Unit 5 – Solving Quadratic Equations

Workbook

MPM2D

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

W1 –Solving Quadratics by Factoring Unit 5
MPM2D
Jensen

1) Solve

a) $(x+1)(x+2)=0$

$x+1=0$ $x+2=0$

$x_1=-1$ $x_2=-2$

b) $(x+3)(x-1)=0$

$x+3=0$ $x-1=0$

$x_1=-3$ $x_2=1$

c) $x(4-x)=0$

$x_1=0$ $4-x=0$

$4=x_2$

d) $(2x+1)(x-3)=0$

$2x+1=0$ $x-3=0$

$2x=-1$ $x_2=3$

$x_1=\frac{-1}{2}$

2) Solve and check

a) $x^2+7x+12=0$ $4\times 3=12$ $4+3=7$

$(x+4)(x+3)=0$

$x+4=0$ $x+3=0$

$x_1=-4$ $x_2=-3$

b) $x^2-x-6=0$ $-3\times 2=-6$ $-3+2=-1$

$(x-3)(x+2)=0$

$x-3=0$ $x+2=0$

$x_1=3$ $x_2=-2$

c) $x^2-8x+16=0$ $-4\times -4=16$ $-4+-4=-8$

$(x-4)(x-4)=0$

$(x-4)^2=0$

$x-4=0$

$x=4$

f) $x^2-7x=18$

$x^2-7x-18=0$ $-9\times 2=-18$ $-9+2=-7$

$(x-9)(x+2)=0$

$x-9=0$ $x+2=0$

$x_1=9$ $x_2=-2$

3) Solve

a) $2a^2 + 3a - 2 = 0$

$4 \times -1 = -4$, $4 + -1 = 3$

$2a^2+4a-1a-2=0$

$2a(a+2)-1(a+2)=0$

$(a+2)(2a-1)=0$

$a+2=0$ $\quad 2a-1=0$

$a_1=-2$ $\quad 2a=1$

$a_2=\frac{1}{2}$

b) $3s^2 - 4s + 1 = 0$

$-3 \times -1 = 3$, $-3 + -1 = -4$

$3s^2-3s-1s+1=0$

$3s(s-1)-1(s-1)=0$

$(s-1)(3s-1)=0$

$s-1=0$ $\quad 3s-1=0$

$s_1=1$ $\quad 3s=1$

$s_2=\frac{1}{3}$

c) $2t^2 + 11t + 5 = 0$

$10 \times 1 = 10$, $10 + 1 = 11$

$2t^2+10t+1t+5=0$

$2t(t+5)+1(t+5)=0$

$(t+5)(2t+1)=0$

$t+5=0$ $\quad 2t+1=0$

$t_1=-5$ $\quad 2t=-1$

$t_2=\frac{-1}{2}$

d) $3x^2 + 7x - 6 = 0$

$9 \times -2 = -18$, $9 + -2 = 7$

$3x^2+9x-2x-6=0$

$3x(x+3)-2(x+3)=0$

$(x+3)(3x-2)=0$

$x+3=0$ $\quad 3x-2=0$

$x_1=-3$ $\quad 3x=2$

$x_2=\frac{2}{3}$

e) $3 = 4m^2 - 4m$

$-6 \times 2 = -12$, $-6 + 2 = -4$

$0=4m^2-4m-3$

$0=4m^2-6m+2m-3$

$0=2m(2m-3)+1(2m-3)$

$0=(2m-3)(2m+1)$

$2m-3=0$ $\quad 2m+1=0$

$2m=3$ $\quad 2m=-1$

$m_1=\frac{3}{2}$ $\quad m_2=\frac{-1}{2}$

f) $10y^2 - 16y = -6$

$-5 \times -3 = 15$, $-5 + -3 = -8$

$10y^2-16y+6=0$

$2(5y^2-8y+3)=0$

$5y^2-5y-3y+3=0$

$5y(y-1)-3(y-1)=0$

$(y-1)(5y-3)=0$

$y-1=0$ $\quad 5y-3=0$

$y_1=1$ $\quad 5y=3$

$y_2=\frac{3}{5}$

g) $x^2 + 2x = 0$

$x(x+2)=0$

$x_1=0$ $\quad x+2=0$

$x_2=-2$

h) $3x^2 + 2x = 0$

$x(3x+2)=0$

$x_1=0$ $\quad 3x+2=0$

$3x=-2$

$x_2=\frac{-2}{3}$

i) $5x^2 - 20x = 0$

$5x(x-4) = 0$

$x_1 = 0$ $\quad x-4=0$

$x_2 = 4$

j) $0 = 4x + 3x^2$

$0 = x(4+3x)$

$x_1 = 0$ $\quad 4+3x = 0$

$3x = -4$

$x_2 = \frac{-4}{3}$

k) $x^2 - 25 = 0$

$(x-5)(x+5) = 0$

$x-5=0$ $\quad x+5=0$

$x_1 = 5$ $\quad x_2 = -5$

l) $x^2 + 4 = 16$

$x^2 = 12$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

$x \simeq \pm 3.46$

m) $x^2 - 2x - 11 = 4$

$x^2 - 2x - 15 = 0$ $\quad -5 \times 3 = -15$, $-5 + 3 = -2$

$(x-5)(x+3) = 0$

$x-5=0$ $\quad x+3=0$

$x_1 = 5$ $\quad x_2 = -3$

n) $5x^2 = 2x$

$5x^2 - 2x = 0$

$x(5x-2) = 0$

$x_1 = 0$ $\quad 5x-2=0$

$5x = 2$

$x_2 = \frac{2}{5}$

o) $(x+4)^2 = 4$

$x+4 = \pm\sqrt{4}$

$x+4 = \pm 2$

$x+4=2$ $\quad x+4=-2$

$x_1 = -2$ $\quad x_2 = -6$

p) $(x-6)^2 - 8x = 0$

$x^2 - 12x + 36 - 8x = 0$

$x^2 - 20x + 36 = 0$ $\quad -2 \times -18 = 36$, $-2 + -18 = -20$

$(x-2)(x-18) = 0$

$x_1 = 2$ $\quad x_2 = 18$

4) The area of the rectangle shown in the diagram is 36 cm^2. What are its dimensions?

$x(x-5) = 36$

$x^2 - 5x - 36 = 0$

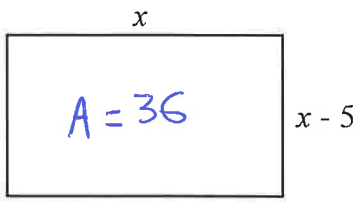
$(x-9)(x+4) = 0$

$x - 9 = 0 \quad x + 4 = 0$

$x_1 = 9 \quad \cancel{x_2 = -4}$ reject

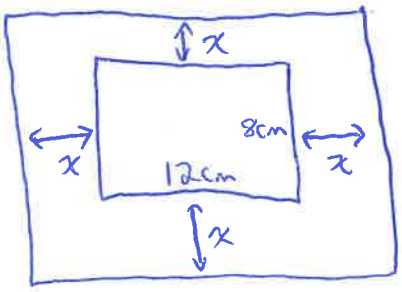
$\frac{-9}{} \times \frac{4}{} = -36$

$\frac{-9}{} + \frac{4}{} = -5$



The dimensions are 9cm and 4cm.

5) A photograph measuring 12 cm by 8 cm is to be surrounded by a mat before framing. The width of the mat is to be the same on all sides of the photograph. The area of the mat is to equal the area of the photograph. Find the width of the mat.



Area of mat = Area of photo

$(12+2x)(8+2x) - 12(8) = 12(8)$

$96 + 24x + 16x + 4x^2 - 96 = 96$

$4x^2 + 40x - 96 = 0$

~~$2(2x^2 + 20x$~~ $4(x^2 + 10x - 24) = 0$

$4(x+12)(x-2) = 0$

$x + 12 = 0 \quad x - 2 = 0$

$\cancel{x = -12} \quad x = 2$ reject

$\frac{12}{} \; x - \frac{2}{} = -24$

$\frac{12}{} + \frac{-2}{} = 10$

The width is 2cm

6) Three times the square of an integer is 432. Find the integer.

$3x^2 = 432$

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

7) A regular polygon with n sides has $\frac{n(n-3)}{2}$ diagonals. Find the number of sides of a regular polygon that has 44 diagonals.

(2) $44 = \frac{n(n-3)}{2}$ (2)

$88 = n^2 - 3n$

$0 = n^2 - 3n - 88$

$0 = (n-11)(n+8)$

$-11 \times 8 = -88$

$-11 + 8 = -3$

$n-11=0 \quad n+8=0$

$n_1 = 11$ $\quad n_2 = -8$ reject

11 sides

Answers

1)a) $x = -2, -1$ **b)** $x = -3,1$ **c)** $x = 0,4$ **d)** $x = -\frac{1}{2}, 3$
2)a) $x = -4, -3$ **b)** $x = -2,3$ **c)** $x = 4$ **d)** $x = -2,9$
3)a) $a = -2, \frac{1}{2}$ **b)** $s = \frac{1}{3}, 1$ **c)** $t = -5, -\frac{1}{2}$ **d)** $x = -3, \frac{2}{3}$ **e)** $m = -\frac{1}{2}, \frac{3}{2}$ **f)** $y = \frac{3}{5}, 1$ **g)** $x = -2,0$
h) $x = -\frac{2}{3}, 0$ **i)** $x = 0,4$ **j)** $x = -\frac{4}{3}, 0$ **k)** $x = -5,5$ **l)** $x = \pm\sqrt{12} = \pm 2\sqrt{3} \cong \pm 3.46$ **m)** $x = -3,5$
n) $x = 0, \frac{2}{5}$ **o)** $x = -6, -2$ **p)** $x = 2,18$
4) 9 cm by 4 cm
5) 2 cm
6) 12 or -12
7) 11

W2 –Solving Quadratics by Completing the Square Unit 5
MPM2D
Jensen

1) Solve each equation by completing the square. Round answers to 2 decimal places where necessary.

a) $x^2 + 14x - 38 = 0$

$(x^2+14x+49-49)-38=0$

$(x^2+14x+49)-49-38=0$

$(x+7)^2-87=0$

$(x+7)^2=87$

$x+7=\pm\sqrt{87}$

$x=-7\pm\sqrt{87}$

$x_1\simeq -16.33$ $x_2\simeq 2.33$

b) $x^2 + 6x - 59 = 0$

$(x^2+6x+9-9)-59=0$

$(x^2+6x+9)-9-59=0$

$(x+3)^2-68=0$

$(x+3)^2=68$

$x+3=\pm\sqrt{68}$

$x=-3\pm\sqrt{68}$

$x_1\simeq -11.25$ $x_2\simeq 5.25$

c) $x^2 + 14x - 51 = 0$

$(x^2+14x+49-49)-51=0$

$(x^2+14x+49)-49-51=0$

$(x+7)^2-100=0$

$(x+7)^2=100$

$x+7=\pm\sqrt{100}$

$x+7=\pm 10$

$x+7=10$ $x+7=-10$

$x_1=3$ $x_2=-17$

d) $x^2 - 12x + 11 = 0$

$(x^2-12x+36-36)+11=0$

$(x^2-12x+36)-36+11=0$

$(x-6)^2-25=0$

$(x-6)^2=25$

$x-6=\pm\sqrt{25}$

$x-6=\pm 5$

$x-6=5$ $x-6=-5$

$x_1=11$ $x_2=1$

e) $x^2 + 6x + 8 = 0$

$(x^2+6x+9-9)+8=0$

$(x^2+6x+9)-9+8=0$

$(x+3)^2-1=0$

$(x+3)^2=1$

$x+3=\pm\sqrt{1}$

$x+3=1$ $x+3=-1$

$x_1=-2$ $x_2=-4$

f) $x^2 - 12x + 23 = 0$

$(x^2-12x+36-36)+23=0$

$(x^2-12x+36)-36+23=0$

$(x-6)^2-13=0$

$(x-6)^2=13$

$x-6=\pm\sqrt{13}$

$x=6\pm\sqrt{13}$

$x_1\simeq 2.39$ $x_2\simeq 9.61$

g) $x^2 - 6x = -91$

$(x^2-6x+9-9) = -91$

$(x^2-6x+9)-9 = -91$

$(x-3)^2 = -82$

$x-3 = \pm\sqrt{-82}$

∞ no real solutions

h) $8x^2 + 16x = 42$

$8(x^2+2x) = 42$

$8(x^2+2x+1-1) = 42$

$8(x^2+2x+1)-8 = 42$

$8(x+1)^2 = 50$

$(x+1)^2 = \frac{50}{8}$

$x+1 = \pm\sqrt{\frac{25}{4}}$

$x+1 = \frac{5}{2}$ $\quad$ $x+1 = -\frac{5}{2}$

$x = \frac{3}{2}$ $\quad$ $x = -\frac{7}{2}$

i) $4x^2 + 4x + 36 = 0$

$4(x^2+x+9) = 0$

$x^2+x+9 = 0$

$(x^2+x+\frac{1}{4}-\frac{1}{4}) = -9$

$(x^2+x+\frac{1}{4})-\frac{1}{4} = -9$

$(x+\frac{1}{2})^2 = -\frac{35}{4}$

$x+\frac{1}{2} = \pm\sqrt{\frac{-35}{4}}$

∞ no real solutions

j) $3x^2 + 5x - 4 = 0$

$3(x^2+\frac{5}{3}x)-4 = 0$

$3(x^2+\frac{5}{3}x+\frac{25}{36}-\frac{25}{36}) = 4$

$3(x^2+\frac{5}{3}x+\frac{25}{36})-\frac{25}{12} = 4$

$3(x+\frac{5}{6})^2 = \frac{73}{12}$

$(x+\frac{5}{6})^2 = \frac{73}{36}$

$x+\frac{5}{6} = \pm\sqrt{\frac{73}{36}}$

$x+\frac{5}{6} = \sqrt{\frac{73}{36}}$ $\quad$ $x+\frac{5}{6} = -\sqrt{\frac{73}{36}}$

$x \simeq 0.59$ $\quad$ $x \simeq -2.26$

Answers

1)a) $x = -16.33, 2.33$ **b)** $x = -11.25, 5.25$ **c)** $x = -17, 3$ **d)** $x = 1, 11$ **e)** $x = -4, -2$ **f)** $x = 2.39, 9.61$
g) no real solutions **h)** $x = -\frac{7}{2}, \frac{3}{2}$ **i)** no real solutions **j)** $x = -2.26, 0.59$

W3 –Solving Quadratics using the Quadratic Formula Unit 5
MPM2D
Jensen

1) Use the quadratic formula to solve each equation. Express answers as ~~exact~~ approximate roots.

a) $7x^2 + 24x + 9 = 0$

$x = \frac{-24 \pm \sqrt{(24)^2 - 4(7)(9)}}{2(7)}$

$x = \frac{-24 \pm \sqrt{324}}{14}$

$x = \frac{-24 + 18}{14}$ $\quad x = \frac{-24 - 18}{14}$

$x_1 = -\frac{3}{7}$ $\quad x_2 = -3$

b) $2x^2 + 4x - 7 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{-4 \pm \sqrt{72}}{4}$

$x = \frac{-4 + \sqrt{72}}{4}$ $\quad x = \frac{-4 - \sqrt{72}}{4}$

$x_1 \simeq 1.12$ $\quad x_2 \simeq -3.12$

c) $4x^2 - 12x + 9 = 0$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$

$x = \frac{12 \pm \sqrt{0}}{8}$

$x = \frac{12}{8}$

$x = \frac{3}{2}$

d) $2x^2 - 7x = -4$

$2x^2 - 7x + 4 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$

$x = \frac{7 \pm \sqrt{17}}{4}$

$x = \frac{7 + \sqrt{17}}{4}$ $\quad x = \frac{7 - \sqrt{17}}{4}$

$x_1 \simeq 2.78$ $\quad x_2 \simeq 0.72$

e) $3x^2 + 5x = 1$

$3x^2 + 5x - 1 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{37}}{6}$

$x_2 \simeq \frac{-5 + \sqrt{37}}{6}$ $\quad x_2 = \frac{-5 - \sqrt{37}}{6}$

$x_1 \simeq 0.18$ $\quad x_2 \simeq -1.85$

f) $16x^2 + 24x = -9$

$16x^2 + 24x + 9 = 0$

$x = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$

$x = \frac{-24 \pm \sqrt{0}}{32}$

$x = \frac{-24}{32}$

$x = -\frac{3}{4}$

2) Use the quadratic formula to solve. Express your answers ~~as exact roots and~~ as approximate roots, rounded to the nearest hundredth.

a) $3x^2 + 14x + 5 = 0$

$x = \frac{-14 \pm \sqrt{(14)^2 - 4(3)(5)}}{2(3)}$

$x = \frac{-14 \pm \sqrt{136}}{6}$

$x_1 = \frac{-14 + \sqrt{136}}{6}$ $\quad x_2 = \frac{-14 - \sqrt{136}}{6}$

$x_1 \simeq -0.39$ $\quad x_2 \simeq -4.28$

b) $8x^2 + 12x + 1 = 0$

$x = \frac{-12 \pm \sqrt{(12)^2 - 4(8)(1)}}{2(8)}$

$x = \frac{-12 \pm \sqrt{112}}{16}$

$x_1 = \frac{-12 + \sqrt{112}}{16}$ $\quad x_2 = \frac{-12 - \sqrt{112}}{16}$

$x_1 \simeq -0.09$ $\quad x_2 \simeq -1.41$

c) $4x^2 - 7x - 1 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(4)(-1)}}{2(4)}$

$x = \frac{7 \pm \sqrt{65}}{8}$

$x_1 = \frac{7 + \sqrt{65}}{8}$ $\quad x_2 = \frac{7 - \sqrt{65}}{8}$

$x_1 \simeq 1.88$ $\quad x_2 \simeq -0.13$

d) $10x^2 - 45x - 7 = 0$

$x = \frac{45 \pm \sqrt{(-45)^2 - 4(10)(-7)}}{2(10)}$

$x = \frac{45 \pm \sqrt{2305}}{20}$

$x_1 = \frac{45 + \sqrt{2305}}{20}$ $\quad x_2 = \frac{45 - \sqrt{2305}}{20}$

$x_1 \simeq 4.65$ $\quad x_2 \simeq -0.15$

e) $-5x^2 + 16x - 2 = 0$

$x = \frac{-16 \pm \sqrt{(16)^2 - 4(-5)(-2)}}{2(-5)}$

$x = \frac{-16 \pm \sqrt{216}}{-10}$

$x_1 = \frac{-16 + \sqrt{216}}{-10}$ $\quad x_2 = \frac{-16 - \sqrt{216}}{-10}$

$x_1 \simeq 0.13$ $\quad x_2 \simeq 3.07$

f) $-6x^2 + 17x + 5 = 0$

$x = \frac{-17 \pm \sqrt{(17)^2 - 4(-6)(5)}}{2(-6)}$

$x = \frac{-17 \pm \sqrt{409}}{-12}$

$x_1 = \frac{-17 + \sqrt{409}}{-12}$ $\quad x_2 = \frac{-17 - \sqrt{409}}{-12}$

$x_1 \simeq -0.27$ $\quad x_2 \simeq 3.1$

g) $x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(2)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{17}}{2}$

$x_1 = \frac{-5+\sqrt{17}}{2}$ $\quad x_2 = \frac{-5-\sqrt{17}}{2}$

$x_1 \simeq -0.44$ $\quad x_2 \simeq -4.56$

h) $5x^2 - 3x + 2 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(2)}}{2(5)}$

$x = \frac{3 \pm \sqrt{-31}}{10}$

∴ no real solutions

3) Describe the roots of the equation $ax^2 + bx + c = 0$ in each of the following situations. Explain and justify your reasoning, and give examples to support your answers.

a) $b^2 - 4ac < 0$

No real solutions. The square root of a negative number is NOT a real number. You get no real solutions if the quadratic opens up and has its vertex above the x-axis OR if the quadratic opens down and has its vertex below the x-axis.

b) $b^2 - 4ac = 0$

1 real solution. In the QF, adding and subtracting 0 gives the same result. You get 1 solution when the vertex is ON the x-axis.

c) $b^2 - 4ac > 0$ and is a perfect square

You get 2 solutions that are rational numbers. If this happens, solving by factoring would also work.

d) $b^2 - 4ac > 0$ and is NOT a perfect square

You get 2 solutions that are irrational numbers. If this happens, solving by factoring would NOT work. QF must be used.

4) Use the discriminant to determine the number of roots for each quadratic equation.

a) $x^2 - 10x + 25 = 0$

$b^2-4ac=(-10)^2-4(1)(25)$

$=0$

$\therefore$ 1 solution

b) $3x^2 + 4x + \frac{4}{3} = 0$

$b^2-4ac=(4)^2-4(3)(\frac{4}{3})$

$=0$

$\therefore$ 1 solution

c) $2x^2 - 8x + 9 = 0$

$b^2-4ac=(-8)^2-4(2)(9)$

$=-8$

$\therefore$ no real solutions

d) $-2x^2 + 0.75x + 5 = 0$

$b^2-4ac=(0.75)^2-4(-2)(5)$

$=40.5625$

$\therefore$ 2 solutions.

Answers

1)a) $-3, \frac{-3}{7}$ **b)** $\frac{-4\pm\sqrt{72}}{4} = \frac{-2\pm3\sqrt{2}}{2}$ **c)** $\frac{3}{2}$ **d)** $\frac{7\pm\sqrt{17}}{4}$ **e)** $\frac{-5\pm\sqrt{37}}{6}$ **f)** $\frac{-3}{4}$

2)a) $\frac{-7\pm\sqrt{34}}{3}$; $-0.39, -4.28$ **b)** $\frac{-3\pm\sqrt{7}}{4}$; $-0.09, -1.41$ **c)** $\frac{7\pm\sqrt{65}}{8}$; $1.88, -0.13$

d) $\frac{45\pm\sqrt{2305}}{20}$; $4.65, -0.15$ **e)** $\frac{-16\pm\sqrt{216}}{-10} = \frac{8\pm3\sqrt{6}}{5}$; $0.13, 3.07$ **f)** $\frac{17\pm\sqrt{409}}{12}$; $3.1, -0.27$

g) $\frac{-5\pm\sqrt{17}}{2}$; $-0.44, -4.56$ **h)** no real solutions

3)a) no real solutions **b)** 1 real solution **c)** 2 real rational solutions **d)** 2 real irrational solutions

4)a) one **b)** one **c)** none **d)** two

W4 –Quadratics in Standard Form Unit 5
MPM2D
Jensen

1) Find the x-intercepts and the vertex of each parabola. Then, sketch its graph.

a) $y = x^2 - 6x + 8$

$0 = (x-4)(x-2)$

$x-4=0 \quad x-2=0$

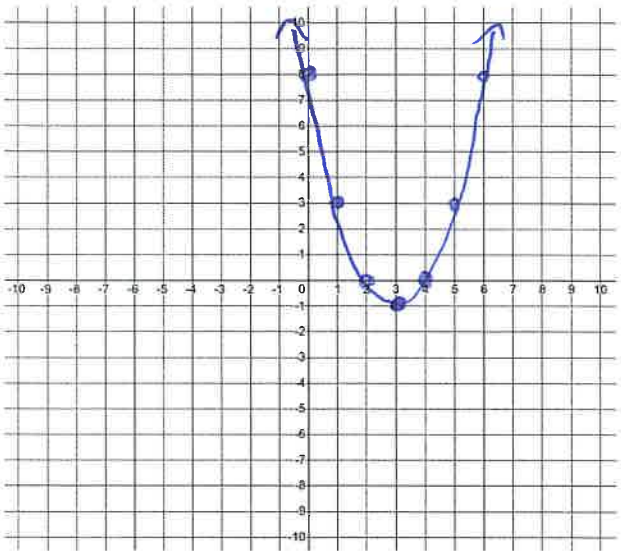
$x=4 \quad x=2$

x-int: $x=2,4$

x-vertex $= \frac{2+4}{2} = 3$

y-vertex $= (3)^2-6(3)+8 = -1$

Vertex: $(3,-1)$



b) $y = -x^2 - 4x + 5$

$y = -(x^2+4x-5)$

$0 = -(x+5)(x-1)$

$x+5=0 \quad x-1=0$

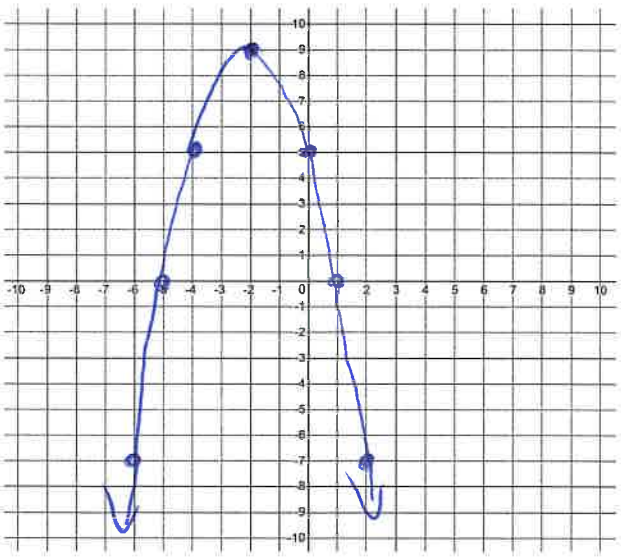
$x=-5 \quad x=1$

x-int: $x=-5,1$

x-Vertex $= \frac{-5+1}{2} = -2$

y-vertex $= -(-2)^2-4(-2)+5 = 9$

Vertex: $(-2,9)$



c) $y = x^2 - 9$

$0 = (x-3)(x+3)$

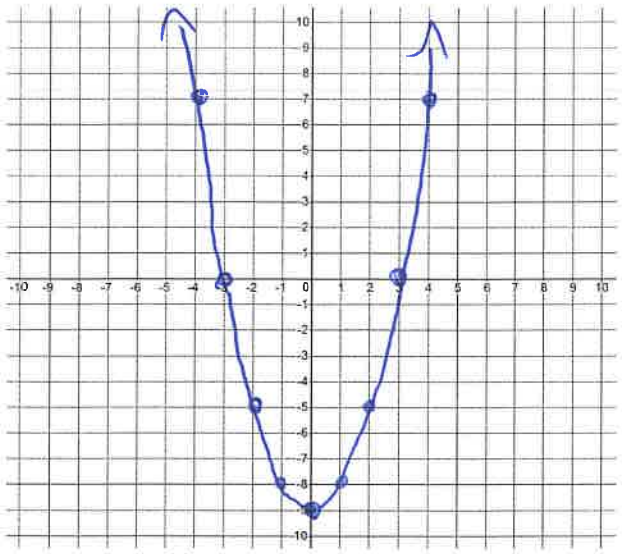
$x-3=0 \quad x+3=0$

$x=3 \quad x=-3$

$x\text{-vertex} = \frac{3+(-3)}{2} = 0$

$y\text{-vertex} = (0)^2 - 9 = -9$

vertex: $(0, -9)$



d) $y = x^2 - 12x + 36$

$0 = (x-6)^2$

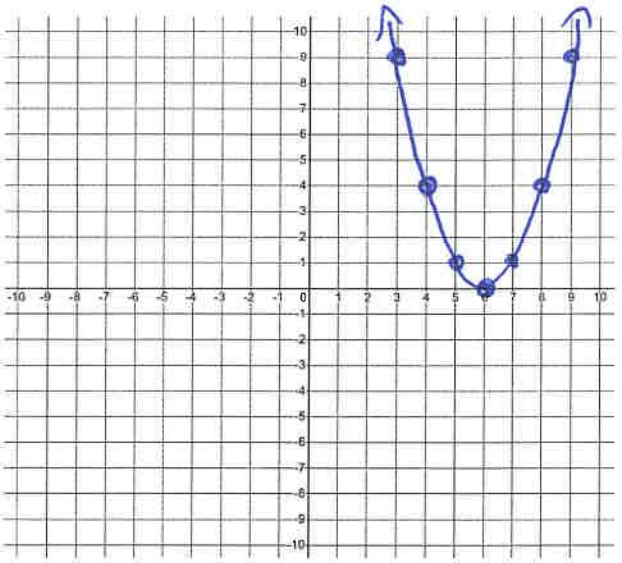
$x-6=0$

$x=6$

$x\text{-vertex} = 6$

$y\text{-vertex} = (6)^2 - 12(6) + 36 = 0$

vertex: $(6, 0)$



e) $y = 2x^2 - 3x + 4$

$0 = 2x^2 - 3x + 4$

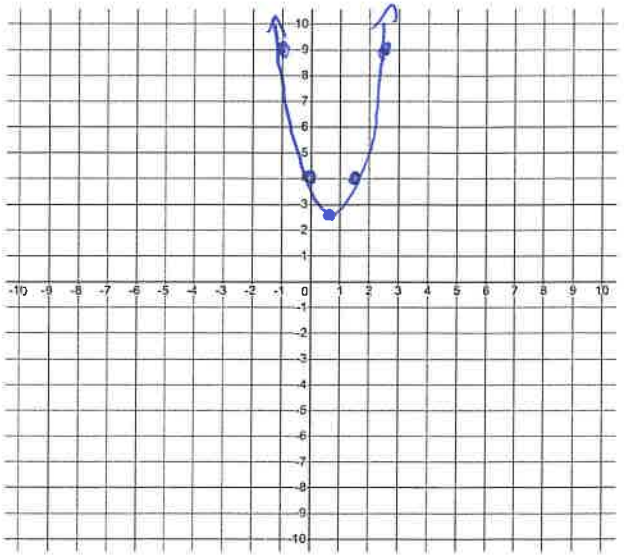
$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$

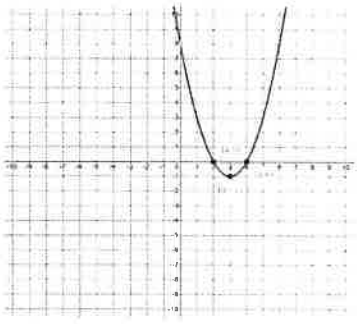
$x = \frac{3 \pm \sqrt{-23}}{4}$ so no x-int.

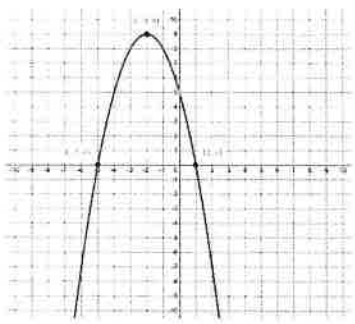
$x\text{-vertex} = \frac{3}{2(2)} = \frac{3}{4} = 0.75$

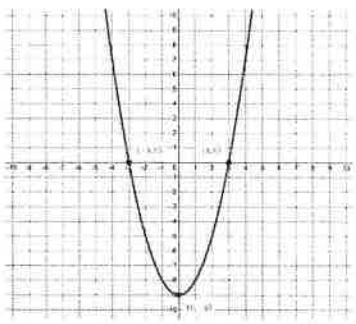
$y\text{-vertex} = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 4 = \frac{23}{8} = 2.875$

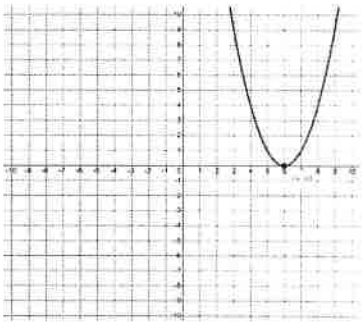
vertex: $(0.75, 2.875)$

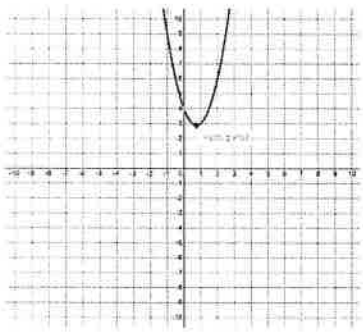


1)a) 

b) 

c) 

d) 

e) 

W5 – Applications of Quadratics Unit 5
MPM2D
Jensen

1) A model rocket is launched from the deck in Jim's backyard and the path followed by the rocket can be modelled by the relation $h = -5t^2 + 100t + 15$, where h, in meters, is the height that the model rocket reaches after t seconds.

a) What is the height of the deck?

$h = -5(0)^2 + 100(0) + 15$

$h = 15\,m$

b) What is the height of the model rocket after 2 s?

$h = -5(2)^2 + 100(2) + 15 = 195\,m$

c) What is the maximum height reached by the model rocket?

$x\text{-vertex} = \frac{-100}{2(-5)} = 10$ seconds

$y\text{-vertex} = -5(10)^2 + 100(10) + 15 = 515\,m$

the max height is 515 m.

d) How long did the model rocket take to reach this height?

10 seconds

e) How long was the model rocket above 200 m?

$200 = -5t^2 + 100t + 15$

$0 = -5t^2 + 100t - 185$

$0 = -5(t^2 - 20t + 37)$

$0 = t^2 - 20t + 37$

$t = \frac{20 \pm \sqrt{(-20)^2 - 4(1)(37)}}{2(1)}$

$t = \frac{20 \pm \sqrt{252}}{2}$

$t = 2.06, 17.94$

Time above 200m = $17.94 - 2.06 = \boxed{15.885}$

f) How long the model rocket was in the air.

$0 = -5t^2 + 100t + 15$

$0 = -5(t^2 - 20t - 3)$

$0 = t^2 - 20t - 3$

$t = \frac{20 \pm \sqrt{(-20)^2 - 4(1)(-3)}}{2(1)}$

$t = \frac{20 \pm \sqrt{412}}{2}$

$t \approx \cancel{-0.15}$ (reject), 20.15

It was in the air for 20.15 seconds

2) A harbour ferry service has about 240 000 riders per day for a fare of $2. The port authority wants to increase the fare to help with increasing operational costs. Research has shown that for every $0.10 increase in the fare the number of riders will drop by 10 000. ~~The port authority established a relation defined by $R = -1000p^2 + 4000p + 480000$, where R represents the revenue from fares and p represents the number of $0.10 increases in the fare.~~

a) What increase in the fare will maximize the revenue?

let n = # of $0.10 price increases

$R = (\text{cost})(\#\text{sold})$

$R = (2 + 0.1n)(240000 - 10000n)$

$0 = (2 + 0.1n)(240000 - 10000n)$

$0 = (2 + 0.1n)$ $\quad$ $0 = (240000 - 10000n)$

$-2 = 0.1n$ $\quad$ $10\,000n = 240\,000$

$\frac{-2}{0.1} = n$ $\quad$ $n = \frac{240\,000}{10\,000}$

$n = -20$ $\quad$ $n = 24$

x-vertex $= \frac{-20 + 24}{2} = 2$ price increases

y-vertex $= [2 + 0.1(2)][240000 - 10000(2)]$

$= (2.2)(220000)$

$= \$484000$

To maximize revenues, they should increase the price by $0.20 up to $2.20 per ticket.

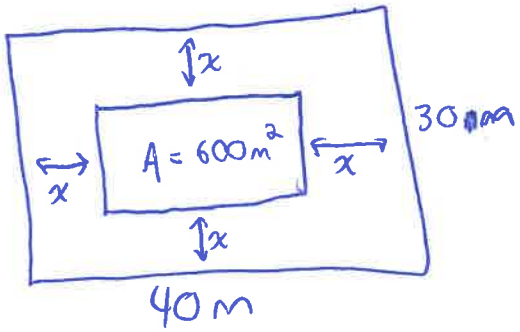
b) What is the new fare?

$220.

c) What is the revenue that will be received from the new fare?

$484000

3) A rectangular lawn measures 30 m by 40 m. Jason is cutting the lawn from the outside perimeter in toward the center by cutting strips along the entire perimeter first, then continuing as he cuts toward the center. How wide is the strip that has been cut along the outside when the area is half cut?



$(30-2x)(40-2x) = \frac{1}{2}(30)(40)$

$1200-60x-80x+4x^2 = 600$

$4x^2-140x+600 = 0$

$4(x^2-35x+150) = 0$

$x^2-35x+150 = 0$

$(x-5)(x-30) = 0$

$x-5=0 \quad x-30=0$

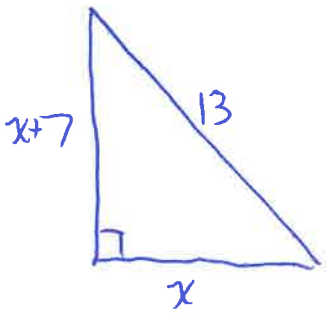
$x=5 \quad \cancel{x=30}$

reject

domain: $0<x<15$

The strip is 5m wide.

4) The hypotenuse of a right triangle measures 13 cm. The legs of the triangle differ by 7 cm. Find the length of each leg.



$(x)^2 + (x+7)^2 = 13^2$

$x^2 + x^2 + 14x + 49 = 169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$x^2 + 7x - 60 = 0$

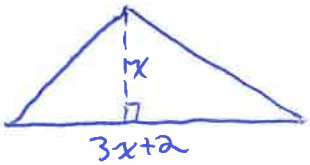
$(x+12)(x-5) = 0$

$x+12 = 0 \quad x-5 = 0$

~~$x = -12$~~ $\quad x = 5$

The legs are 5cm and 12cm

5) A triangle has an area of 308 cm^2. If the base is 2 cm more than three times the height of the triangle, find the base and height of the triangle.



height $= x$

base $= 3x+2$

$A = \frac{1}{2}bh$

$308 = \frac{1}{2}(3x+2)(x)$

$308 = \frac{3}{2}x^2 + x$

$0 = \frac{3}{2}x^2 + x - 308$

$0 = \frac{1}{2}(3x^2 + 2x - 616)$

$0 = 3x^2 + 2x - 616$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-616)}}{2(3)}$

$x = \frac{-2 \pm 86}{6}$

$x = -14, 14$

The dimensions are:

$h = 14$ cm

$b = 3(14)+2 = 44$ cm

6) The sum of the squares of four consecutive integers is 630. Find the integers.

$(x)^2 + (x+1)^2 + (x+2)^2 + (x+3)^2 = 630$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + x^2 + 6x + 9 = 630$

$4x^2 + 12x - 616 = 0$

$4(x^2 + 3x - 154) = 0$

$x^2 + 3x - 154 = 0$

$(x+14)(x-11) = 0$

$x+14 = 0 \quad x-11 = 0$

$x = -14 \quad x = 11$

The integers are either

$-14, -13, -12, -11$

OR

$11, 12, 13, 14$

7) Twice the width of a rectangle is 3 m more than the length. If the area of the rectangle is 209 m^2, find the dimensions of the rectangle.

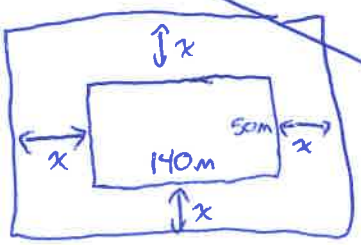
length $= 2x - 3$
width $= x$

$209 = x(2x-3)$
$209 = 2x^2 - 3x$
$0 = 2x^2 - 3x - 209$
$0 = 2x^2 - 22x + 19x - 209$
$0 = 2x(x-11) + 19(x-11)$
$0 = (x-11)(2x+19)$
$x - 11 = 0 \quad 2x + 19 = 0$
$x = 11 \quad x = -9.5$

$A = 209\ m^2$ | x width
$2x-3$ length

The dimensions are:
width $= 11$ m
length $= 2(11) - 3 = 19$ m

8) The playing field at the local high school measures 140 m by 50 m. By increasing this rectangular area by the same amount on all sides, the new area will be exactly double the area of the field. By how much was each dimension increased, to the nearest meter?

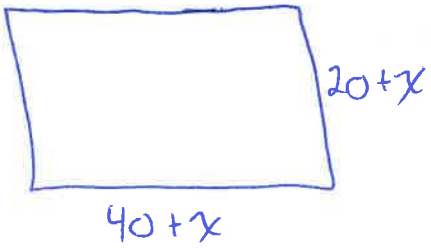


$(140+2x)(50+2x) = 2(50)(140)$
$7000 + 280x + 100x + 4x^2 = 14000$
$4x^2 + 380x - 7000 = 0$
$4(x^2 + 95x - 1750) = 0$

$(140+x)(50+x) = 2(50)(140)$
$7000 + 140x + 50x + x^2 = 14000$
$x^2 + 190x - 7000 = 0$

Answers

1)a)15 m **b)**195 m **c)** 515 m **d)**10 s **e)** 15.874 s **f)**20.15 s
2)a) \$0.20 **b)** \$2.20 **c)** \$484 000
3) 5 m
4) 12 cm and 5 cm
5) base 44 cm, height 14 cm
6) 11, 12, 13, 14 or −14, −13, −12, −11
7) width 11 m, length 19 m
8) 16 m

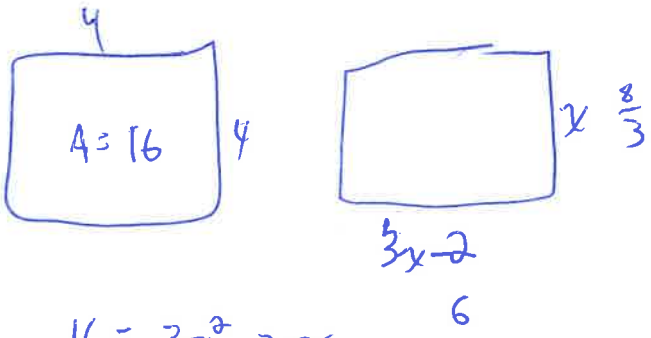


$(40+2x)(20+2x) = 2(40)(20)$

$800+80x+40x+4x^2 = 1600$

$4x^2+120x-800=0$

$x^2+30x-200$



$16 = 3x^2-2x$

$0 = 3x^2-2x-16$

$0 = 3x^2-8x+6x-16$

$0 = x(3x-8)+2(3x-8)$