ **c)** $0 = 2x^2 + 2x + 7$

Another piece of useful information about the discriminant:

If $b^2 - 4ac$ is a ____________________________ number, you will get rational solutions which means the quadratic is factorable. If you aren't sure if a quadratic is factorable, just check to see if $b^2 - 4ac$ is a perfect square number $(0,1,4,9,16,25,36, \ldots)$

L4 –Quadratics in Standard Form Unit 5
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Part 1: Vertex from Standard Form Quadratic

Remember that parabolas are symmetrical about the axis of symmetry which is a vertical line that passes through the vertex. Because of this symmetry property, you can find the x-coordinate of the vertex by averaging the x-intercepts.

From quadratic formula we know that the x-intercepts of a standard form quadratic are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Therefore, the x-coordinate of the vertex is:

Conclusion:

From the standard form equation of a quadratic, $y = ax^2 + bx + c$, you can determine the x-coordinate of the vertex using the formula:

$$x - vertex =$$

Example 1: Find the vertex of the following quadratics

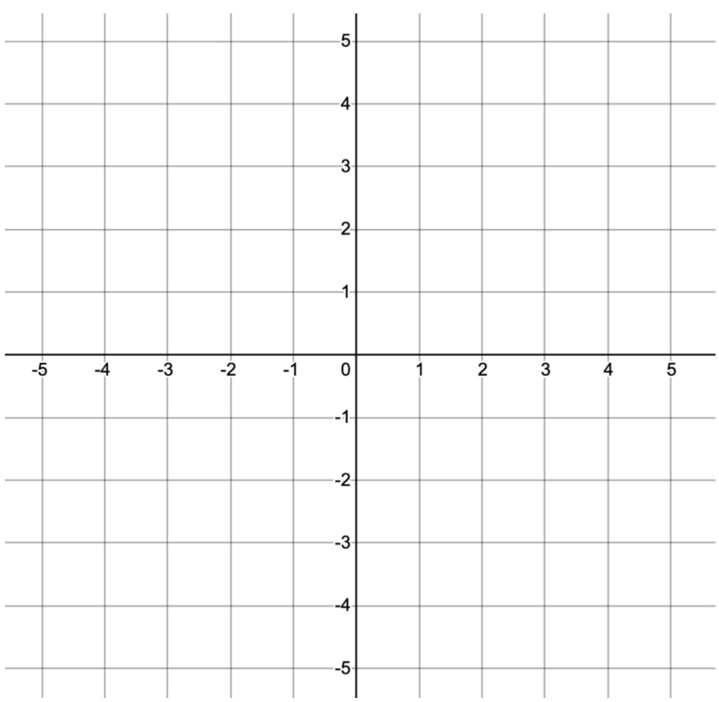
a) $y = x^2 - 6x + 11$

b) $y = -3x^2 + 2x - 1$

Part 2: Putting it all together

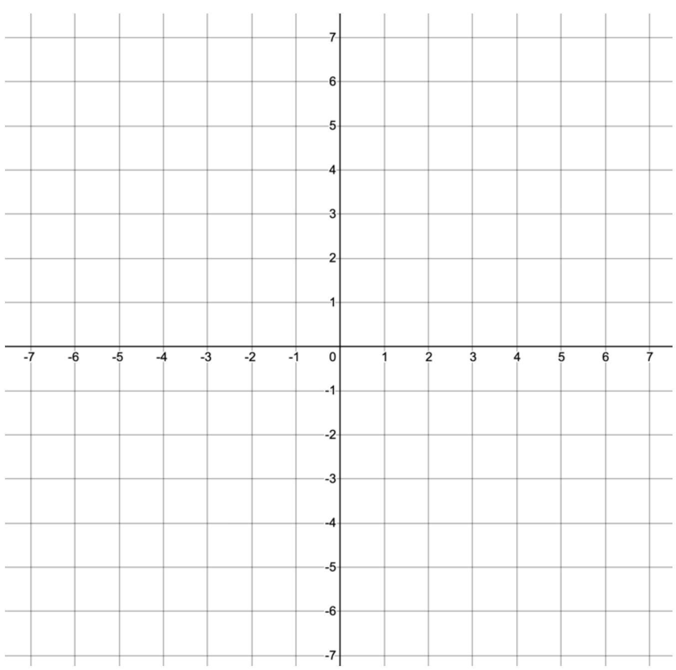
Example 2: For the quadratic $y = -5x^2 + 8x - 3$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points



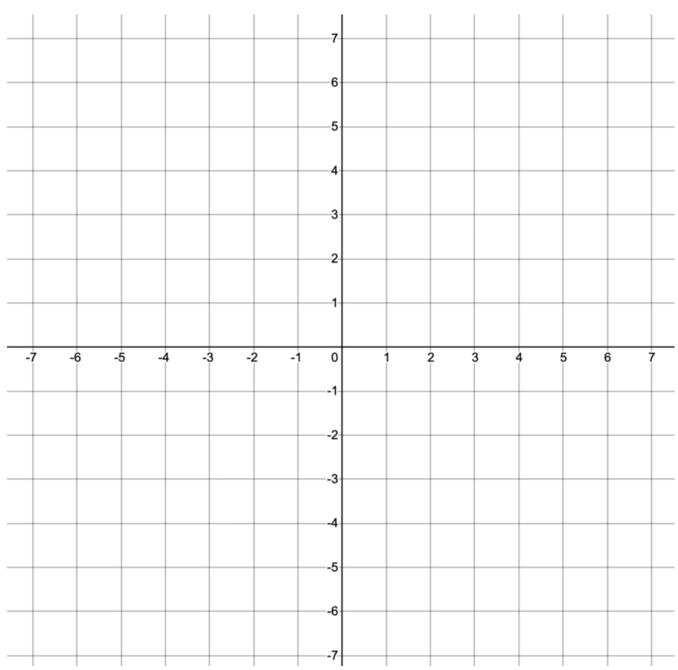
Example 3: For the quadratic $y = 2x^2 - 8x + 11$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points



Example 4: For the quadratic $y = x^2 - 10x + 25$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points



L5 – Applications of Quadratics Unit 5
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Example 1: The equation $h(t) = -4.9t^2 + 60t + 3$ represents the path of a rocket where h is height in meters and t is time in seconds after is has been launched.

a) What is the height of the rocket when it is launched?
b) How long does it take the rocket to land on the ground?
c) What is the maximum height of the rocket?
d) When is the rocket 4 meters above the ground?

Example 2: One leg of a right triangle is 1 cm longer than the other leg. The length of the hypotenuse is 9 cm greater than that of the shorter leg. Find the length of the three sides.

Example 3: The length of a rectangle is 16cm greater than its width. The area is $35cm^2$. Find the dimensions of the rectangle.

Example 4: The path of a soccer ball after it is kicked from a height of 0.5 meters above the ground is given by the equation $h(d) = -0.1d^2 + d + 0.5$, where h is the height in meters, and d is the horizontal distance in meters.

a) How far has the soccer ball travelled horizontally when it lands on the ground?
b) Find the horizontal distance when the soccer ball is at a height of 2.6 meters above the ground.
c) What is the max height of the ball?

Example 5: A sporting goods store sells 90 ski jackets in a season for \$200 each. Each \$10 decrease in the price would result in five more jackets being sold. At what price should they sell the jackets in order to obtain a maximum revenue? What is the max revenue?