**Unit 5 Pretest Review** Unit 5
MPM2D
*Jensen*

**1)** Solve each of the following quadratics using the most appropriate method. Round answers to 2 decimal places when necessary.

**a)** $0 = x^2 + 7x + 5$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{29}}{2}$

$x_1 = \frac{-7 + \sqrt{29}}{2}$ $x_2 = \frac{-7 - \sqrt{29}}{2}$

$\boxed{x_1 \simeq -0.81}$ $\boxed{x_2 \simeq -6.19}$

**b)** $x^2 + 5x = -4$

$x^2 + 5x + 4 = 0$

$\frac{4}{4} \times \frac{1}{1} = 4$
$+ = 5$

$(x+4)(x+1) = 0$

$x+4 = 0$ $x+1 = 0$

$\boxed{x_1 = -4}$ $\boxed{x_2 = -1}$

**c)** $6x^2 + x = 1$

$6x^2 + x - 1 = 0$

$\frac{3}{3} \times \frac{-2}{-2} = -6$
$+ = 1$

$6x^2 + 3x - 2x - 1 = 0$

$3x(2x+1) - 1(2x+1) = 0$

$(2x+1)(3x-1) = 0$

$2x+1 = 0$ $3x - 1 = 0$

$2x = -1$ $3x = 1$

$\boxed{x_1 = \frac{-1}{2}}$ $\boxed{x_2 = \frac{1}{3}}$

**d)** $4a^2 + 12a = -9$

$4a^2 + 12a + 9 = 0$

$\frac{6}{6} \times \frac{6}{6} = 36$
$+ = 12$

$4a^2 + 6a + 6a + 9 = 0$

$2a(2a+3) + 3(2a+3) = 0$

$(2a+3)(2a+3) = 0$

$(2a+3)^2 = 0$

$2a + 3 = 0$

$2a = -3$

$\boxed{a = \frac{-3}{2}}$

**e)** $0 = -2x^2 + 4x + 7$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(7)}}{2(-2)}$

$x = \frac{-4 \pm \sqrt{72}}{-4}$

$x_1 = \frac{-4 + \sqrt{72}}{-4}$ $x_2 = \frac{-4 - \sqrt{72}}{-4}$

$\boxed{x_1 \simeq -1.12}$ $\boxed{x_2 \simeq 3.12}$

**f)** $x^2 - 2x + 3 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-8}}{2}$

$\boxed{\text{No real solutions}}$

**g)** $x^2 + 4x - 21 = 0$

$\underline{7} \times \underline{-3} = -21$

$\underline{7} + \underline{-3} = 4$

$(x+7)(x-3)=0$

$x+7=0 \quad x-3=0$

$\boxed{x_1=-7} \quad \boxed{x_2=3}$

**h)** $-x^2 + 5x + 6 = 0$

$-(x^2-5x-6)=0$

$\underline{-6} \times \underline{1} = -6$

$\underline{-6} + \underline{1} = -5$

$-(x-6)(x+1)=0$

$x-6=0 \quad x+1=0$

$\boxed{x_1=6} \quad \boxed{x_2=-1}$

**i)** $0 = 3x^2 + 6x + 4$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{-12}}{6}$$

$\boxed{\text{No real solutions}}$

j) $x^2 + 11 = 155$

$x^2 - 144 = 0$

$(x)^2 - (12)^2 = 0$

$(x-12)(x+12)=0$

$x-12=0 \quad x+12=0$

$\boxed{x_1=12} \quad \boxed{x_2=-12}$

Alternate Method

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

$x_1 = 12 \quad x_2 = -12$

**k)** $8x^2 = 4x$

$8x^2 - 4x = 0$

$4x(2x-1)=0$

$4x=0 \quad 2x-1=0$

$2x=1$

$\boxed{x_1=0} \quad \boxed{x_2=\frac{1}{2}}$

**l)** $3x^2 - x - 7 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(3)(-7)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{85}}{6}$$

$x_1 = \frac{1+\sqrt{85}}{6} \quad x_2 = \frac{1-\sqrt{85}}{6}$

$\boxed{x_1 \simeq 1.70} \quad \boxed{x_2 \simeq -1.37}$

**2)** Use the discriminant to determine the number of solutions each quadratic equation would have.

**a)** $x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4)$
$= 0$

$\boxed{\therefore 1 \text{ solution}}$

**b)** $-2x^2 + 3x - 8 = 0$

$b^2 - 4ac = (3)^2 - 4(-2)(-8)$
$= -55$

$\boxed{\therefore \text{no real solutions}}$

**c)** $2x^2 + 3x - 8 = 0$

$b^2 - 4ac = (3)^2 - 4(2)(-8)$
$= 73$

$\boxed{\therefore 2 \text{ solutions}}$

**3)** Describe the roots of the equation $ax^2 + bx + c = 0$ in each of the following situations. Explain and justify your reasoning.

**a)** $b^2 - 4ac < 0$

No real solutions. The square root of a negative number is NOT a real number. You get no real solutions if the quadratic opens up and has its vertex above the x-axis OR if the quadratic opens down and has its vertex below the x-axis.

**b)** $b^2 - 4ac = 0$

1 real solution. In the QF, adding and subtracting 0 gives the same result. You get 1 solution when the vertex is ON the x-axis.

**c)** $b^2 - 4ac > 0$ and is a perfect square

You get 2 solutions that are rational numbers. If this happens, solving by factoring would also work.

**d)** $b^2 - 4ac > 0$ and is NOT a perfect square

You get 2 solutions that are irrational numbers. If this happens, solving by factoring would NOT work. QF must be used.

**4)** Determine the vertex of each of the following quadratics.

**a)** $y = 2x^2 - 20x + 7$

$x\text{-vertex} = \frac{20}{2(2)} = 5$

$y\text{-vertex} = 2(5)^2 - 20(5) + 7 = -43$

$\boxed{\text{The vertex is } (5, -43)}$

**b)** $y = 3x^2 + 12x - 4$

$x\text{-vertex} = \frac{-12}{2(3)} = -2$

$y\text{-vertex} = 3(-2)^2 + 12(-2) - 4 = -16$

$\boxed{\text{The vertex is } (-2, -16)}$

**5)** Find the $x$-intercepts and the vertex of each parabola. Then, sketch its graph.

**a)** $y = x^2 + 8x + 12$

x-int

$0 = x^2 + 8x + 12$

$0 = (x+6)(x+2)$

$x+6=0 \quad x+2=0$

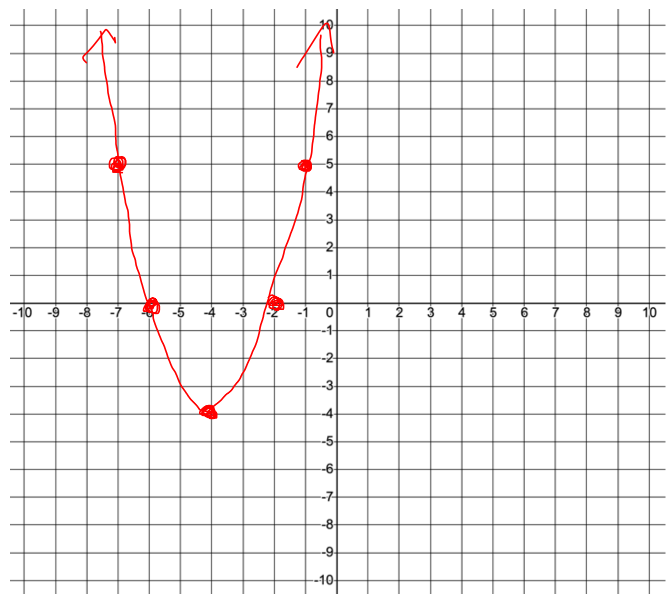
$\boxed{x_1 = -6 \quad x_2 = -2}$

vertex

$x\text{-vertex} = \frac{-6+(-2)}{2} = -4$

$y\text{-vertex} = -4$

$\boxed{\text{vertex is } (-4, -4)}$



**b)** $y = -2x^2 - 6x + 3$

x-int

$0 = -2x^2 - 6x + 3$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(-2)(3)}}{2(-2)}$

$x = \frac{6 \pm \sqrt{60}}{-4}$

$x_1 = \frac{6 + \sqrt{60}}{-4} \qquad x_2 = \frac{6 - \sqrt{60}}{-4}$

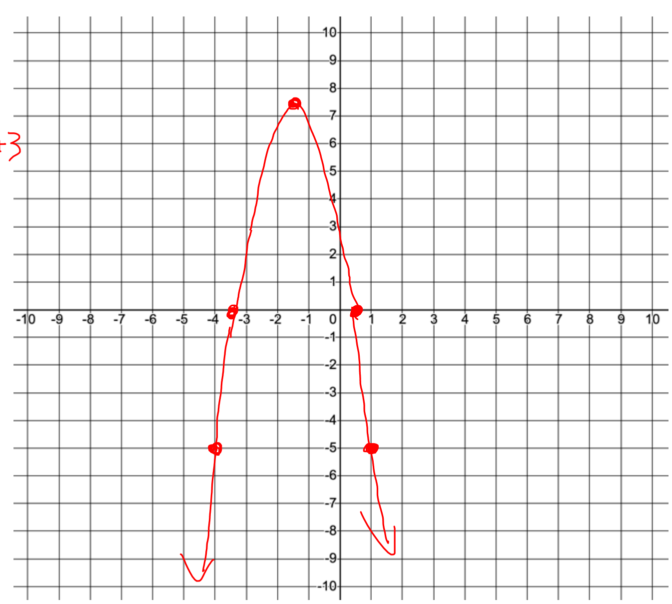
$\boxed{x_1 \simeq -3.44 \qquad x_2 \simeq 0.44}$

vertex

$x\text{-vertex} = \frac{6}{2(-2)} = -1.5$

$y\text{-vertex} = -2(-1.5)^2 - 6(-1.5) + 3$
$= 7.5$

$\boxed{\text{vertex is } (-1.5, 7.5)}$



**6)** Angie sold 1200 tickets for the holiday concert at \$20 per ticket. Her committee is planning to increase the prices this year. Their research shows that for each \$2 increase in the price of a ticket, 60 fewer tickets will be sold.

**a)** Determine the revenue relation that describes the ticket sales.

$$R = (\text{price})(\#\text{sold})$$

$$\boxed{R = (20+2n)(1200-60n)}$$

**b)** What should the selling price per ticket be to maximize revenue?

$$0 = \underbrace{(20+2n)}_{\text{Price}}\underbrace{(1200-60n)}_{\#\text{sold}} \qquad x\text{-vertex} = \frac{-10+20}{2} = 5$$

$$20+2n=0 \qquad 1200-60n=0$$

$$2n=-20 \qquad 1200=60n$$

$$n=-10 \qquad n=20$$

5 price increases will generate a max revenue.

the new selling price $= 20+2(5) = \$30$

**c)** How many tickets will be sold at the maximum revenue?

$$\#\text{sold} = 1200 - 60(5) = \boxed{900 \text{ tickets}}$$

**d)** What is the maximum revenue?

$$R = [20+2(5)][1200-60(5)]$$

$$= (30)(900)$$

$$\boxed{= \$27000}$$

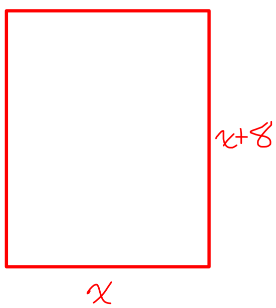
**7)** The path of a golf ball can be modelled by the equation $h = -2d^2 + 12d - 13$, where $d$ represents the horizontal distance, in metres, that the ball travels and $h$ represents the height of the ball, in metres, above the ground. What is the maximum height of the golf ball and at what horizontal distance does it occur?

$x\text{-vertex} = \frac{-12}{2(-2)} = 3$

$y\text{-vertex} = -2(3)^2 + 12(3) - 13 = 5$

$\boxed{\text{A max height of 5 m occurs at a horizontal distance of 3 m.}}$

**8)** The area of the front cover of a daily journal is 273 cm$^2$, and the length is 8 cm greater than the width. What are the dimensions of the cover?




length = $x+8$

width = $x$

$A = lw$

$273 = x(x+8)$

$273 = x^2 + 8x$

$0 = x^2 + 8x - 273$

$0 = (x+21)(x-13)$

$21 \times -13 = -273$

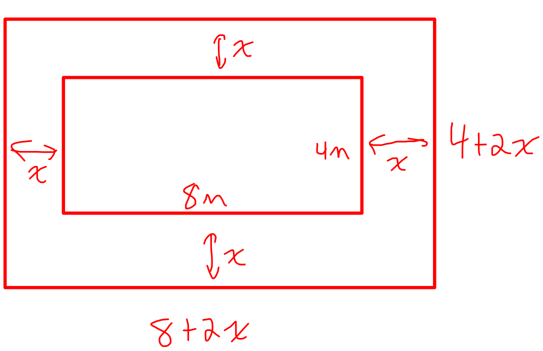
$21 + -13 = 8$

$x + 21 = 0 \quad x - 13 = 0$

$\cancel{x = -21} \quad x = 13$

$\boxed{\text{The dimensions are 13 cm by 21 cm.}}$

**9)** A rectangular lawn measuring 8 meters by 4 meters is surrounded by a flower bed of uniform width. The combined area of the lawn and the flower bed is 165 m$^2$. What is the width of the flower bed?




$165 = (8+2x)(4+2x)$

$165 = 32 + 16x + 8x + 4x^2$

$0 = 4x^2 + 24x - 133$

$x = \frac{-24 \pm \sqrt{(24)^2 - 4(4)(-133)}}{2(4)}$

$x = \frac{-24 \pm \sqrt{2704}}{8}$

$x = \frac{-24+52}{8} \qquad x = \frac{-24-52}{8}$

$x = 3.5\text{ m} \qquad \cancel{x = -9.5}$

## Answers

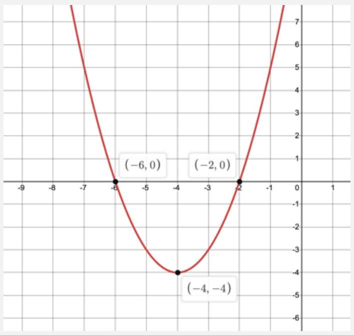
**1)a)** $x = -6.19, -0.81$ **b)** $x = -4, -1$ **c)** $x = -\frac{1}{2}, \frac{1}{3}$ **d)** $x = -\frac{3}{2}$ **e)** $x = -1.12, 3.12$ **f)** no real solutions **g)** $x = -7, 3$ **h)** $x = -1, 6$ **i)** no real solutions **j)** $x = -12, 12$ **k)** $x = 0, \frac{1}{2}$ **l)** $x = -1.37, 1.70$

**2)a)** 1 solution **b)** no real solutions **c)** 2 solutions

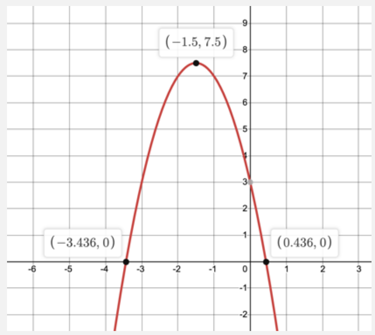
**3)a)** no real solutions **b)** 1 real solution **c)** 2 real rational solutions **d)** 2 real irrational solutions

**4)a)** $(5, -43)$ **b)** $(-2, -16)$

**5)a)**




**b)**




**6)a)** $R = (20 + 2x)(1200 - 60x)$ **b)** \$30 **c)** 900 **d)** \$27 000

**7)** The maximum height of 5 m occurs at a horizontal distance of 3 m.

**8)** 13 cm by 21 cm

**9)** 3.5 m